

TRANSFORMATIONS OF TRANSFORMATIONS

NOTES

We have been studying four types of transformations. Each type can be identified by looking at a specific part of the function equation. The table below lists each type of transformation and identifies where to look.

| FLIP | STRETCH/COMPRESSION | | HORIZONTAL | | VERTICAL | |
|---------------------------------------|--|---------------------------------|---|------------|--|-----------|
| Sign at the beginning of the equation | Number at the beginning of the equation. | | Number inside parentheses, absolute value bars or under the radical sign. | | Number outside the parentheses, absolute value bars or radical sign. | |
| – | Stretch $n > 1$ × | Compression $0 < n < 1$ × | Left + | Right – | Up + | Down – |

Let's look at the function $g(x) = -4(x - 2)^3 + 5$. Four transformations have been applied to the parent function $f(x) = x^3$.

| | | | |
|------|-------------|-------|----|
| Flip | Stretch | Right | Up |
| – | Factor of 4 | 2 | 5 |

By looking at specific parts of this equation we can determine that the function was flipped over the x-axis, stretched by a factor of 4, shifted right 2 and up 5. Now let's take the function $g(x) = -4(x - 2)^3 + 5$ and apply **more** transformations to it. For example, what would this equation look like if we wanted to flip it back over the x-axis, stretch by a factor of 2, shift it left 1 and down 2?

| | Flip | Str/Cmp | L/R | U/D |
|----------------------|--------|-----------------|-------------|------------|
| Where to look | Sign | Number in front | Inside | outside |
| Current Value | – | 4 | $(x - 2)^3$ | +5 |
| What to do | Change | Multiply by 2 | Add 1 | Subtract 2 |
| New value | + | 8 | $(x - 1)^3$ | +3 |

So the new equation is $p(x) = 8(x - 1)^3 + 3$

More Examples

| Original Equation $y = 4x^2 + 4$ | Transformations Flip over x axis, shift right 3 and down 2 | Parts Effected <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th></th> <th>F</th> <th>S/C</th> <th>L/R</th> <th>U/D</th> </tr> </thead> <tbody> <tr> <td>C</td> <td>+</td> <td>4</td> <td>0</td> <td>4</td> </tr> <tr> <td>T</td> <td>–</td> <td></td> <td>–3</td> <td>–2</td> </tr> <tr> <td>N</td> <td>–</td> <td>4</td> <td>–3</td> <td>2</td> </tr> </tbody> </table> | | F | S/C | L/R | U/D | C | + | 4 | 0 | 4 | T | – | | –3 | –2 | N | – | 4 | –3 | 2 | Transformed Equation $y = -4(x - 3)^2 + 2$ |
|--|--|--|-----|-----|-----|-----|-----|---|---|---------------|----|----|---|---|---------------|----|----|---|---|---------------|----|----|--|
| | F | S/C | L/R | U/D | | | | | | | | | | | | | | | | | | | |
| C | + | 4 | 0 | 4 | | | | | | | | | | | | | | | | | | | |
| T | – | | –3 | –2 | | | | | | | | | | | | | | | | | | | |
| N | – | 4 | –3 | 2 | | | | | | | | | | | | | | | | | | | |
| $y = \frac{1}{2}\sqrt{x + 1} - 2$ | Stretch by a factor of 4, left 5 and up 3. | <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th></th> <th>F</th> <th>S/C</th> <th>L/R</th> <th>U/D</th> </tr> </thead> <tbody> <tr> <td>C</td> <td>+</td> <td>$\frac{1}{2}$</td> <td>+1</td> <td>–2</td> </tr> <tr> <td>T</td> <td></td> <td>4</td> <td>+5</td> <td>+3</td> </tr> <tr> <td>N</td> <td>+</td> <td>2</td> <td>+6</td> <td>+1</td> </tr> </tbody> </table> | | F | S/C | L/R | U/D | C | + | $\frac{1}{2}$ | +1 | –2 | T | | 4 | +5 | +3 | N | + | 2 | +6 | +1 | $y = 2(x + 6)^2 + 1$ |
| | F | S/C | L/R | U/D | | | | | | | | | | | | | | | | | | | |
| C | + | $\frac{1}{2}$ | +1 | –2 | | | | | | | | | | | | | | | | | | | |
| T | | 4 | +5 | +3 | | | | | | | | | | | | | | | | | | | |
| N | + | 2 | +6 | +1 | | | | | | | | | | | | | | | | | | | |
| $y = - x - 7 $ | Flip over x axis, compression by factor of $\frac{1}{3}$ up 8 | <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th></th> <th>F</th> <th>S/C</th> <th>L/R</th> <th>U/D</th> </tr> </thead> <tbody> <tr> <td>C</td> <td>–</td> <td>1</td> <td>–7</td> <td>0</td> </tr> <tr> <td>T</td> <td>–</td> <td>$\frac{1}{3}$</td> <td>0</td> <td>+8</td> </tr> <tr> <td>N</td> <td>+</td> <td>$\frac{1}{3}$</td> <td>–7</td> <td>+8</td> </tr> </tbody> </table> | | F | S/C | L/R | U/D | C | – | 1 | –7 | 0 | T | – | $\frac{1}{3}$ | 0 | +8 | N | + | $\frac{1}{3}$ | –7 | +8 | $y = \frac{1}{3} x - 7 + 8$ |
| | F | S/C | L/R | U/D | | | | | | | | | | | | | | | | | | | |
| C | – | 1 | –7 | 0 | | | | | | | | | | | | | | | | | | | |
| T | – | $\frac{1}{3}$ | 0 | +8 | | | | | | | | | | | | | | | | | | | |
| N | + | $\frac{1}{3}$ | –7 | +8 | | | | | | | | | | | | | | | | | | | |

TRANSFORMATIONS OF TRANSFORMATIONS

PRACTICE PROBLEMS - Fill in the missing column.

| | Original Equation | Transformations | New Equation |
|----|-------------------------|---|-------------------------------|
| 1 | $f(x) = x^2$ | Stretch by a factor of 3 Left 3 Down 2 | |
| 2 | $y = \sqrt{x}$ | Flip over x axis Right 2 Up 1 Compression by factor of $\frac{2}{7}$ | |
| 3 | $y = x $ | | $y = -\frac{1}{2} x - 2 + 1$ |
| 4 | $y = 3\sqrt{x} + 2$ | Left 7 Down 3 Compression by factor of $\frac{2}{3}$ | |
| 5 | $f(x) = 0.5 x - 2 $ | Stretch by factor of 2 Right 2 Down 3 | |
| 6 | $g(x) = -x^3 - 2$ | Flip over x-axis Right 4 Up 2 | |
| 7 | $g(x) = -x^3 - 2$ | | $q(x) = (x - 3)^3 - 1$ |
| 8 | $f(x) = x - 1 + 3$ | | $p(x) = - x + 2 + 1$ |
| 9 | $y = 4\sqrt{x - 1} + 7$ | Stretch by a factor of 2 Flip over the x axis Left 2 Up 3 | |
| 10 | $f(x) = -(x + 3)^2 - 1$ | Right 10 Up 5 Stretch by a factor of 4 | |
| 11 | $g(x) = x^3 + 7$ | Flip over x-axis Right 10 Compression by a factor of .01 | |
| 12 | $y = 4 x + 2 $ | | $y = 2 x - 2 $ |