Triangle Congruence Theorems

You look like twins... Are you? Let's have a little look at that there DNA shall we?

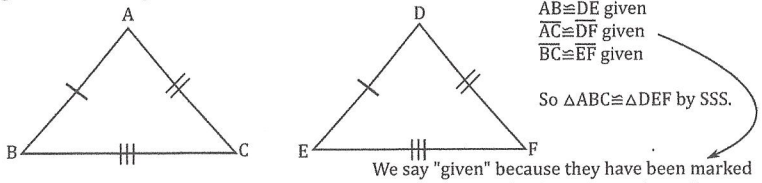
Remember back when we talked about what congruent meant? No? Okay, remember it means two shapes that are exactly the same size and shape...like identical twins. Okay, so two people run into each other walking down the sidewalk, they look exactly alike. Are they long lost identical twins, or just a freak coincidence of genetic mayhem? Well, there's a good way to find out... You guessed it! A DNA test will confirm if these two are twins or just scary.

The triangle congruence theorems are no different than a DNA test... Okay, well there is less blood and bodily fluids involved... but other than that they perform the same function... confirming identical triangle twins.

They are SSS, SAS, ASA, AAS, and HL (for right triangles only).

Huh? Okay, we will look at an example of each before we get into practicing.

SSS stands for Side, Side, Side. If three sides of one triangle are congruent to three sides of the other triangle then the triangles are congruent. Take a look...

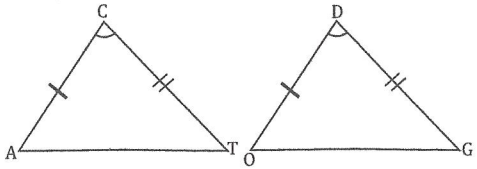


$\overline{AB} \cong \overline{DE}$ given
$\overline{AC} \cong \overline{DF}$ given
$\overline{BC} \cong \overline{EF}$ given

So $\triangle ABC \cong \triangle DEF$ by SSS.

We say "given" because they have been marked on the diagram like that info was "given" to us.

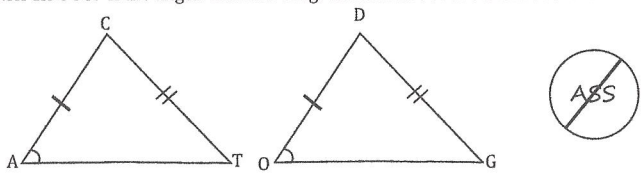
SAS stands for Side, Angle, Side. If two sides of a triangle and the included angle (means the one between the two sides) are congruent to the two sides, and an included angle of another triangle are congruent, then the triangles themselves are congruent. Take another look...



$\overline{AC} \cong \overline{OD}$ given
$\angle C \cong \angle D$ given
$\overline{CT} \cong \overline{DG}$ given

So $\triangle ABC \cong \triangle DEF$ by SAS.

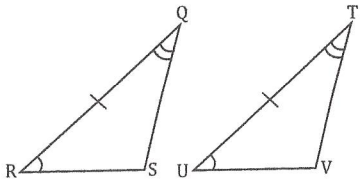
WATCH OUT! If the angles that are congruent are not between the two sides like this...



then it's not SAS, it's ASS. Don't say ASS! Don't use ASS! It is not a congruence property! Don't use SSA either, because it is just ASS backwards.

Ahem....now that we have that cleared up...

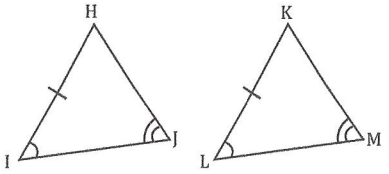
ASA stands for Angle, Side, Angle. If two angles of a triangle and the included side (means the one between the two angles) are congruent to the two angles and an included side of another triangle, then the triangles themselves are congruent. More looking for you...



$\angle R \cong \angle U$ given
$\overline{RQ} \cong \overline{UT}$ given
$\angle Q \cong \angle T$ given

So $\triangle QRS \cong \triangle TUV$ by ASA.

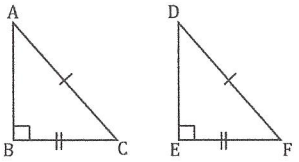
AAS stands for Angle, Angle, Side. It is really just a form of ASA. Think about it, If you have two angles in a triangle you can always find the third using the Triangle Sum Theorem. If you have AAS, you can find the third A, and get to ASA. Fortunately, we don't have to actually do this because it can be as hard as it sounds. Here is an example of AAS.



$\angle I \cong \angle L$ given
$\angle J \cong \angle M$ given
$\overline{IH} \cong \overline{LK}$ given

So $\triangle HIJ \cong \triangle KLM$ by AAS.

HL stands for Hypotenuse, Leg. What kind of triangle has a hypotenuse? A right triangle of course. This special congruence property is for right triangles only. If the hypotenuse and one leg of a right triangle is congruent to the hypotenuse and leg of another, then the right triangles are congruent. This is actually a shortcut for SSS because if you have two sides of a right triangle you can find the third using the Pythagorean theorem, thus arriving at SSS. Fortunately, you don't have to do that either.



$\overline{AC} \cong \overline{DF}$ given
$\overline{BC} \cong \overline{EF}$ given

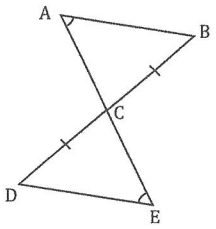
So $\triangle ABC \cong \triangle DEF$ by HL.

Make sure when you do this one that you actually have a hypotenuse and a leg and not two legs. (If you do have two legs the right angle will be between them giving you SAS. Trust me, you'll see.)

Great, these really don't get too hard except sometimes you have to come up with some parts that are congruent, like alternate interior angles, or shared sides. We will start with some simple examples. When we get to the little bit harder ones, I'll walk you through how to find the missing pieces.

To the twin mobile!...Okay, bad example.

25.



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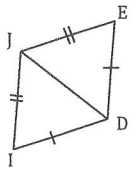

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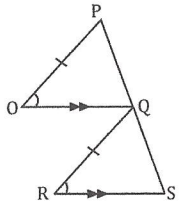

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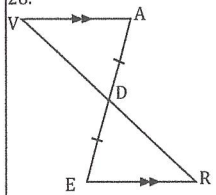

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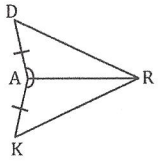

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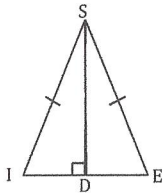

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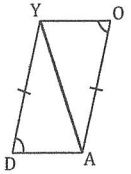

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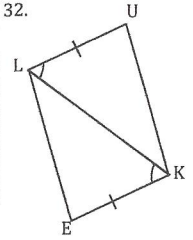
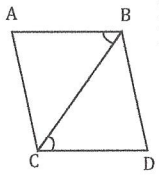


Bubble the correct answer choice from each item above.

#25	#26	#27	#28	#29	#30	#31	#32
○A.	○A.	○A.	○A.	○A.	○A.	○A.	○A.
○B.	○B.	○B.	○B.	○B.	○B.	○B.	○B.
○C.	○C.	○C.	○C.	○C.	○C.	○C.	○C.
○D.	○D.	○D.	○D.	○D.	○D.	○D.	○D.

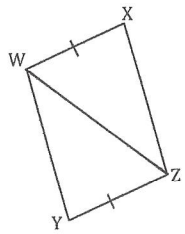
What additional information is needed to show the triangles are congruent by the given property?

1. SAS



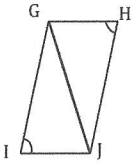
A. $\overline{AC}\cong\overline{CD}$
B. $\angle A\cong\angle D$
C. $\overline{AB}\cong\overline{CD}$
D. $\angle ABC\cong\angle DBC$

2. SSS



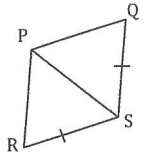
A. $\overline{XZ}\cong\overline{YW}$
B. $\angle X\cong\angle Y$
C. $\overline{YZ}\cong\overline{XZ}$
D. $\overline{WZ}\cong\overline{XZ}$

3. AAS



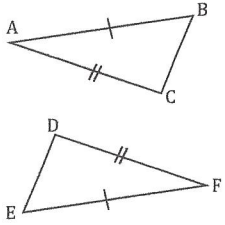
A. $\angle IGJ\cong\angle HGI$
B. $\overline{IG}\cong\overline{HJ}$
C. $\angle IGJ\cong\angle HJG$
D. $\angle HJG\cong\angle HGJ$

4. SAS



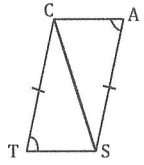
A. $\angle QPS\cong\angle SQP$
B. $\angle QSP\cong\angle RSP$
C. $\angle QSP\cong RPS$
D. $\angle R\cong\angle Q$

5. SAS



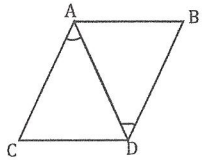
A. $\angle A\cong\angle E$
B. $\angle C\cong\angle D$
C. $\angle B\cong\angle E$
D. $\angle A\cong\angle F$

6. SAS



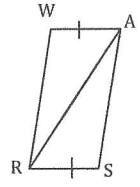
A. Nothing
B. $\angle TCS\cong\angle ASC$
C. $\angle A\cong\angle T$
D. $\angle TSC\cong\angle ACS$

7. AAS



A. $\angle C\cong\angle B$
B. $\overline{CD}\cong\overline{AB}$
C. $\angle BAD\cong\angle ABD$
D. $\angle C\cong\angle BDA$

8. SSS



A. $\overline{WR}\cong\overline{AR}$
B. $\overline{AS}\cong\overline{AR}$
C. $\overline{WR}\cong\overline{AS}$
D. $\angle RAS\cong\angle ARW$

Bubble the correct answer choice from each item above.

#1	#2	#3	#4	#5	#6	#7	#8
○A.	○A.	○A.	○A.	○A.	○A.	○A.	○A.
○B.	○B.	○B.	○B.	○B.	○B.	○B.	○B.
○C.	○C.	○C.	○C.	○C.	○C.	○C.	○C.
○D.	○D.	○D.	○D.	○D.	○D.	○D.	○D.

11

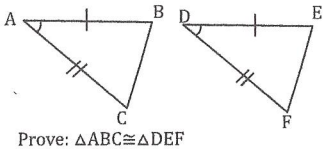
Proofs Involving Congruent Triangles

First, let's analyze some proofs.

This is easy! All you have to do is explain in plain English what is going on in the proofs. We'll look at some examples first.

AE. 1.

Given: $\overline{AB}\cong\overline{DE}$, $\overline{AC}\cong\overline{DF}$, and $\angle A\cong\angle D$



Prove: $\triangle ABC\cong\triangle DEF$

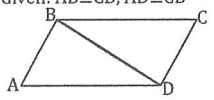
Statements	Reasons
1. $\overline{AB}\cong\overline{DE}$	1. Given
2. $\overline{AC}\cong\overline{DF}$	2. Given
3. $\angle A\cong\angle D$	3. Given
4. $\triangle ABC\cong\triangle DEF$	4. SAS

Analysis:

Working backward we must ask the key question, "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC\cong\triangle DEF$, by some property, but which one? To find out, start working forward. Listing all of the given information gives us a pair of angles $\angle A$ and $\angle D$ sandwiched between a pair of congruent sides $\overline{AB}\cong\overline{DE}$ and $\overline{AC}\cong\overline{DF}$. So this means we have $\triangle ABC\cong\triangle DEF$ by the SAS theorem which is B2: and the proof is complete.

AE. 2.

Given: $\overline{AB}\cong\overline{CD}$, $\overline{AD}\cong\overline{CB}$



Prove: $\triangle ABD\cong\triangle BCD$

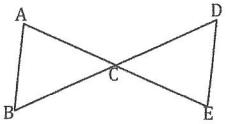
Statements	Reasons
1. $\overline{AB}\cong\overline{CD}$	1. Given
2. $\overline{AD}\cong\overline{CB}$	2. Given
3. $\overline{BD}\cong\overline{BD}$	3. Reflexive property
4. $\triangle ABD\cong\triangle CDB$	4. SSS

Analysis:

Working backward, we must ask the key question "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC\cong\triangle BCD$ bys ome property, but which one? Then start working forward. Listing all of the given information gives us two pairs of sides $\overline{AB}\cong\overline{CD}$ and $\overline{AD}\cong\overline{CB}$, but this is not enough. We need another pair of sides or an angle between them. Looking now at the diagram we have $\overline{BD}\cong\overline{BD}$ as a shared line. So this brings us to say $\triangle ABC\cong\triangle BCD$ by SSS which is B1 and the proof is complete.

AE. 3.

Given: $\overline{AE}$ Bisects $\overline{BD}$, $\angle B\cong\angle D$



Prove: $\triangle ABC\cong\triangle DBC$

Statements	Reasons
1. $\angle B\cong\angle D$	1. Given
2. $\overline{AC}$ Bisects $\overline{BD}$	2. Given
3. $\overline{BC}\cong\overline{DC}$	3. Definition of Bisect
4. $\angle ACB\cong\angle DCE$	4. Vertical angles
5. $\triangle ABC\cong\triangle DBC$	5. ASA

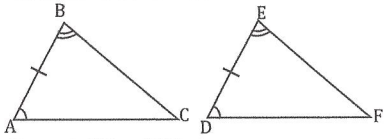
Analysis:

Working backward we must ask the key question, "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC\cong\triangle BCD$ by some property, but which one? Then start working forward. Listing all of the given information gives us a pair of angles $\angle B$ and $\angle D$, and $\overline{BD}$ and $\overline{AE}$ bisects $\overline{BD}$. If AE bisects $\overline{BD}$ then $\overline{BD}$ is cut in half at C so $\overline{BC}\cong\overline{DC}$! This is not enough though. Looking at the diagram we see vertical angles $\angle ACB\cong\angle DCE$, which gives us $\triangle ABC\cong\triangle BCD$ by the property ASA . This is B1 and the proof is complete.

For these fill in any missing statements or reasons.

1.

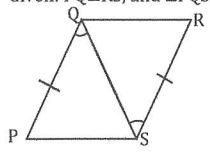
Given: $\overline{AB}\cong\overline{DE}$, $\angle B\cong\angle E$, and $\angle A\cong\angle D$



Prove: $\triangle ABC\cong\triangle DBC$

Statements	Reasons
1. $\overline{AB}\cong\overline{DE}$	1. Given
2.	2.Given
3. $\angle A\cong\angle D$	3.
4. $\triangle ABC\cong\triangle DEF$	4.

2. Given: $\overline{PQ}\cong\overline{RS}$, and $\angle PQS\cong\angle RSQ$

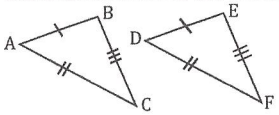


Prove: $\triangle ABC\cong\triangle DBC$

Statements	Reasons
1.	1. Given
2.	2. Given
3. $\overline{QS}\cong\overline{QS}$	3.
4. $\triangle PQS\cong\triangle RSQ$	4.

3.

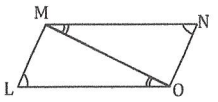
Given: $\overline{AB}\cong\overline{DE}$, $\overline{AC}\cong\overline{DF}$, and $\overline{BC}\cong\overline{EF}$



Prove: $\triangle ABD\cong\triangle DEF$

Statements	Reasons
1. $\overline{AB}\cong\overline{DE}$	1.
2.	2.
3.	3.
4.	4. SSS

4. Given: $\angle L\cong\angle N$, $\angle LOM\cong\angle NMO$

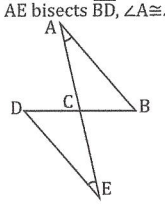


Prove: $\triangle LMO\cong\triangle NOM$

Statements	Reasons
1.	1.
2.	2. Given
3.	3. Reflexive Property
4. $\triangle LMO\cong\triangle NOM$	4.

5.

Given: $\overline{AE}$ bisects $\overline{BD}$, $\angle A\cong\angle E$

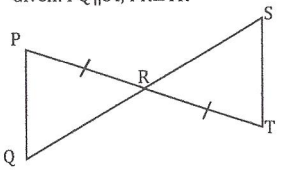


Prove: $\triangle ABC\cong\triangle EDC$

Statements	Reasons
1. $\angle A\cong\angle E$	1.
2.	2. Given
3.	3. Definition of Bisect
4. $\angle ACB\cong\angle DCE$	4.
5. $\triangle ABC\cong\triangle EDC$	5.

6.

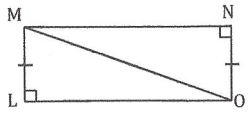
Given: $\overline{PQ}\parallel\overline{ST}$, $\overline{PR}\cong\overline{TR}$



Prove: $\triangle PQR\cong\triangle TSR$

Statements	Reasons
1. $\overline{PR}\cong\overline{TR}$	1.
2.	2. Given
3. $\angle P\cong\angle T$	3.
4. $\angle ACB\cong\angle DCE$	4.
5.	5. ASA

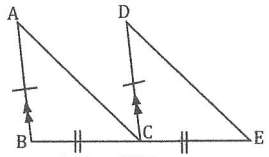
13. Given: $\overline{LM} \cong \overline{NO}$



Prove: $\triangle LMO \cong \triangle NOM$

Statements	Reasons
1. $\overline{LM} \cong \overline{NO}$	1.
2.	2.
3.	3.

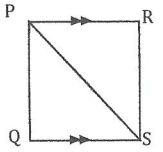
14. Given: $\overline{AB} \cong \overline{DC}$, $\overline{AB} \| \overline{DC}$, and $\overline{BC} \cong \overline{CE}$



Prove: $\triangle ABC \cong \triangle DCE$

Statements	Reasons
1. $\overline{AB} \cong \overline{DC}$	1. Given
2.	2. Given
3.	3. Given
4.	4. Corresponding Angles
5. $\triangle ABC \cong \triangle DCE$	5.

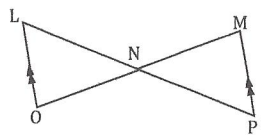
15. Given: $\overline{PR} \| \overline{QS}$, $\angle QPS \cong \angle RSP$



Prove: $\triangle PQS \cong \triangle SRP$

Statements	Reasons
1. $\overline{PR} \| \overline{QS}$	1.
2. $\angle QPS \cong \angle RSP$	2.
3. $\angle PSQ \cong \angle SPR$	3. Alternate Interior
4.	4. Reflexive Property
5. $\triangle ABD \cong \triangle CDB$	5.

16. Given: $\overline{LP}$ bisects $\overline{MO}$, $\overline{LO} \| \overline{MP}$

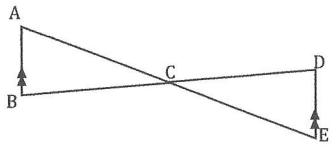


Prove: $\triangle LNO \cong \triangle MNP$

Statements	Reasons
1.	1. Given
2.	2. Given
3. $\overline{LN} \cong \overline{PN}$	3.
4.	4. Alternate Interior
5.	5. Vertical Angles
6.	6. ASA

17.

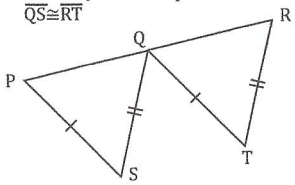
Given: $\overline{AE}$ bisects $\overline{BD}$, $\overline{AB} \| \overline{DE}$



Prove: $\triangle ABC \cong \triangle DCE$

Statements	Reasons
1. $\overline{AE}$ bisects $\overline{BD}$	1.
2.	2. Given
3. $\overline{BC} \cong \overline{DC}$	3.
4. $\angle ACB \cong \angle DCB$	4.
5.	5. Alternate Interior
6.	6. ASA

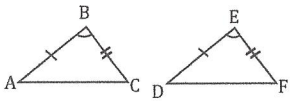
18. Given: Q is the midpoint of $\overline{PR}$. $\overline{PS} \cong \overline{QT}$ and $\overline{QS} \cong \overline{RT}$



Prove: $\triangle PQS \cong \triangle RQT$

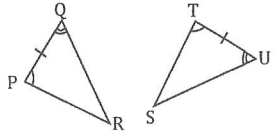
Statements	Reasons
1.	1. Given
2.	2. Given
3. $\overline{QS} \cong \overline{RT}$	3.
4.	4. Midpoint
5. $\triangle ABC \cong \triangle DBC$	5.

19. Given: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\angle B \cong \angle E$



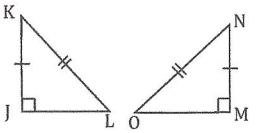
Prove: $\triangle ABC \cong \triangle DEF$

20. Given: $\overline{PQ} \cong \overline{TU}$, $\angle P \cong \angle T$, and $\angle Q \cong \angle U$



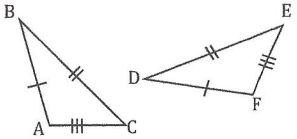
Prove: $\triangle PQR \cong \triangle TUS$

21. Given: $JK \cong MN$, $KL \cong NO$



Prove: $\triangle JKL \cong \triangle MNO$

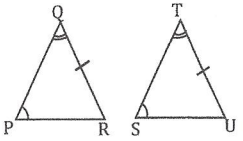
22. Given: $\overline{AB} \cong \overline{DF}$, $\overline{BC} \cong \overline{DE}$, and $\overline{AC} \cong \overline{EF}$



Prove: $\triangle ABD \cong \triangle FDE$

23.

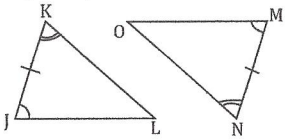
Given: $\angle P \cong \angle S$, $\angle Q \cong \angle T$, and $\overline{QR} \cong \overline{TU}$



Prove: $\triangle PQR \cong \triangle STU$

24.

Given: $\angle J \cong \angle M$, $\overline{JK} \cong \overline{MN}$ and $\angle K \cong \angle N$



Prove: $\triangle JKL \cong \triangle MNO$

111 Proofs Involving CPCTC

How to fix your car....

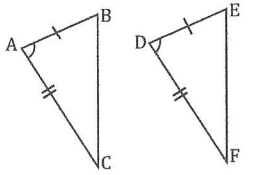
Okay, remember that to use CPCTC (Corresponding Parts of Congruent Triangles are Congruent), it's like saying that the carburetor from a '57 Chevy will be the same as the carburetor from another '57 Chevy. But, if you have two carburetors from two unknown cars, who knows if they are same or not? Okay, maybe a experienced mechanic could tell, but not me.

So remember... BEFORE YOU USE CPCTC YOU MUST PROVE THAT THE TRIANGLES IN QUESTION ARE CONGRUENT FIRST!!!

Let's analyze a couple of these, and then we will get to practicing...

Ex. 1.

Given: $\overline{AB}\cong\overline{DE}$, $\angle A\cong\angle D$, and $\overline{AC}\cong\overline{DF}$



Prove: $\angle C\cong\angle F$

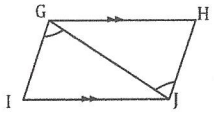
Statements	Reasons
1. $\overline{AB}\cong\overline{DE}$	1. Given
2. $\angle A\cong\angle D$	2. Given
3. $\overline{AC}\cong\overline{DF}$	3. Given
B2:4. $\triangle ABC\cong\triangle DEF$	4. SAS
B1:5. $\angle C\cong\angle F$	5. CPCTC

Analysis:

Working backwards, the statement "Prove: $\angle C\cong\angle F$" begs the key question ,"How can we show two segments from two different triangles are congruent?" The answer? CPCTC. This means that if we can prove the triangles are congruent then $\angle C\cong\angle F$ because they are corresponding parts of congruent triangles. So B1: $\angle C\cong\angle F$ by CPCTC. This then begs the second key question, "How can I show two triangles are congruent?" This means B2 will be $\triangle ABC\cong\triangle DEF$ by some congruence property. Which one? Now we work forward and see we have. $\overline{AB}\cong\overline{DE}$, $\angle A\cong\angle D$, and $\overline{AC}\cong\overline{DF}$ which are all given, This means $\triangle ABC\cong\triangle DEF$, which is B2, and the proof is complete.

Ex. 2.

Given: $\overline{GH}\|\overline{IJ}$, $\angle IGJ\cong\angle HJG$



Prove: $\overline{IG}\cong\overline{HJ}$

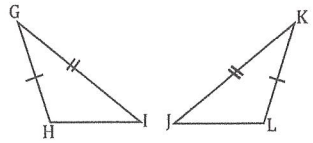
Statements	Reasons
1. $\overline{GH}\|\overline{IJ}$	1. Given
2. $\angle IGJ\cong\angle HJG$	2. Given
3. $\angle HGJ\cong\angle IJG$	3. Alternate Interior
4. $\overline{GJ}\cong\overline{GJ}$	4. Reflexive Property
B2: 5. $\triangle IGJ\cong\triangle HJG$	5. ASA
B1: 6. $\overline{IG}\cong\overline{HJ}$	6. CPCTC

Analysis:

Working backwards, the statement "Prove: $\overline{IG}\cong\overline{HJ}$" begs the key question ,"How can we show two segments from two different triangles are congruent?" The answer? CPCTC. This means that if we can prove the triangles are congruent then $\overline{IG}\cong\overline{HJ}$ because they are corresponding parts of congruent triangles. So B1 $\overline{IG}\cong\overline{HJ}$ by CPCTC. This then begs the second key question, "How can I show two triangles are congruent?" This means B2 will be $\triangle IGJ\cong\triangle HJG$ by some congruence property. Which one? Now we work forward and see we have $\overline{GH}\|\overline{IJ}$, and $\angle IGJ\cong\angle HJG$, Given. We know when we see parallel lines we look for alternate interior or corresponding angles. We have alternate interior angles $\angle HGJ\cong\angle IJG$. We also have $\overline{GJ}$ as a shared side. This means $\triangle IGJ\cong\triangle HJG$ by ASA, which is B2, and the proof is complete.

Fill in the missing information in each proof.

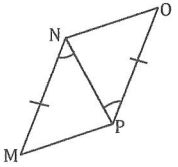
4. Given: $\overline{GH}\cong\overline{KL}$, $\angle G\cong\angle K$, and $\overline{GI}\cong\overline{KJ}$



Prove: $\overline{HI}\cong\overline{LJ}$

Statements	Reasons
1. $\overline{GH}\cong\overline{KL}$	1. Given
2.	2. Given
3. $\overline{GI}\cong\overline{KJ}$	3.
4.	4. SAS
5. $\overline{HI}\cong\overline{LJ}$	5.

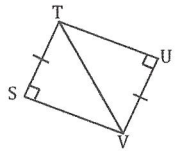
5. Given: $\angle MNP\cong\angle OPN$, and $\overline{MN}\cong\overline{OP}$



Prove: $\overline{MP}\cong\overline{NO}$

Statements	Reasons
1.	1. Given
2. $\overline{MN}\cong\overline{OP}$	2.
3. $\overline{NP}\cong\overline{NP}$	3.
4. $\triangle MNP\cong\triangle OPN$	4.
5.	5. CPCTC

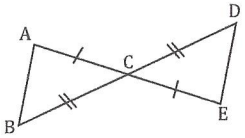
6. Given: $\overline{ST}\cong\overline{VU}$



Prove: $\angle SVT\cong\angle UTV$

Statements	Reasons
1.	1. Given
2.	2. Reflexive Property
3.	3. HL
4. $\angle SVT\cong\angle UTV$	4.

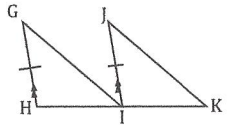
7. Given: $\overline{AC}\cong\overline{CE}$, $\overline{DC}\cong\overline{BC}$



Prove: $\angle B\cong\angle D$

Statements	Reasons
1.	1.
2.	2. Given
3. $\angle ACB\cong\angle DCE$	3.
4. $\triangle ABC\cong\triangle DEF$	4.
5. $\angle B\cong\angle D$	5.

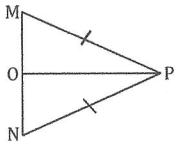
8. Given: $\overline{GH}\|\overline{JI}$, I is the midpoint of HK and $\overline{GH}\cong\overline{JI}$



Prove: $\angle G\cong\angle J$

Statements	Reasons
1. $\overline{GH}\|\overline{JI}$	1.
2. I is the midpoint of $\overline{HK}$	2.
3.	3. Given
4. $\overline{HI}\cong\overline{IK}$	4.
5.	5. Corresponding
6.	6. SAS
7. $\angle G\cong\angle J$	7.

9. Given: $\overline{MP}\cong\overline{NP}$, $\overline{MN}\perp\overline{OP}$

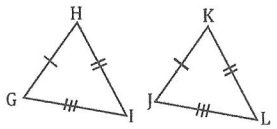


Prove: $\overline{MO}\cong\overline{ON}$

Statements	Reasons
1.	1. Given
2. $\overline{MN}\perp\overline{OP}$	2.
3. $\overline{OP}\cong\overline{OP}$	3.
4. $\triangle MOP\cong\triangle NOP$	4.
5.	5.

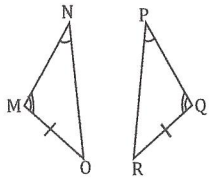
Write a two column proof for each.

16. Given: $\overline{GH} \cong \overline{JK}$, $\overline{HI} \cong \overline{KL}$, and $\overline{IG} \cong \overline{LJ}$



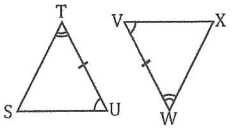
Prove: $\angle I \cong \angle L$

17. Given: $\angle N \cong \angle P$, $\angle M \cong \angle Q$, and $\overline{MO} \cong \overline{QR}$



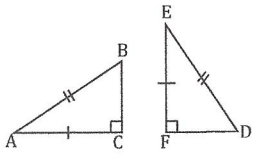
Prove: $\angle O \cong \angle R$

18.
Given: $\angle U \cong \angle V$, $\angle T \cong \angle W$, and $\overline{TU} \cong \overline{VW}$



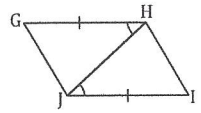
Prove: $\angle S \cong \angle X$

19. Given: $\overline{AC} \cong \overline{EF}$, and $\overline{AB} \cong \overline{ED}$



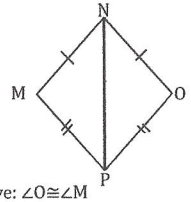
Prove: $\overline{BC} \cong \overline{FD}$

20.
Given: $\overline{GH} \cong \overline{JI}$, $\angle GHJ \cong \angle IJH$



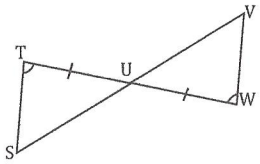
Prove: $\overline{GJ} \cong \overline{HI}$

21. Given: $\overline{MN} \cong \overline{NO}$, $\overline{MP} \cong \overline{OP}$



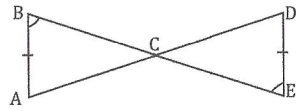
Prove: $\angle O \cong \angle M$

22. Given: $\overline{TU} \cong \overline{WU}$, $\angle T \cong \angle W$



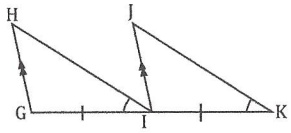
Prove: $\overline{TS} \cong \overline{WV}$

23. Given: $\overline{AB} \cong \overline{DE}$, $\angle B \cong \angle E$



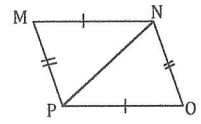
Prove: $\overline{AC} \cong \overline{DC}$

24. Given: $\overline{HG} \| \overline{JI}$, $\overline{GI} \cong \overline{IK}$, and $\angle HIG \cong \angle JKI$



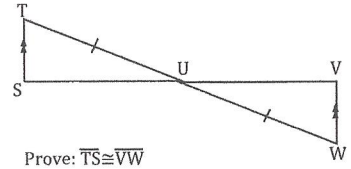
Prove: $\angle C \cong \angle F$

25. Given: $\overline{MN} \cong \overline{PO}$, $\overline{MP} \cong \overline{NO}$



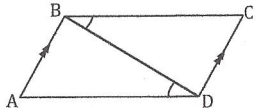
Prove: $\angle M \cong \angle O$

26. Given: $\overline{TS} \| \overline{VW}$, $\overline{TU} \cong \overline{WU}$



Prove: $\overline{TS} \cong \overline{VW}$

27. Given: $\overline{AB} \| \overline{DE}$, $\angle CBD \cong \angle ADB$



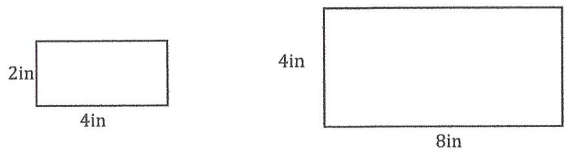
Prove: $\overline{BC} \cong \overline{AD}$

IV

Similarity and Proportions

Honey I shrunk and/or enlarged the whatever!....

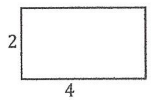
Huh? Maybe I'm older than I thought... Oh well... Similarity is almost the same thing as congruence except similar figures are the same shape but not necessarily the same size. Here's what I mean... Have you ever been into a store and seen a model? (No gentlemen, not that kind of model.) Like a model car, or a model airplane or even one of those "look at the new buildings we are going to build" kind of models you see in public buildings sometimes. That is similar! So the model corvette and real corvette are the same shape but different sizes. Here is another example...

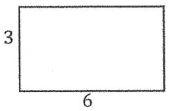


Notice that these two rectangles are the same shape, but one is bigger than the other. How much bigger? Well, notice that if you multiply the sides of the first one by 2 you get the sides of the second one. This leads us to the Ratio of Similarity. All this is, is the ratio of the sides of the smaller shape to the sides of the larger shape. These two have a ratio of 2/4 or 1/2. So the smaller one is half the size of the larger... Let's practice finding the ratio of similarity.

Each pair of figures is similar. Find the ratio of similarity.

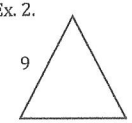
Ex. 1.

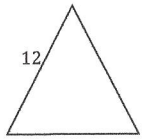




$\frac{2}{3}$

Ex. 2.





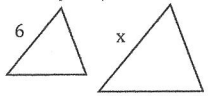
$\frac{9}{12} = \frac{3}{4}$

Okay, so what is this good for anyway? Well, let's say that you have a model car and the ratio between the model and the real thing is 1/8. Say your model is 2ft long. How long is the real car? Well... all you have to do is set up and solve a proportion like this...

$\frac{1}{8} = \frac{2}{x} =$ $1(x)=2(8)$ $x=16$ So the real car is 16 feet long! (Maybe it's a Mini.)

Now when you set up the proportion, you must make sure that if you have the smaller number on top for your ratio, the other fraction must always have the smaller number on top, too. I like to always put the small number on top so I don't make the mistake of flipping one of the ratios. Let's look at two more examples...

Ex. 3. These figures are similar. The ratio of similarity is 2/5. Find x.



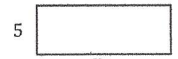
$\frac{2}{5} = \frac{6}{x} =$

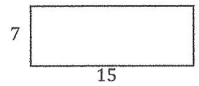
$2(x)=5(6)$

$\frac{2x}{2}=\frac{30}{2}$

$x=15$

Ex. 4. These figures are similar. Find x.





$\frac{5}{7} = \frac{x}{15} =$

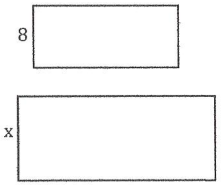
$7(x)=5(15)$

$\frac{7x}{7}=\frac{75}{7}$

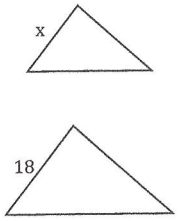
$x=10.71$

Each pair of figures is similar. Find the missing length.

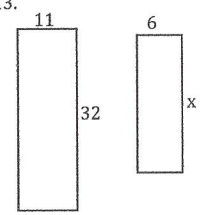
9. Ratio of similarity =2/3.



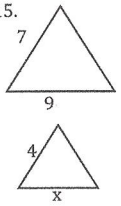
11. Ratio of similarity =2/7.



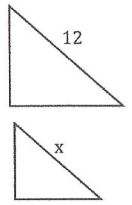
13.



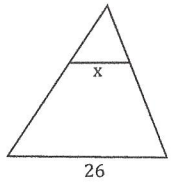
15.



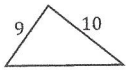
10. Ratio of similarity =4/5.

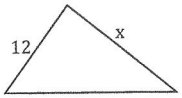


12. Ratio of similarity =5/13.

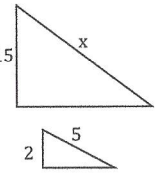


14.





16.



Bubble all the correct answers from above. Don't bubble incorrect answers.

○4.15 ○17.45 ○14.45 ○13.33 ○ 20 ○ 15 ○ 10 ○ 12 ○ 9.6 ○ 5.14 ○ 37.5 ○ 5.14 ○ 13 ○ 8.6

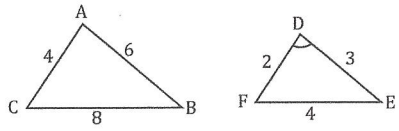
Similar Triangles

The same but not quite....

Triangle similarity and triangle congruence are almost the same thing. That's a big "almost" though. Congruent triangles have the same size and shape, and similar triangles have the same shape but DIFFERENT sizes.

Triangle similarity also has "DNA tests" (Just like congruence had SSS, SAS, ASA, etc.) to tell whether or not they are in fact similar. They are AAA, AA, SAS, and SSS. We will of course go over these now and explain the difference between them and their congruency counterparts.

SSS is first. It means that if all three sides of two triangles have the same RATIO then the triangles are similar. Let's see...



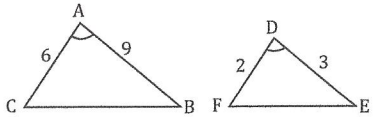
$\frac{DF}{AC} = \frac{2}{4} = \frac{1}{2}$ So... $\frac{DF}{AC} = \frac{DE}{AB} = \frac{EF}{BC}$

$\frac{DE}{AB} = \frac{3}{6} = \frac{1}{2}$

$\frac{EF}{BC} = \frac{4}{8} = \frac{1}{2}$ So... $\triangle ABC \sim \triangle DEF$ by SSS

By the way, "~" means "similiar" or "is similar to."

SAS is next. It means that if two sides of two triangles have the same RATIO and the angle between the sides are CONGRUENT, then the triangles are similar. Let's see...



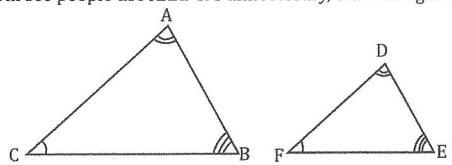
$\frac{DF}{AC} = \frac{2}{6} = \frac{1}{3}$ So... $\frac{DF}{AC} = \frac{DE}{AB}$

$\frac{DE}{AB} = \frac{3}{9} = \frac{1}{3}$ and... $\angle A \cong \angle D$

So... $\triangle ABC \sim \triangle DEF$ by SAS

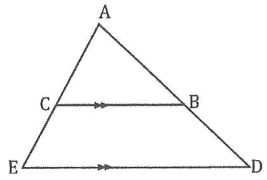
I can't stress enough that the angles that are congruent must be BETWEEN the sides that have equal ratios. If not, you get ASS and its backward evil twin SSA, and they don't work for similarity either!

AAA and AA go together. If all three angles of one triangle are congruent to all three angles of another then the triangles are similar. But what about AA? Well, if you have two angles inside a triangle can you not solve for the third because their sum is 180°? Of course you can! So having 2 angles is the same as having three angles for similarity. I like to always use AA but sometimes you will see people use AAA. It's unnecessary, but I thought I would warn you. Let's see this one...



$\angle C \cong \angle F$
$\angle A \cong \angle D$
$\angle B \cong \angle E$

$\triangle ABC \sim \triangle DEF$ by AAA
or...
$\triangle ABC \sim \triangle DEF$ by AA



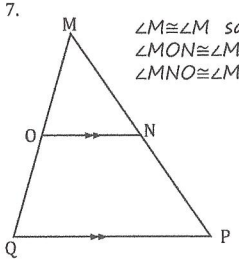
$\angle ABC \cong \angle ADE$ Corresponding Angles

$\angle ACB \cong \angle AED$ Corresponding Angles

$\triangle ABC \sim \triangle ADE$ by AA

To the similarity mobile! Let's practice...

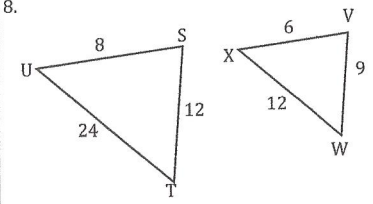
7.



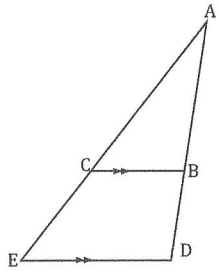
$\angle M \cong \angle M$ same angle
$\angle MON \cong \angle MQP$ corresponding
$\angle MNO \cong \angle MPQ$ corresponding

$\triangle MNO \sim \triangle MPQ$ by AAA
or...
$\triangle MNO \sim \triangle MPQ$ by AA

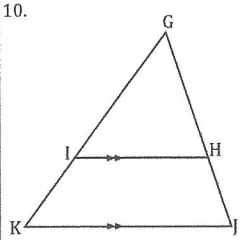
8.



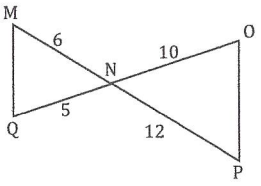
9.



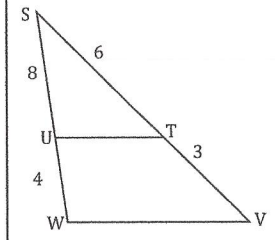
10.



11.



12.

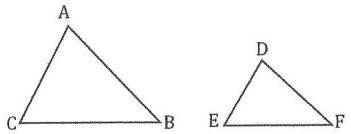


Bubble the correct answer choice from each item above.

○A.	○A.	○A.	○A.	○A.	○A.
○B.	○B.	○B.	○B.	○B.	○B.
○C.	○C.	○C.	○C.	○C.	○C.
○D.	○D.	○D.	○D.	○D.	○D.

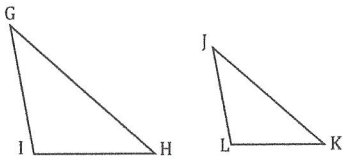
Choose the best choice.

25. $\frac{AB}{DF} = \frac{BC}{EF}$ What additional information is necessary to show $\Delta ABC \sim \Delta DEF$ by SSS?



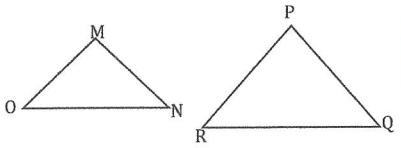
A. $\frac{AB}{AC} = \frac{BC}{EF}$
B. $\frac{AC}{DE} = \frac{BC}{EF}$
C. $\frac{AC}{DF} = \frac{AC}{EF}$
D. $\frac{CB}{AB} = \frac{EF}{DF}$

26. $\frac{GI}{JL} = \frac{IH}{LK}$ What additional information is necessary to show $\Delta GHI \sim \Delta JKL$ by SAS?



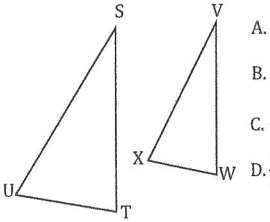
A. $\angle G \cong \angle J$
B. $\angle H \cong \angle K$
C. $\angle I \cong \angle L$
D. $\angle G \cong \angle H$

27. $\angle M \cong \angle P$. What additional information is necessary to show that $\Delta MNO \sim \Delta PQR$ by AA?



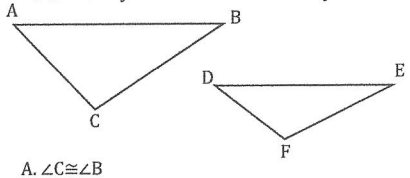
A. $\angle M \cong \angle O$
B. $\angle P \cong \angle R$
C. $\angle N \cong \angle P$
D. $\angle N \cong \angle Q$

28. $\frac{SU}{VX} = \frac{UT}{VW}$ What additional information is necessary to show $\Delta STU \sim \Delta VWX$ by SSS?



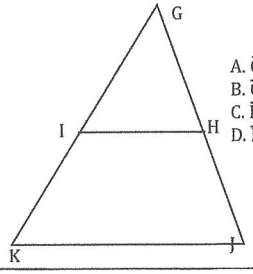
A. $\frac{ST}{VW} = \frac{SU}{VX}$
B. $\frac{ST}{VW} = \frac{SU}{ST}$
C. $\frac{SU}{ST} = \frac{ST}{SU}$
D. $\frac{VW}{XV} = \frac{VX}{XV}$

29. $\angle C \cong \angle F$. What additional information is necessary to show $\Delta ABC \sim \Delta DEF$ by AA?



A. $\angle C \cong \angle B$
B. $\angle E \cong \angle F$
C. $\angle A \cong \angle D$
D. $\angle E \cong \angle C$

30. What additional information is necessary to show $\Delta GHI \sim \Delta GJK$ by AA?

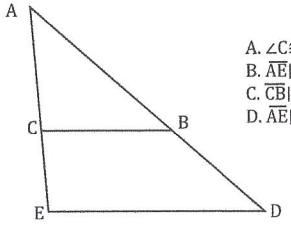


A. $\overline{GI} \cong \overline{GH}$
B. $\overline{GK} \| \overline{GJ}$
C. $\overline{IH} \cong \overline{KJ}$
D. $\overline{IH} \| \overline{KJ}$

Bubble the correct answer choice from each item above.

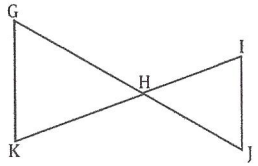
#25.	#26.	#27.	#28.	#29.	#30.
○A.	○A.	○A.	○A.	○A.	○A.
○B.	○B.	○B.	○B.	○B.	○B.
○C.	○C.	○C.	○C.	○C.	○C.
○D.	○D.	○D.	○D.	○D.	○D.

31. What additional information is necessary to show $\Delta ABC \sim \Delta ADE$ by AA?



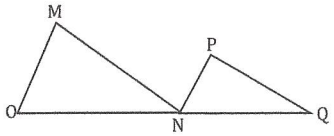
A. $\angle C \cong \angle A$
B. $\overline{AE} \| \overline{AD}$
C. $\overline{CB} \| \overline{ED}$
D. $\overline{AE} \| \overline{CB}$

32. What additional information is necessary to show $\Delta GHK \sim \Delta IHJ$ by AA?



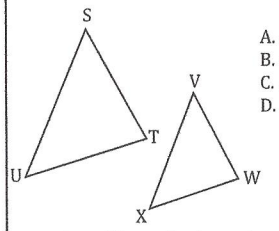
A. $\angle G \cong \angle IHJ$
B. $\angle K \cong \angle G$
C. $\overline{GK} \| \overline{IK}$
D. $\overline{GK} \| \overline{IJ}$

33. OM||NP. What additional information is necessary to show $\Delta MNO \sim \Delta PQN$ by SAS?



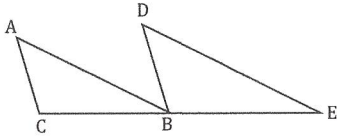
A. $\frac{MN}{PQ} = \frac{NO}{QN}$
B. $\frac{OM}{MN} = \frac{NP}{NQ}$
C. $\frac{OM}{NP} = \frac{MN}{PQ}$
D. $\frac{OM}{NP} = \frac{ON}{NQ}$

34. $\frac{ST}{VW} = \frac{TU}{WX}$ What additional information is necessary to show $\Delta STU \sim \Delta VWX$ by SAS?



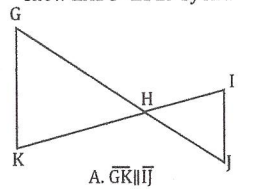
A. $\angle W \cong \angle V$
B. $\angle T \cong \angle W$
C. $\angle V \cong \angle S$
D. $\angle U \cong \angle X$

35. AB||DE. What additional information is necessary to show $\Delta ABC \sim \Delta DEF$ by SAS?



A. $\frac{AB}{DE} = \frac{BC}{EB}$
B. $\frac{AC}{DB} = \frac{AB}{DE}$
$\frac{BC}{EB} = \frac{AC}{DB}$
$\frac{AC}{AB} = \frac{BD}{DE}$

36. What additional information is necessary to show $\Delta ABC \sim \Delta DEF$ by AA?



A. $\overline{GK} \| \overline{IJ}$
B. $\angle K \cong \angle IJH$
C. $\overline{GK} \| \overline{GJ}$
D. $\overline{GH} \| \overline{JH}$

Bubble the correct answer choice from each item above.

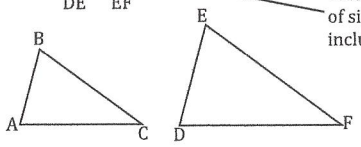
#31.	#32.	#33.	#34.	#35.	#36.
○A.	○A.	○A.	○A.	○A.	○A.
○B.	○B.	○B.	○B.	○B.	○B.
○C.	○C.	○C.	○C.	○C.	○C.
○D.	○D.	○D.	○D.	○D.	○D.

VI Proofs Involving Similar Triangles

We all look the same, don't we?

Okay, so if you can do proofs involving congruent triangles, then proving triangles are similar will be a piece of cake. Why? Because they are basically the same except we are proving that the triangles are exactly the same shape but different sizes. Remember for similarity we have the SSS, SAS, and AA (and AAA) theorems. There is a notation thing with similarity. The way that most proofs demonstrate that two pairs of sides have the same ratio is to write them as in the following example...

Given: $\frac{AB}{DE}=\frac{BC}{EF}$, $\angle B\cong\angle E$

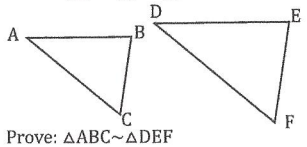


This means that $\overline{AB}$ and $\overline{DE}$ have the same ratio of similarity as $\overline{BC}$ and $\overline{EF}$. (Combined with the included angles gives us SAS similarity.)

Ready? Let's do some more analysis and then practice....

AE. 1.

Given: $\frac{AB}{DE}=\frac{AC}{DF}=\frac{BC}{EF}$



Prove: $\triangle ABC\sim\triangle DEF$

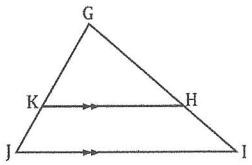
Statements	Reasons
1. $\frac{AB}{DE}=\frac{AC}{DF}=\frac{BC}{DF}$	1. Given
2. $\triangle ABC\sim\triangle DEF$	2. SSS

Analysis:

Working backward we must ask the key question. "How can we show two triangles are similar?" The answer? Use a similarity property such as SSS, SAS, or AA (AAA). That leads us to B1: by one of these properties. But which one? We need to start working forward. We see we have given AB/DE, AC/DF, and BC/EF. This gives us $\triangle ABC\sim\triangle DEF$ by SSS, which is B1, and the proof is complete!

AE. 2.

Given: JI||KH



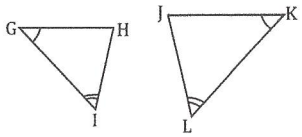
Prove: $\triangle JGI\sim\triangle KGH$

Statements	Reasons
1. JI||KH	1. Given
2. $\angle J\cong\angle GKH$	2. Corresponding Angles
3. $\angle I\cong\angle GHK$	3. Corresponding Angles
4. $\triangle JGI\sim\triangle KGH$	4. AA

Analysis:

Working backward we must ask the key question. "How can we show two triangles are similar?" The answer? Use a similarity property such as SSS, SAS, or AA (AAA). That leads us to B1: $\triangle JGI\sim\triangle KGH$ by one of these properties. But which one? We need to start working forward. Parallel lines... and when we see parallel lines we should look for corresponding angles or alternate interior angles. We see we have the corresponding angles $\angle J\cong\angle GKH$, and $\angle I\cong\angle GHK$. This gives us $\triangle JGI\sim\triangle KGH$ by AA, which is B1, and the proof is complete!

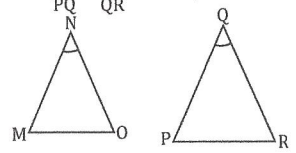
4. Given: $\angle G\cong\angle K$, and $\angle I\cong\angle L$



Prove: $\triangle GHI\sim\triangle KJL$

Statements	Reasons
1. $\angle G\cong\angle K$	1.
2.	2. Given
3. $\triangle GHI\sim\triangle KJL$	3.

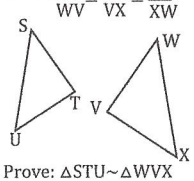
5. Given: $\frac{MN}{PQ}=\frac{NO}{QR}$, $\angle N\cong\angle Q$



Prove: $\triangle MNO\sim\triangle PQR$

Statements	Reasons
1. $\frac{MN}{PQ}=\frac{NO}{QR}$	1.
2.	2. Given
3. $\triangle MNO\sim\triangle PQR$	3.

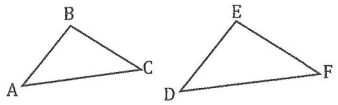
6. Given: $\frac{ST}{WV}=\frac{TU}{VX}=\frac{US}{XW}$



Prove: $\triangle STU\sim\triangle WVX$

Statements	Reasons
1.	1. Given
2.	2. SSS

7. Given: $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$

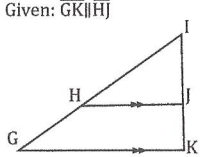


Prove: $\triangle ABC\sim\triangle DEF$

Statements	Reasons
1.	1. Given
2. $\triangle ABC\sim\triangle DEF$	2. SSS

8.

Given: $\overline{GK}\|\overline{HJ}$

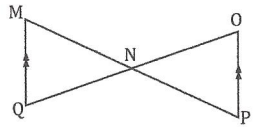


Prove: $\triangle GIK\sim\triangle HIJ$

Statements	Reasons
1.	1. Given
2.	2. Corresponding Angles
3. $\angle G\cong\angle JHI$	3.
4. $\triangle GIK\sim\triangle HIJ$	4.

9.

Given: $\overline{MQ}\|\overline{OP}$

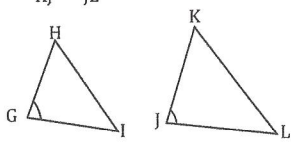


Prove: $\triangle MNQ\sim\triangle PON$

Statements	Reasons
1.	1. Given
2. $\angle QMN\cong\angle OPN$	2.
3.	3. Vertical Angles
4. $\triangle GIK\sim\triangle HIJ$	4.

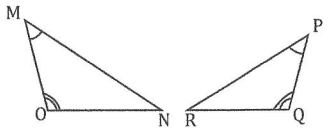
16.

Given: $\frac{GH}{KJ} = \frac{GI}{JL}$, $\angle G \cong \angle J$



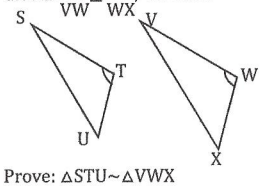
Prove: $\triangle GHI \sim \triangle JKL$

17. Given: $\angle M \cong \angle P$, $\angle O \cong \angle Q$



Prove: $\triangle OMN \sim \triangle DBC$

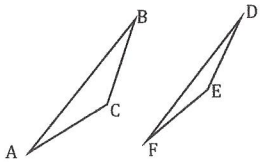
18.
Given: $\frac{ST}{VW} = \frac{TU}{WX}$, $\angle T \cong \angle W$



Prove: $\triangle STU \sim \triangle VWX$

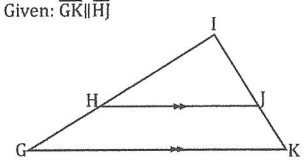
19.

Given: $\frac{AB}{FD} = \frac{BC}{DE} = \frac{CA}{EF}$



Prove: $\triangle ABC \sim \triangle FDE$

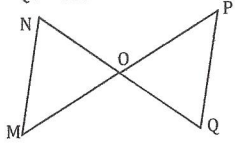
20.
Given: $\overline{GK} \| \overline{HJ}$



Prove: $\triangle GIK \sim \triangle HIJ$

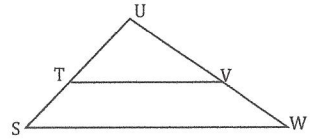
21.

Given: $\frac{NO}{QO} = \frac{PO}{MO}$



Prove: $\triangle MNO \sim \triangle PQO$

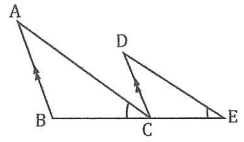
22.
Given: $\angle S \cong \angle UTV$



Prove: $\triangle SUW \sim \triangle TUV$

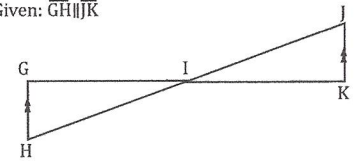
23.

Given: $\overline{AB} \| \overline{DC}$, $\angle ACB \cong \angle E$



Prove: $\triangle ABC \sim \triangle DCE$

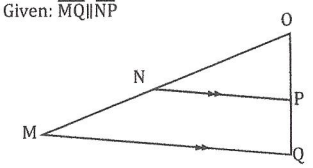
24.
Given: $\overline{GH} \| \overline{JK}$



Prove: $\triangle GHI \sim \triangle KJI$

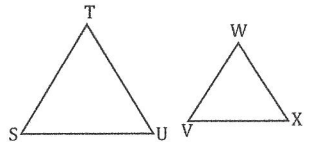
25.

Given: $\overline{MQ} \| \overline{NP}$



Prove: $\triangle QMO \sim \triangle PNO$

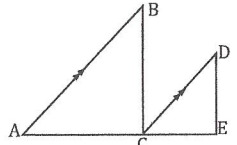
26.
Given: $\triangle ABD$ and $\triangle BCD$ are equilateral



Prove: $\triangle STU \sim \triangle VWX$

27.

Given: $\frac{AB}{DC} = \frac{AC}{CE}$, $\overline{AB} \| \overline{CD}$



Prove: $\triangle ABC \sim \triangle CDE$