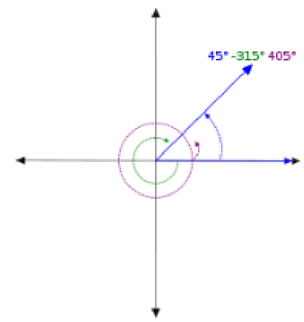
Math III Trigonometry Unit – Study Guide

Angles and Measure

<u>Angles in the Coordinate Plane</u>

Vertex is located at the origin
Initial Side is located on the positive x-axis
Terminal side rotates in either a counterclockwise (positive) or clockwise (negative) direction from the Initial Side.
Co-Terminal Angles share the same terminal side.
The angle represented to the left shows three angles that are Co-terminal, 45°, -315° and 405°



Radians VS Degrees	Convert the following to radians	Convert the following to degrees
When converting angle measures multiply the angle measure by $\frac{180}{\pi}$ when converting to degrees $\frac{\pi}{180}$ when converting to radians	1. 270° 3. 75°	2. $\frac{7\pi}{4}$ 4. $\frac{15\pi}{6}$

Unit Circle

<u>Construction</u>

Angles used in the circle are based on the common trianges 30-60-90 and 45-45-90.

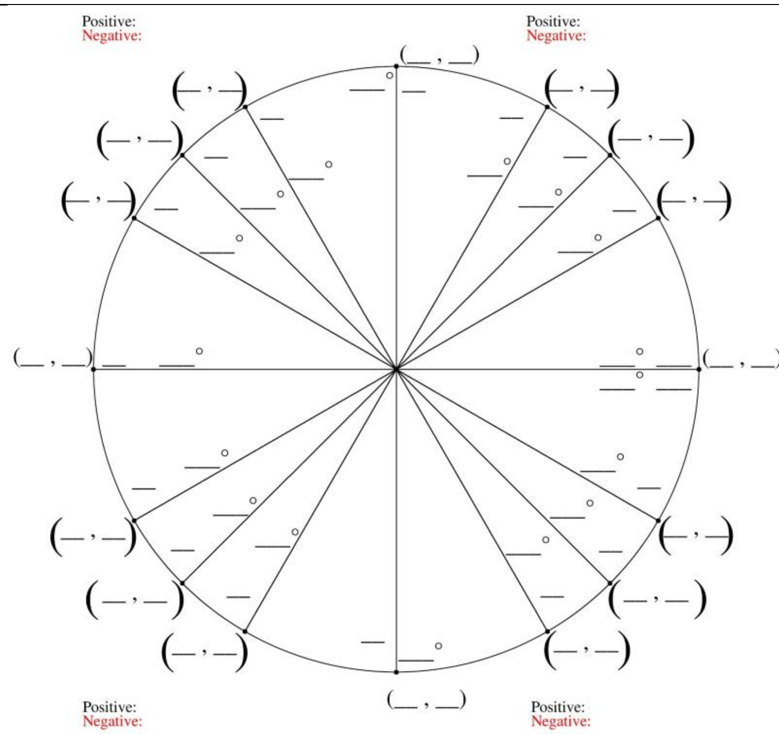
Write in Degree measures first
Convert each to radians

Fill in the coordinates 90° increments

Focus on the first quadrant first. Remember all the remaining coordinates are fractions with a denominator of 2.

Fill in the numerators by using the 123,123 method. Then take the square root of each numerator.

Now use the "boxes" to complete the rest of the circle.



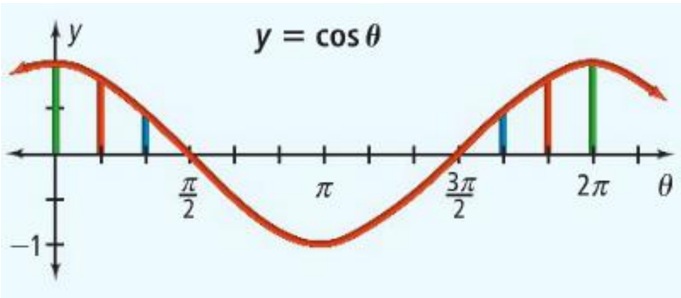
Using the Unit Circle		
$\cos\theta = x\ coordinate$ $\sin\theta = y\ coordiante$ $\tan\theta = \frac{y}{x}$	5. Find $\sin 60$ 7. Find $\cos -270$	6. Find $tan\frac{3\pi}{4}$ 8. Find $sin\frac{4\pi}{3}$

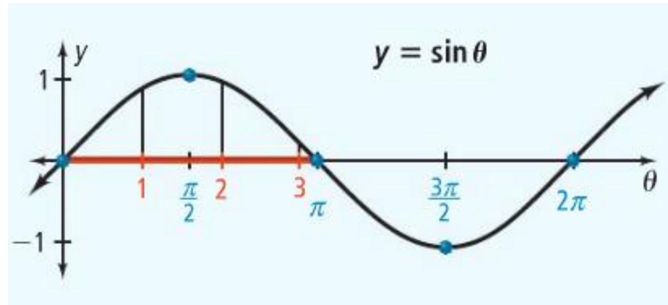
The Six Trigonometric Functions							
$\sin\theta$	$\frac{opposite}{hypotenuse}$	$\frac{1}{\csc\theta}$		$\csc\theta$	$\frac{hypotenuse}{opposite}$	$\frac{1}{\sin\theta}$	
$\cos\theta$	$\frac{adjacent}{hypotenuse}$	$\frac{1}{\sec\theta}$		$\sec\theta$	$\frac{hypotenuse}{adjacent}$	$\frac{1}{\cos\theta}$	
$\tan\theta$	$\frac{opposite}{adjacent}$	$\frac{1}{\cot\theta}$	$\frac{\sin\theta}{\cos\theta}$	$\cot\theta$	$\frac{adjacent}{opposite}$	$\frac{1}{\tan\theta}$	$\frac{\cos\theta}{\sin\theta}$

Pythagorean Identities	
$sin^2\theta + cos^2\theta = 1$	To derive the other two versions of the Pythagorean Identity first divide each term my $sin^2\theta$ and then simplify. Repeat the process for the third version by dividing each term by $cos^2\theta$.

Trig Expressions and Identities	
<u>Steps for Simplifying - </u>Objective: One trigonometric function. Convert all functions into sine and cosine using the identities from the table above. Try combining fractions that are added or subtracted into one fraction. Separate fractions with one term in the denominator into multiple fractions and simplify the smaller fractions. Look for opportunities to substitute $sin^2\theta + cos^2\theta$ for 1 in the expression. <u>Solving</u> – Objective, get both side the same Remember there is a brick wall between each side of the equation. Start with the more complicated side. Simplify the expression using the methods described above.	9. Simplify $\frac{\sin\theta}{\csc\theta}$ 10. Simplify $\sec\theta - \tan\theta\sin\theta$ 11. Verify the identity $cos^2\theta - sin^2\theta = 1 - 2sin^2\theta$ 12.Verify the identity $\frac{1+cos\theta}{\sin\theta} = \csc\theta + \cot\theta$ 13.Verify the identity $\frac{\tan\theta}{\sec\theta} + \frac{\cot\theta}{\csc\theta} = \sin\theta + \cos\theta$

Graphs of Trig Functions





Parent Function	**Transformed Function**
$y = \sin x$	$y = a \sin b(x - h) + k$
$y = \cos x$	$y = a \cos b(x - h) + k$

- $|a|$ = amplitude (vertical stretch or shrink)
- $\frac{2\pi}{b}$ = period (when x is in radians and $b > 0$)
- h = phase shift, or horizontal shift
- k = vertical shift

<u>Analyzing Trigonometric Equations</u>

Use the standard form of a sine/cosine function pictured to the left to determine specific components of the graph of the function.

For example find thecomponents of the function.

$$y = 3\sin 4(\theta - 1) + 7$$

$Amplitude = |3|,$ Note: since a is positive, there is no flip across the x axis.

$Period = \frac{2\pi}{4} = \frac{\pi}{2}$

$Phase\ Shift = 1, shift\ right\ 1\ unit$

$Vertical\ Shift = Up\ 7\ units$

Identify Amplitude, Period, Phase Shift and Vertical Shift for the following functions.

14. $sin2\theta$

15. $2\cos(x - 4) + 5$

16. $-\frac{1}{2}\sin \pi x - 1$

Write the equation for the given componets of the function.

17. sine curve with amplitude = 4, period = 2 and vertical shift up one unit.

18. cosine curve with amplitude = 1, period = 4π and flipped over the x axis.

19. sine curve with amplitude 3, period 2π, phase shift = -2 and vertical shift down 1.

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Analyzing Trigonometric Graphs

Amplitude is the half the distance between the max y value and the min y value.

Period is the distance on the x axis it takes for the function to repeat a full pattern. Remember the following patterns

Sine, no flip:	Zero, Max, Zero Min, Zero
Sine flipped:	Zero, Min, Zero, Max, Zero
Cosine, no flip	Max, Zero, Min, Zero, Max
Cosine, flipped	Min, Zero, Max, Zero, Min

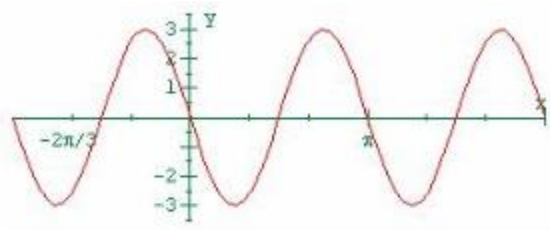
Phase shift, determine which function you are starting with, cosine or sine. Cosine normally starts at y=1 and Sine starts a y=0. Looking at the graph, determine if you have shifted left or right from the normal starting position. The opposite of the x value of the initial point is your phase shift (or horizontal shift)

Vertical Shift, normally the sine and cosine functions are centered around the x axis. To find the center line of a function subtract the max y value from the min y value. This will give you the value of your vertical shift.

To write the equation use the information determined above. If you have a flip remember to put a negative infront of the equation.

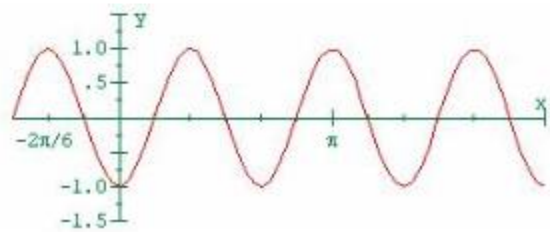
Write the equation for the following **sine** graphs.

20.



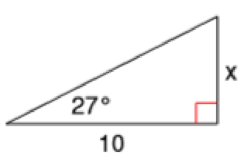
Write the equation for the following **cosine** graphs.

21.



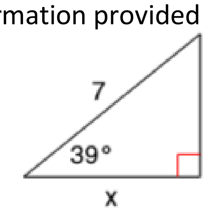
Using Trig functions to solve problems

If you have a right triangle, you can use trig functions to find missing parts of the traingle. Be sure to correctly identify the trig function you can use by the information provided and asked for. For example



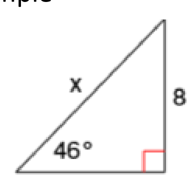
You have the **opposite** side and need the **adjacent** side. Use the **TANGENT** function. Solve the equation

$$\tan(27) = \frac{x}{10}$$
$$10\tan(27) = x$$



You have the **hypotenuse** and need the **adjacent** side. Use the **COSINE** function. Solve the equation

$$\cos(39) = \frac{x}{7}$$
$$7\cos(39) = x$$

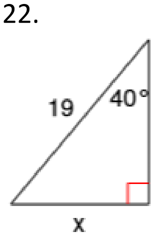


You have the **opposite** side and need the **hypotenuse**. Use the **SINE** function. Solve the equation

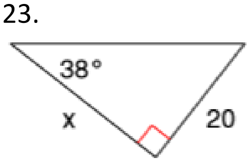
$$\sin(46) = \frac{8}{x}$$
$$x = \frac{8}{\sin(46)}$$

Find the value of x for each of the following triangles.

22.



23.



24.