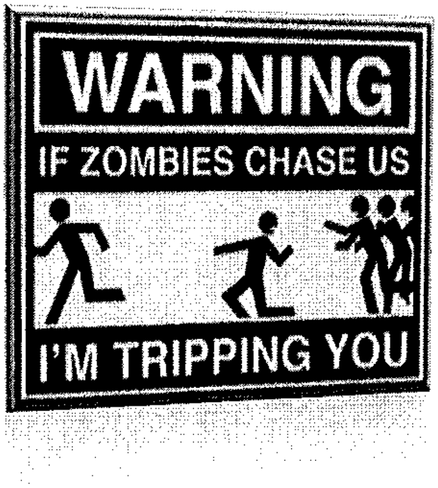
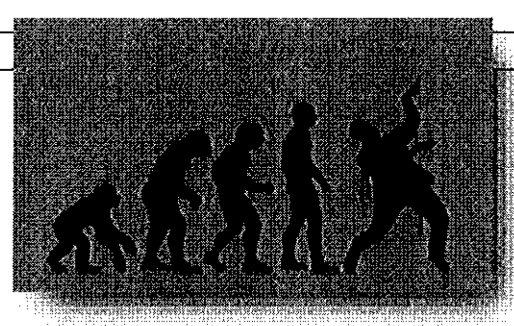


Logs and Exponentials:
Unit Schedule

When			Topics/Student Objectives
4/13	Monday	1	**Growth and Decay** Use Exponential functions to create growth and decay models. Use growth and decay models to predict outcomes. Use periodic and continuous interest formulas to calculate information about financial investments.
4/14	Tuesday	2	**Inverse Functions** Given a function, find the inverse of that function. Recognize that a relation is an inverse of a function based on a table of values and/or a graph. Use properties of inverse functions to determine if functions are inverses of each other.
4/15	Wednesday	3	**Definition and Properties Logs** Use the relationship between the log and exponential function to convert between forms. Use the properties of logarithms to expand and contract logarithmic statements
4/16	Thursday	4	**Properties of Logs** (continued) Use the properties of logarithms to expand and contract logarithmic statements. **Quiz – Growth and Decay word problems and Inverse Functions**
4/17	Friday	5	**Solving Equations – Common Logs** Solve equations in which the variable is in the exponent. Solve equations in which the variable is the argument of a log function.
4/20	Monday	6	**Solving Equations – Common Logs** Solve exponential and logarithmic equations, extra practice. **Quiz – Definitions and Properties of Log functions**
4/21	Tuesday	7	**Properties of Natural Logarithms** Use properties of Natural Logs to expand and condense expressions
4/22	Wednesday	8	**Solving Equations – Natural Logs** Solve equations involving the constant e in which the variable is the exponent or the argument of a natural log statement.
4/23	Thursday		Review
4/24	Friday		**Unit Test**



Exponential Models

Clues in the word problems tell you which formula to use. If there's no mention of compounding, use a growth or decay model. If your interest is compounded, check for the word continuous. That's your clue to use the "Pert" Formula.

Simple Interest Growth	Simple Interest Decay	Compound Interest	Continuously Compounded Interest
$A(t) = a(1+r)^t$	$A(t) = a(1-r)^t$	$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$	$A(t) = Pe^{rt}$

$A(t)$	Amount after time t.	a	Initial amount	r	Rate expressed as a decimal
t	Time	n	Number of interest payments in one year	P	Initial investment

Growth
Example baseball card bought for $150 **increases in value** at a rate of 3% each year. How much is the card worth in 10 years?

$$A = 150(1+.03)^{10}$$

1.) The yellow bellied sapsucker has a population growth rate of approximately 4.7% If the population was 8,530 in 2000 and this growth rate continues, about how many yellow bellied sapsuckers will there be in 2006?

Decay
You bought a new Ford truck for $40,000 yesterday. The truck **depreciates** a rate of 11% each year. How much is your truck worth 8 years from now?

$$A = 40000(1-.11)^{8}$$

2.) Amy Farah Fowler bought a new car for $25,000. Suppose the car depreciates at a rate of 13% per year. How much will the car be worth in 4 years?

Compound Interest
Your favorite Aunt gives you a quick pick. It's your lucky day! You win $1500. You give $500 to your Aunt and put the rest in a savings account that pays 3% interest **compounded *monthly***. How much money will you have in 10 years?

$$A = 1000\left(1+\frac{.03}{12}\right)^{(12)(10)}$$

3.) If you put $2400 in an account that pays 6.2% interest compounded quarterly. How much will you have in eight years?

Continuous Compounding
Your Aunt decides to deposit the$500 you gave her into a savings account at her bank. This account pays 3.5% interest and compounds **continuously**. How much money will she have in this account in 8 years?

$$A = 500e^{(.035)(8)}$$

4.) If you put the same $2400 in an account that pays 5.7% interest compounded continuously. How much will you have in eight years?

(

Inverse Functions

To find the inverse of a function,

1. Switch x and y values
2. Solve for y

Inverse notation: $f^{-1}(x)$

Find the inverse of each function:

5.) $f(x) = 2x^2 - 8$

6.) $f(x) = \frac{x}{4} + 3$

For logs and exponents, put the equation in the "other form". Then switch x and y, solve for y.

$y=\log_4(16x)$	Find the inverse
$x=\log_4(16y)$	Switch x and y
$4^x=16y$	Put in exponent form
$4^x/16=y$	Solve for y
$4^{x-2}=y$	Simplify if possible

$y=4^x$	Find the inverse
$x=4^y$	Switch x and y
$y=\log_4 x$	Put in log form

7.) $f(x) = 3^{x-2}$

8.) $f(x) = \log(2x - 1)$

Definition of Logarithms

THE Relationship

$$If\ y = b^x, then\ log_b y = x$$

Write $6^2 = 36$ in log form

$log_6 36 = 2$

Write $log_2 64 = 6$ in exponential form

$2^6 = 64$

Write the following in log form:

9.) $6^2 = 36$

10.) $5^3 = 125$

11.) $2^4 = 32$

Write the following in exponential form:

12.) $\log_2 8 = 3$

13.) $\log_3 81 = 4$

14.) $\log_4 16 = 2$

Change of base formula

$$log_b x = \frac{\log x}{\log b}$$

Evaluate $log_6 32$

$$log_6 32 = \frac{\log 32}{\log 6} = 1.9343$$

Common log

log_{10} *is written as log*

Natural Log

log_e *is written as ln*

15.) Evaluate $log_2 8$

16.) Evaluate $log_{0.25} 0.0625$

2

Properties of Logarithms

PROPERTIES	EX 1: Condense the following into one log statement.	EX2: Expand the expression $log\frac{x}{yz^2}$
$log_b b = 1$ $lob_b 1 = 0$ $log_b mn = log_b m + lob_b n$ $log_b \frac{m}{n} = log_b m - log_b n$ $log_b m^n = n\ log_b m$ *To condense log statements, they must have the same base.*	$3\ log_4 x + 2\ log_4 y$ Step 1: Move the constants in front of the log statements into the exponent position. $log_4 x^3 + log_4 y^2$ Step 2: Combine the arguments. Change subtraction to multiplication and addition to multiplication. $log_4 x^3 y^2$	Step 1: Deal with the division operation first. Split the argument into two logs. $\log x - log\ yz^2$ Step 2: Split any statements with multiplication into addition operations. Be sure to distribute the negative from the division. $\log x - (\log y + \log z^2)$ $\log x - \log y - \log z^2$ Step 3: Move any exponents in front of the log statement. $\log x - \log y - 2\log z$

Condense the following Log Statements

17.) $\log_5 4 + \log_5 3$	18.) $\frac{1}{3}\log 3x + \frac{2}{3}\log 3x$	19.) $\log_3 2x - 5\log_3 y$

20.) $\log_5 y - 4(\log_5 r + 2\log_5 t)$

Expand the following Log Statements

21.) $\log 6x^3 y$	22.) $\log_2 \frac{x}{yz}$	23.) $\log\sqrt{\frac{2rst}{5w}}$

Solve Exponential and Logarithmic Equations		
To **solve** an **exponential equation**, take the log of both sides, and **solve** for the variable. To **solve** a **logarithmic equation**, rewrite the **equation** in exponential form and **solve** for the variable. Other helpful properties: $log_b b^x = x$ $b^{log_b x} = x$	Solve the equation $3^{x-2} + 5 = 74$. $3^{x-2} = 69.$ Subtract 5 from both sides. $\log(3^{x-2}) = \log 69$ Take the log of both sides $(x-2)\log 3 = \log 69$ Simplify the left side $x - 2 = \frac{\log 69}{\log 3}$ Evaluate logs $x - 2 = 3.85$ Solve for x $x = 5.85$	Solve the equation $log_2 4x = 5$ $4x = 2^5$ Put in exponential form. $4x = 32$ Simplify right side $x = 16$ Divide both sides by log 4.

Solve the following equations

24.) $8^{n+1} = 3$	25.) $10^{3y} = 5$	26.) $4^x - 5 = 12$
27.) log (2x + 5) = 3	28.) log 4x = 2	29). 2 log (2x + 5) = 4

Laws of Exponents

Law	Example
$x^1 = x$	$6^1 = 6$
$x^0 = 1$	$7^0 = 1$
$x^{-1} = 1/x$	$4^{-1} = 1/4$
$x^m x^n = x^{m+n}$	$x^2x^3 = x^{2+3} = x^5$
$x^m/x^n = x^{m-n}$	$x^6/x^2 = x^{6-2} = x^4$
$(x^m)^n = x^{mn}$	$(x^2)^3 = x^{2\times3} = x^6$
$(xy)^n = x^n y^n$	$(xy)^3 = x^3y^3$
$(x/y)^n = x^n/y^n$	$(x/y)^2 = x^2 / y^2$
$x^{-n} = 1/x^n$	$x^{-3} = 1/x^3$
And the law about Fractional Exponents:	
$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$	$x^{\frac{2}{3}} = \sqrt[3]{x^2} = (\sqrt[3]{x})^2$

Warm-Ups

Day 1	Day 2
Day 3	**Day 4**

Warm-Ups

Day 5	Day 6
Day 7	Day 8

Name ______________________ Class __________ Date __________

7-1 Reteaching

Exploring Exponential Models

- The general form of an exponential function is $y = ab^x$, where a is the initial amount and b is the growth or decay factor.
- To find b, use the formula $b = 1 + r$, where r is the constant rate of growth or decay. If r is a rate of growth, it will be positive. If r is a rate of decay, it will be negative. Therefore, if b is greater than 1, the function models growth. If b is between zero and 1, the function models decay. When you see words like increase or appreciation, think growth. When you see words like decrease or depreciation, think decay.
- For an exponential function, the y-intercept is always equal to the value of a.

Problem

Carl's weight at 12 yr is 82 lb. Assume that his weight increases at a rate of 16% each year. Write an exponential function to model the increase. What is his weight after 5 years?

Step 1 Find a and b.

$a = 82$ *a* is the original amount.

$b = 1 + 0.16$ *b* is the growth or decay factor. Since this problem models growth, *r* will be positive. Make sure to rewrite the rate, *r*, as a decimal.

$= 1.16$

Step 2 Write the exponential function.

$y = ab^x$ Use the formula.

$y = 82(1.16)^x$ Substitute.

Step 3 Calculate.

$y = 82(1.16)^5$ Substitute 5 for *x*.

$y \approx 172.228$ Use a calculator.

Carl will weigh about 172 lb in 5 years.

Exercises

Determine whether the function represents exponential growth or exponential decay. Then find the y-intercept.

1. $y = 8000(1.15)^x$

2. $y = 20(0.75)^x$

3. $y = 15\left(\frac{1}{2}\right)^x$

4. $f(x) = 6\left(\frac{5}{2}\right)^x$

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Name________________________________ Class ____________ Date ____________

7-1 Reteaching (continued)

Exploring Exponential Models

You can use the general form of an exponential function to solve word problems involving growth or decay.

Problem

A motorcycle purchased for $9000 today will be worth 6% less each year. How much will the motorcycle be worth at the end of 5 years?

Step 1 Find a and b.

$a = 9000$	*a* is the original amount.
$b = 1 + (-0.06)$	*b* is the growth or decay factor. Since this problem models decay, *r* will be negative. Make sure to rewrite the rate, *r*, as a decimal.
$= 0.94$	

Step 2 Write the exponential function.

$y = ab^x$	Use the formula.
$y = 9000(0.94)^x$	Substitute.

Step 3 Calculate.

$y = 9000(0.94)^5$	Substitute 5 for *x*.
$y \approx 6605.13$	Use a calculator.

The motorcycle will be worth about $6605.13 after 5 years.

Exercises

Write an exponential function to model each situation. Find each amount after the specified time.

5. A tree 3 ft tall grows 8% each year. How tall will the tree be at the end of 14 yr? Round the answer to the nearest hundredth.

6. The price of a new home is $126,000. The value of the home appreciates 2% each year. How much will the home be worth in 10 yr?

7. A butterfly population is decreasing at a rate of 0.82% per year. There are currently about 100,000 butterflies in the population. How many butterflies will there be in the population in 250 years?

8. A car depreciates 10% each year. If you bought this car today for $5000, how much will it be worth in 7 years?

Math III

Logs and Exponents

Exponential Growth and Decay Problems

Name: ______________________ **Date:** ____________ **Period:** ____________

The formulas you need to solve these problems can be found on page 1 of your study guide.

1. Determine the balance of an account that starts with $100, has an annual rate of 4%, and the money is left in the account for 12 years.

2. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

3. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacteria which can double every hour, how many bacteria will we have by the end of one day?

4. Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

5. The population of Winnemucca, Nevada, can be modeled by P=6191(1.04)t where t is the number of years since 1990. What was the population in 1990? By what percent did the population increase by each year?

6. You have inherited land that was purchased for $30,000 in 1960. The value of the land increased by approximately 5% per year. What is the approximate value of the land in the year 2011?

7. During normal breathing, about 12% of the air in the lungs is replaced after one breath. Write an exponential decay model for the amount of the original air left in the lungs if the initial amount of air in the lungs is 500 mL. How much of the original air is present after 240 breaths?

8. An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 29%. How much ibuprofen is left after 6 hours?

9. You deposit $1600 in a bank account. Find the balance after 3 years for each of the following situations:

   a. The account pays 2.5% annual interest compounded monthly.

   b. The account pays 1.75% annual interest compounded quarterly.

   c. The account pays 4% annual interest compounded yearly.

10. You buy a new computer for $2100. The computer decreases by 50% annually. When will the computer have a value of $600?

11. If you invest $2500 in an account, what is the balance in the account and the amount of interest after 4 years if you earn:

   a. 1.7% interest compounded annually?

   b. 1.5% compounded monthly?

   c. 1.2% compounded daily?

   d. 0.7% compounded continuously?

12. A loan shark lends a gambler $1,000.00 to cover a debt. He charges 35% annual interest compounded continuously. How much does the gambler owe the loan shark at the end of one year? Two years?

13. The value of a $25,000 car depreciates at a rate of 12% per year. What will the car be worth in 5 years?

Name________________________ Class __________ Date__________

6-7 Reteaching

Inverse Relations and Functions

- Inverse operations "undo" each other. Addition and subtraction are inverse operations. So are multiplication and division. The inverse of cubing a number is taking its cube root.
- If two functions are inverses, they consist of inverse operations performed in the opposite order.

Problem

What is the inverse of the relation described by $f(x) = x + 1$?

$f(x) = x + 1$	
$y = x + 1$	Rewrite the equation using *y*, if necessary.
$x = y + 1$	Interchange *x* and *y*.
$x - 1 = y$	Solve for *y*.
$y = x - 1$	The resulting function is the inverse of the original function.

So, $f^{-1}(x) = x - 1$.

Exercises

Find the inverse of each function.

1. $y = 4x - 5$

2. $y = 3x^3 + 2$

3. $y = (x + 1)^3$

4. $y = 0.5x + 2$

5. $f(x) = x + 3$

6. $f(x) = 2(x - 2)$

7. $f(x) = \frac{x}{5}$

8. $f(x) = 4x + 2$

9. $y = x$

10. $y = x - 3$

11. $y = \frac{x-1}{2}$

12. $y = x^3 - 8$

13. $f(x) = \sqrt{x+2}$

14. $f(x) - \frac{2}{3}x - 1$

15. $f(x) = \frac{x+3}{5}$

16. $f(x) = 2(x - 5)^2$

17. $y = \sqrt{x} + 4$

18. $y = 8x + 1$

Name ______________________ Class __________ Date __________

7-3 Reteaching

Logarithmic Functions as Inverses

A logarithmic function is the inverse of an exponential function.

To evaluate logarithmic expressions, use the fact that $x = \log_b y$ is the same as $y = b^x$. Keep in mind that $x = \log y$ is another way of writing $x = \log_{10} y$.

Problem

What is the logarithmic form of $6^3 = 216$?

Step 1 Determine which equation to use.

The equation is in the form $b^x = y$.

Step 2 Find x, y, and b.

$b = 6$, $x = 3$, and $y = 216$

Step 3 Because $y = b^x$ is the same as $x = \log_b y$, rewrite the equation in logarithmic form by substituting for x, y, and b.

$3 = \log_6 216$

Exercises

Write each equation in logarithmic form.

1. $4^{-3} = \frac{1}{64}$

2. $5^{-2} = \frac{1}{25}$

3. $8^{-1} = \frac{1}{8}$

4. $11^0 = 1$

5. $6^1 = 6$

6. $6^{-3} = \frac{1}{216}$

7. $17^0 = 1$

8. $17^1 = 17$

Problem

What is the exponential form of $4 = \log_5 625$?

Step 1 Determine which equation to use.

The equation is in the form $x = \log_b y$.

Step 2 Find x, y, and b.

$x = 4$, $b = 5$, and $y = 625$

Step 3 Because $x = \log_b y$ is the same as $y = b^x$, rewrite the equation in exponential form by substituting for x, y, and b.

$625 = 5^4$

Name ______________________ Class ____________ Date ____________

7-3 Reteaching (continued)

Logarithmic Functions as Inverses

Exercises

Write each equation in exponential form.

9. $3 = \log_2 8$

10. $2 = \log_5 25$

11. $\log 0.1 = -1$

12. $\log 7 \approx 0.845$

13. $\log 1000 = 3$

14. $-2 = \log 0.01$

15. $\log_3 81 = 4$

16. $\log_{49} 7 = \frac{1}{2}$

17. $\log_8 \frac{1}{4} = -\frac{2}{3}$

18. $\log_2 128 = 7$

19. $\log_5 \frac{1}{625} = -4$

20. $\log_6 36 = 2$

Problem

What is the value of $\log_4 32$?

$x = \log_4 32$	Write the equation in logarithmic form $x = \log_b y$.
$32 = 4^x$	Rewrite in exponential form $y = b^x$.
$2^5 = (2^2)^x$	Rewrite each side of the equation with like bases in order to solve the equation.
$2^5 = 2^{2x}$	Simplify.
$5 = 2x$	Set the exponents equal to each other.
$x = \frac{5}{2}$	Solve for *x*.
$\log_4 32 = \frac{5}{2}$	

Exercises

Evaluate the logarithm.

21. $\log_2 64$

22. $\log_2 64$

23. $\log_3 3^4$

24. $\log 10$

25. $\log 0.1$

26. $\log 1$

27. $\log_8 2$

28. $\log_{32} 2$

29. $\log_9 3$

Name______________________________ Class ____________ Date ___________

Write each equation in logarithmic form.

1. $9^2 = 81$ **2.** $\frac{1}{64} = \left(\frac{1}{4}\right)^3$ **3.** $8^3 = 512$ **4.** $\left(\frac{1}{3}\right)^{-2} = 9$

5. $2^9 = 512$ **6.** $4^5 = 1024$ **7.** $5^4 = 625$ **8.** $10^{23} = 0.001$

Describe how the graph of each function compares with the graph of the parent function, $y = \log_b x$.

24. $y = \log_3 x - 2$

25. $y = \log_8 (x - 8)$

26. $y = \log_6 (x + 1) - 5$

27. $y = \log_2 (x - 4) + 1$

Write each equation in exponential form.

28. $\log_4 256 = 4$ **29.** $\log_7 1 = 0$ **30.** $\log_2 32 = 5$

31. $\log 10 = 1$ **32.** $\log_5 5 = 1$ **33.** $\log_8 \frac{1}{64} = -2$

Find the inverse of each function.

49. $y = \log_2 x$ **50.** $y = \log_{0.7} x$ **51.** $y = \log_{100} x$

52. $y = \log_8 x$ **53.** $y = \log_2 (4x)$ **54.** $y = \log (x + 4)$

Name ______________________________ Class ____________ Date ____________

7-4 Reteaching

Properties of Logarithms

You can write a logarithmic expression containing more than one logarithm as a single logarithm as long as the bases are equal. You can write a logarithm that contains a number raised to a power as a logarithm with the power as a coefficient. To understand the following properties, remember that logarithms are powers.

Name	Formula	Why?
Product Property	$\log_b mn = \log_b m + \log_b n$	When you multiply two powers, you add the exponents. Example: $2^6 \cdot 2^2 = 2^{(6+2)} = 2^8$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$	When you divide two powers, you subtract the exponents. Example: $\frac{2^6}{2^2} = 2^{(6-2)} = 2^4$
Power Property	$\log_b m^n = n\log_b m$	When you raise a power to a power, you multiply the exponents. Example: $(2^6)^2 = 2^{(6 \cdot 2)} = 2^{12}$

Problem

What is $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27$ written as a single logarithm?

$2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27 = \log_2 6^2 - \log_2 9 + \log_2 27^{\frac{1}{3}}$ Use the Power Property twice.

$= \log_2 36 - \log_2 9 + \log_2 3$ $6^2 = 36, 27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$= (\log_2 36 - \log_2 9) + \log_2 3$ Group two of the logarithms. Use order of operations.

$= \log_2 \frac{32}{9} + \log_2 3$ Quotient Property

$= \log_2 \left(\frac{36}{9} \bullet 3\right)$ Product Property

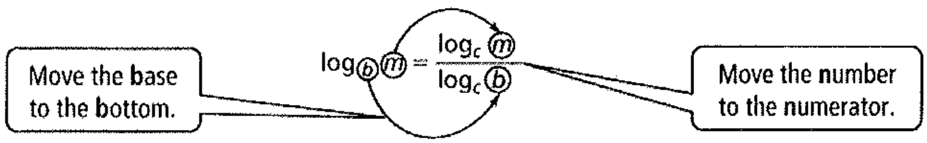
$= \log_2 12$ Simplify.

As a single logarithm, $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27 = \log_2 12$.

7-4 Reteaching (continued)

Properties of Logarithms

To evaluate logarithms with any base, you can rewrite the logarithm as a quotient of two logarithms with the same base.



Problem

What is $\log_4 8$ written as a quotient of two logarithms with base 2? Simplify your answer, if possible.

$\log_4 8 = \dfrac{\log_2 8}{\log_2 4}$ — The base is 4 and the number is 8. Move the base to the bottom and the number to the numerator.

$= \dfrac{3}{2}$ — Evaluate the logarithms in the numerator and the denominator.

Exercises

Write each logarithmic expression as a single logarithm.

1. $\log_3 13 + \log_3 3$
2. $2 \log x + \log 5$
3. $\log_4 2 - \log_4 6$
4. $3\log_3 3 - \log_3 3$
5. $\log_5 8 + \log_5 x$
6. $\log 2 - 2 \log x$
7. $\log_2 x + \log_2 y$
8 $3\log_7 x - 5\log_7 y$
9. $4 \log x + 3 \log x$
10. $\log_5 x + 3\log_5 y$
11. $3\log_2 x - \log_2 y$
12. $\log_2 16 - \log_2 8$

Write each logarithm as a quotient of two common logarithms. Simplify your answer, if possible. (Hint: Common logarithms are logarithms with base 10.)

13. $\log_4 12$
14. $\log_2 1000$
15. $\log_5 16$
16. $\log_{11} 205$
17. $\log_9 32$
18. $\log_{100} 5$

Name______________________________ Class ___________ Date___________

7-4 Practice

Form G

Properties of Logarithms

Write each expression as a single logarithm.

1. $\log_5 4 + \log_5 3$

2. $\log_6 25 - \log_6 5$

3. $\log_2 4 + \log_2 2 - \log_2 8$

4. $5\log_7 x = 2\log_7 x$

5. $\log_4 60 - \log_4 4 + \log_4 x$

6. $\log 7 - \log 3 + \log 6$

7. $2 \log x - 3 \log y$

8. $\frac{1}{2}\log r + \frac{1}{3}\log s - \frac{1}{4}\log t$

9. $\log_3 4x + 2 \log_3 5y$

10. $5 \log 2 - 2 \log 2$

11. $\frac{1}{3}\log 3x + \frac{2}{3}\log 3x$

12. $2 \log 4 + \log 2 + \log 2$

13. $(\log 3 - \log 4) - \log 2$

14. $5 \log x + 3 \log x^2$

15. $\log_6 3 - \log_6 6$

16. $\log 2 + \log 4 - \log 7$

17. $\log_3 2x - 5\log_3 y$

18. $\frac{1}{3}(\log_2 x - \log_2 y)$

19. $\frac{1}{2}\log x + \frac{1}{3}\log y - 2\log z$

20. $3(4 \log t^2)$

21. $\log_5 y - 4(\log_5 r + 2\log_5 t)$

Expand each logarithm. Simplify if possible.

22. $\log xyz$

23. $\log_2 \frac{x}{yz}$

24. $\log 6x^3 y$

25. $\log 7(3x - 2)^2$

26. $\log\sqrt{\frac{2rst}{5w}}$

27. $\log\frac{5x}{4y}$

28. $\log_5 5x^{-5}$

29. $\log\frac{2x^2 y}{3k^3}$

30. $\log_4 (3xyz)^2$

Use the Change of Base Formula to evaluate each expression. Round your answer to the nearest thousandth.

31. $\log_4 32$

32. $\log_3 5$

33. $\log_2 15$

34. $\log_6 17$

35. $\log_6 10$

36. $\log_5 6$

37. $\log_8 1$

38. $\log_9 11$

39. The concentration of hydrogen ions in a batch of homemade ketchup is 10^{-4}. What is the pH level of the ketchup? 17

Doodle Here

Name_________________________ Class ___________ Date ___________

7-5 Reteaching

Exponential and Logarithmic Equations

Use logarithms to solve exponential equations.

Problem

What is the solution of $7 - 5^{2x-1} = 4$?

$7 - 5^{2x-1} = 4$	
$-5^{2x-1} = -3$	First isolate the term that has the variable in the exponent. Begin by subtracting 7 from each side.
$5^{2x-1} = 3$	Multiply each side by –1.
$\log_5 5^{2x-1} = \log_5 3$	Because the variable is in the exponent, use logarithms. Take $\log_5$ of each side because 5 is the base of the exponent.
$(2x - 1)\log_5 5 = \log_5 3$	Use the Power Property of Logarithms.
$2x - 1 = \log_5 3$	Simplify. (Recall that $\log_b b = 1$.)
$2x - 1 = \frac{\log 3}{\log 5}$	Apply the Change of Base Formula.
$2x = \frac{\log 3}{\log 5} + 1$	Add 1 to each side.
$x = \frac{1}{2}\left(\frac{\log 3}{\log 5} + 1\right)$	Divide each side by 2.
$x \approx 0.84$	Use a calculator to find a decimal approximation.

Exercises

Solve each equation. Round the answer to the nearest hundredth.

1. $2^x = 5$ **2.** $10^{2x} = 8$ **3.** $5^{x+1} = 25$

4. $2^{x+3} = 9$ **5.** $3^{2x-3} = 7$ **6.** $4^x - 5 = 3$

7. $5 + 2^{x+6} = 9$ **8.** $4^{3x} + 2 = 3$ **9.** $1 - 3^{2x} = -5$

10. $2^{3x} - 2 = 13$ **11.** $5^{2x+7} - 1 = 8$ **12.** $7 - 2^{x+7} = 5$

7-5 Reteaching (continued)

Exponential and Logarithmic Equations

Use exponents to solve logarithmic equations.

Problem

What is the solution of $8 - \log(4x - 3) = 4$?

$8 - 2\log(4x - 3) = 4$	
$-\log(4x - 3) = -4$	First isolate the term that has the variable in the logarithm. Begin by subtracting 8 from each side.
$\log(4x - 3) = 4$	Multiply each side by –1.
$4x - 3 = 10^4$	Write in exponential form.
$4x - 3 = 10{,}000$	Simplify.
$4x = 10{,}003$	Add 3 to each side.
$x = \frac{10{,}003}{4}$	Solve for *x*.
$x = 2500.75$	Divide.

Exercises

Solve each equation. Round the answer to the nearest thousandth.

13. $\log x = 2$

14. $\log 3x = 3$

15. $\log 2x + 2 = 6$

16. $5 + \log(2x + 1) = 6$

17. $\log 5x + 62 = 62$

18. $6 - \log\frac{1}{2}x = 3$

19. $\log(4x - 3) + 6 = 4$

20. $\frac{2}{3}\log 5x = 2$

21. $2\log 250x - 6 = 4$

22. $5 - 2\log x = \frac{1}{2}$

Name______________________ Class __________ Date __________

7-5 Practice

Form G

Exponential and Logarithmic Equations

Solve each equation.

1. $8^{2x} = 32$

2. $7^{n} = 343$

3. $9^{2x} = 27$

4. $25^{2n+1} = 625$

5. $36^{-2x+1} = 216$

6. $64^{x} = 4096$

31. The equation $y = 281(1.01)^{x}$ is a model for the population of the United States y, in millions of people, x years after the year 2000. Estimate when the United States population will reach 400 million people.

Solve each equation. Check your answers.

32. $\log x = 2$

33. $\log 4x = -1$

34. $\log 3x = 2$

35. $\log 4x = 2$

36. $4 \log x = 4$

37. $8 \log x = 16$

38. $2 \log x = 2$

39. $\log (2x + 5) = 3$

40. $\log (3x - 2) 5=3$

41. $\log (x - 25) = 2$

42. $2 \log (2x + 5) = 4$

43. $3 \log (1 - 2x) = 6$

Solve each equation.

44. $\log x - \log 4 = 3$

45. $\log x - \log 4 = -2$

46. $2 \log x - \log 4 = 2$

47. $\log 3x - \log 5 = 1$

48. $2 \log x - \log 3 = 1$

49. $\log 8 - \log 2x = -1$

50. $2 \log 3x - \log 9 = 1$

51. $2 \log x - \log 5 = -2$

52. $\log (x + 21) + \log x = 2$

53. The function $y = 1000(1.005)^{x}$ models the value of $1000 deposited at an interest rate of 6% per year (0.005 per month) x months after the money is deposited.

a. Use a graph (on your graphing calculator) to predict how many months it will be until the account is worth $1100.

b. Predict how many years it will be until the account is worth $5000.

54. Suppose the population of a country is currently 8,100,000. Studies show this country's population is increasing 2% each year.

a. What exponential function would be a good model for this country's population?

b. Using the equation you found in part (a), how many years will it take for the country's population to reach 9 million? Round your answer to the nearest hundredth.

55. Suppose you deposit $2500 in a savings account that pays you 5% interest per year.

a. How many years will it take for you to double your money?

b. How many years will it take for your account to reach $8,000?

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7-6 Reteaching

Natural Logarithms

The **natural logarithmic function** is a logarithm with base e, an irrational number.

You can write the natural logarithmic function as $y = \log_e x$, but you usually write it as $y = \ln x$.

$y = e^x$ and $y = \ln x$ are inverses, so if $y = e^x$, then $x = \ln y$.

To solve a natural logarithm equation:

- If the term containing the variable is an exponential expression, rewrite the equation in logarithmic form.
- If term containing the variable is a logarithmic expression, rewrite the equation in exponential form.

Problem

What is the solution of $4e^{2x} - 2 = 3$?

Step 1 Isolate the term containing the variable on one side of the equation.

$$4e^{2x} - 2 = 3$$

$$4e^{2x} = 5$$ Add 2 to each side of the equation.

$$e^{2x} = \frac{5}{4}$$ Divide each side of the equation by 4.

Step 2 Take the natural logarithm of each side of the equation.

$$\ln(e^{2x}) = \ln\left(\frac{5}{4}\right)$$

$$2x = \ln\left(\frac{5}{4}\right)$$ Definition of natural logarithm

Step 3 Solve for the variable.

$$x = \frac{\ln\left(\frac{5}{4}\right)}{2}$$ Divide each side of the equation by 2.

$$x \approx 0.112$$ Use a calculator.

Step 4 Check the solution.

$$4e^{2(0.112)} - 2 \stackrel{?}{=} 3$$

$$4e^{0.224} - 2 \stackrel{?}{=} 3$$

$$3.004 \approx 3$$

The solution is $x \approx 0.112$.

7-6 Reteaching (continued)

Natural Logarithms

Problem

What is the solution of $\ln(t-2)^2 + 1 = 6$? Round your answer to the nearest thousandth.

Step 1 Isolate the term containing the variable on one side of the equation.

$\ln(t-2)^2 + 1 = 6$

$\ln(t-2)^2 = 5$ Subtract 1 from each side of the equation.

Step 2 Raise each side of the equation to the base e.

$e^{\ln(t-2)^2} = e^5$

$(t-2)^2 = e^5$ Definition of natural logarithm

Step 3 Solve for the variable.

$t-2 = \pm e^{\frac{5}{2}}$ Take the square root of each side of the equation.

$t = 2 \pm e^{\frac{5}{2}}$ Add 2 to each side of the equation.

$t \approx 14.182$ or -10.182 Use a calculator.

Step 4 Check the solution.

$\ln(14.182-2)^2 \stackrel{?}{=} 5$ $\ln(-10.182-2)^2 \stackrel{?}{=} 5$

$4.9999 \approx 5$ $4.9999 \approx 5$

The solutions are $t \approx 14.182$ and -10.182.

Exercises

Use natural logarithms to solve each equation. Round your answer to the nearest thousandth. Check your answers.

1. $2e^x = 4$ **2.** $e^{4x} = 25$ **3.** $e^x = 72$

4. $e^{3x} = 124$ **5.** $12e^{3x-2} = 8$ **6.** $\frac{1}{2}e^{6x} = 5$

Solve each equation. Round your answer to the nearest thousandth. Check your answers.

7. $\ln(x-3) = 2$ **8.** $\ln 2t = 4$ **9.** $1 + \ln x^2 = 2$

10. $\ln(2x-5) = 3$ **11.** $\frac{1}{3}\ln 2t = 1$ **12.** $\ln(t-4)^2 + 2 = 5$

Name________________________________ Class____________ Date____________

7-6 Practice

Form G

Natural Logarithms

Write each expression as a single natural logarithm.

1. $\ln 16 - \ln 8$

2. $3 \ln 3 + \ln 9$

3. $a \ln 4 - \ln b$

4. $\ln z - 3 \ln x$

5. $\frac{1}{2} \ln 9 + \ln 3x$

6. $4 \ln x + 3 \ln y$

7. $\frac{1}{3} \ln 8 + \ln x$

8. $3 \ln a - b \ln 2$

9. $2 \ln 4 - \ln 8$

Solve each equation. Check your answers. Round your answer to the nearest hundredth.

10. $4 \ln x = -2$

11. $2 \ln (3x - 4) = 7$

12. $5 \ln (4x - 6) = -6$

13. $-7 + \ln 2x = 4$

14. $3 - 4 \ln (8x + 1) = 12$

15. $\ln x + \ln 3x = 14$

16. $2 \ln x + \ln x^2 = 3$

17. $\ln x + \ln 4 = 2$

18. $\ln x - \ln 5 = -1$

19. $\ln e^x = 3$

20. $3 \ln e^{2x} = 12$

21. $\ln e^{x+5} = 17$

22. $\ln 3x + \ln 2x = 3$

23. $5 \ln (3x - 2) = 15$

24. $7 \ln (2x + 5) = 8$

25. $\ln (3x + 4) = 5$

26. $\ln \frac{2x}{41} = 2$

27. $\ln (2x - 1)^2 = 4$

Use natural logarithms to solve each equation. Round your answer to the nearest hundredth.

28. $e^x = 15$

29. $4e^x = 10$

30. $e^x\ 1 - = 50$

31. $4e^{3x21} = 5$

32. $e^{x-4} = 2$

33. $5e^{6x+3} = 0.1$

34. $e^x = 1$

35. $e^{\frac{x}{5}} = 32$

36. $3e^{3x-5} = 49$

37. $7e^{5x+8} = 0.23$

38. $6 - e^{12x} = 5.2$

39. $e^{\frac{x}{5}} = 25$

40. $e^{2x} = 25$

41. $e^{\ln 5x} = 20$

42. $e^{\ln x} = 21$

43. $e^{x+6} + 5 = 1$

Name________________________________ Class __________ Date__________

7-6 Practice (continued)

Form G

Natural Logarithms

The formula $P = 50e^{-\frac{t}{25}}$ gives the power output P, in watts, needed to run a certain satellite for t days. Find how long a satellite with the given power output will operate.

44. 10 W

45. 12 W

46. 14 W

The formula for the maximum velocity v of a rocket is $v = -0.0098t + c \ln R$, where c is the exhaust velocity in km/s, t is the f ring time, and R is the mass ratio of the rocket. A rocket must reach 7.7 km/s to attain a stable orbit 300 km above Earth.

47. What is the maximum velocity of a rocket with a mass ratio of 18, an exhaust velocity of 2.2 km/s, and a firing time of 25 s?

48. Can the rocket in Exercise 47 achieve a stable orbit? Explain your answer.

49. What mass ratio would be needed to achieve a stable orbit for a rocket with an exhaust velocity of 2.5 km/s and a f ring time of 29 s?

50. A rocket with an exhaust velocity of 2.4 km/s and a 28 second f ring time can reach a maximum velocity of 7.8 km/s. What is the mass ratio of the rocket?

By measuring the amount of carbon-14 in an object, a paleontologist can determine its approximate age. The amount of carbon-14 in an object is given by $y = ae^{-0.00012t}$, where a is the amount of carbon-14 originally in the object, and t is the age of the object in years.

51. A fossil of a bone contains 32% of its original carbon-14. What is the approximate age of the bone?

52. A fossil of a bone contains 83% of its original carbon-14. What is the approximate age of the bone?

Simplify each expression.

53. $\ln e^4$

54. $5 \ln e^5$

55. $\frac{\ln e^2}{2}$

56. $\ln e^{100}$