Exponential Models

Clues in the word problems tell you which formula to use. If there's no mention of compounding, use a growth or decay model. If your interest is compounded, check for the word continuous. That's your clue to use the "Pert" Formula.

Simple Interest Growth	Simple Interest Decay	Compound Interest	Continuously Compounded Interest
$A(t) = a(1+r)^t$	$A(t) = a(1-r)^t$	$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$	$A(t) = Pe^{rt}$

$A(t)$	Amount after time t.	a	Initial amount	r	Rate expressed as a decimal
t	Time	n	Number of interest payments in one year	P	Initial investment

Growth Example baseball card bought for $150 **<u>increases in value</u>** at a rate of 3% each year. How much is the card worth in 10 years? $A = 150(1+.03)^{10}$	1.) The yellow bellied sapsucker has a population growth rate of approximately 4.7% If the population was 8,530 in 2000 and this growth rate continues, about how many yellow bellied sapsuckers will there be in 2006?
Decay You bought a new Ford truck for $40,000 yesterday. The truck **<u>depreciates</u>** a rate of 11% each year. How much is your truck worth 8 years from now? $A = 40000(1-.11)^8$	2.) Amy Farah Fowler bought a new car for $25,000. Suppose the car depreciates at a rate of 13% per year. How much will the car be worth in 4 years?
Compound Interest Your favorite Aunt gives you a quick pick. It's your lucky day! You win $1500. You give $500 to your Aunt and put the rest in a savings account that pays 3% interest **<u>compounded *monthly*</u>**. How much money will you have in 10 years? $A = 1000(1+\frac{.03}{12})^{(12)(10)}$	3.) If you put $2400 in an account that pays 6.2% interest compounded quarterly. How much will you have in eight years?
Continuous Compounding Your Aunt decides to deposit the$500 you gave her into a savings account at her bank. This account pays 3.5% interest and compounds **<u>continuously</u>**. How much money will she have in this account in 8 years? $A = 500e^{(.035)(8)}$	4.) If you put the same $2400 in an account that pays 5.7% interest compounded continuously. How much will you have in eight years?

Inverse Functions

To find the inverse of a function,

1. Switch x and y values
2. Solve for y

Inverse notation: $f^{-1}(x)$

Find the inverse of each function:

5.) $f(x) = 2x^2 - 8$

6.) $f(x) = \frac{x}{4} + 3$

For logs and exponents, put the equation in the "other form". Then switch x and y, solve for y.

$y=\log_4(16x)$	Find the inverse
$x=\log_4(16y)$	Switch x and y
$4^x=16y$	Put in exponent form
$4^x/16=y$	Solve for y
$4^{x-2}=y$	Simplify if possible

$y=4^x$	Find the inverse
$x=4^y$	Switch x and y
$y=\log_4 x$	Put in log form

7.) $f(x) = 3^{x-2}$

8.) $f(x) = \log(2x - 1)$

Definition of Logarithms

THE Relationship

$$If\ y = b^x, then\ log_b y = x$$

Write $6^2 = 36$ in log form

$log_6 36 = 2$

Write $log_2 64 = 6$ in exponential form

$2^6 = 64$

Write the following in log form:

9.) $6^2 = 36$

10.) $5^3 = 125$

11.) $2^4 = 32$

Write the following in exponential form:

12.) $\log_2 8 = 3$

13.) $\log_3 81 = 4$

14.) $\log_4 16 = 2$

Change of base formula

$$log_b x = \frac{\log x}{\log b}$$

Evaluate $log_6 32$

$$log_6 32 = \frac{\log 32}{\log 6} = 1.9343$$

Common log

$log_{10}\ is\ written\ as\ log$

Natural Log

$log_e\ is\ written\ as\ ln$

15.) Evaluate $log_2 8$

16.) Evaluate $log_{0.25} 0.0625$

Properties of Logarithms

PROPERTIES

$$log_b b = 1$$
$$lob_b 1 = 0$$

$$log_b mn = log_b m + lob_b n$$

$$log_b \frac{m}{n} = log_b m - log_b n$$

$$log_b m^n = n \, log_b m$$

To condense log statements, they must have the same base.

EX 1: <u>Condense</u> the following into one log statement.

$$3 \, log_4 x + 2 \, log_4 y$$

Step 1: Move the constants in front of the log statements into the exponent position.

$$log_4 x^3 + \, log_4 y^2$$

Step 2: Combine the arguments. Change subtraction to multiplication and addition to multiplication.

$$log_4 x^3 y^2$$

EX2: <u>Expand</u> the expression $log \frac{x}{yz^2}$

Step 1: Deal with the division operation first. Split the argument into two logs.

$$\log \, x - log \, yz^2$$

Step 2: Split any statements with multiplication into addition operations. Be sure to distribute the negative from the division.

$$\log \, x - (\log y + \log z^2)$$
$$\log \, x - \log y - \log z^2$$

Step 3: Move any exponents in front of the log statement.

$$\log \, x - \log y - 2 \log z$$

Condense the following Log Statements

17.) $\log_5 4 + \log_5 3$

18.) $\frac{1}{3} \log 3x + \frac{2}{3} \log 3x$

19.) $\log_3 2x - 5 \log_3 y$

20.) $\log_5 y - 4(\log_5 r + 2 \log_5 t)$

Expand the following Log Statements

21.) $\log 6x^3 y$

22.) $\log_2 \frac{x}{yz}$

23.) $\log \sqrt{\frac{2rst}{5w}}$

Solve Exponential and Logarithmic Equations		
To **solve** an **exponential equation**, take the log of both sides, and **solve** for the variable. To **solve** a **logarithmic equation**, rewrite the **equation** in exponential form and **solve** for the variable. Other helpful properties: $log_b b^x = x$ $b^{log_b x} = x$	Solve the equation $3^{x-2} + 5 = 74$. $3^{x-2} = 69.$ Subtract 5 from both sides. $\log(3^{x-2}) = \log 69$ Take the log of both sides $(x-2)\log 3 = \log 69$ Simplify the left side $x - 2 = \frac{\log 69}{\log 3}$ Evaluate logs $x - 2 = 3.85$ Solve for x $x = 5.85$	Solve the equation $log_2 4x = 5$ $4x = 2^5$ Put in exponential form. $4x = 32$ Simplify right side $x = 16$ Divide both sides by log 4.
Solve the following equations		
24.) $8^{n+1} = 3$	25.) $10^{3y} = 5$	26.) $4^x - 5 = 12$
27.) log (2x + 5) = 3	28.) log 4x = 2	29). 2 log (2x + 5) = 4

Sequences and Series (see last page for complete list of formulas)	
Determine if each sequence is arithmetic or geometric. Then find the 13st term in each sequence. 30). 9, 14, 19, 24…	 31). -1, 6, -36, 216, …
Evaluate the following series. 32.) 13, 15, …, 23	 33.)$\sum_{n=1}^{35}(5n-2)$
32.) A board is made up of 9 squares. A certain number of pennies is placed in each square following a geometric sequence. The first square has 1 penny, the second has 2 pennies, the third has 4 pennies, etc. When every square is filled, how many pennies will be used in total?	

Sequences

ARITHMETIC	GEOMETRIC
Sequences happen when you add numbers. The number added is called the **common difference**. $$d = a_n - a_{n-1}$$	Sequences happen when you multiply numbers. The number multiplied is called the **common ratio**. $$r = \frac{a_n}{a_{n-1}}$$
Explicit Formula of a basic arithmetic sequence $$a_n = a_1 + (n-1)d$$ Where n is the number of the term in the sequence and d is the **common difference**.	**Explicit Formula** of a basic geometric sequence $$a_n = a_1 \times (r^{n-1})$$ Where n is the number of the term in the sequence and r is the **common ratio**.
Recursive Formula of an arithmetic sequence $$a_n = a_{n-1} + d$$ Where n is the number of the term in the sequence and d is the **common difference**.	**Recursive Formula** of an geometric sequence $$a_n = r \times (a_{n-1})$$ Where n is the number of the term in the sequence and d is the **common ratio**.

Series

A series is the sum of the terms in a sequence.

Explicit Formula for the partial sum of an arithmetic sequence $$S_n = \frac{n}{2}(a_1 + a_n)$$	Explicit Formula for the partial sum of a geometric sequence $$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$ $$S_n = \frac{a_1 - a_n(r^n)}{1-r}$$
To find the number of terms in a finite series $$n = \frac{a_n - a_1}{d} + 1$$	To find the number of terms in a finite series $$n = \frac{Log\left(\frac{a_n}{a_1}\right)}{Log(r)} + 1$$

Sigma Notation

The Greek letter sigma means to sum up. The example below is a simple summation.

$$\sum_{n=1}^{4} n = 1 + 2 + 3 + 4$$

When a series is expressed in sigma notation, we translate it into the explicit formula to calculate the sum.

$$\sum_{n=1}^{k} a_n = S_k = \frac{k}{2}(a_1 + a_k)$$	$$\sum_{n=1}^{k} a_n = S_k = a_1\left(\frac{1-r^k}{1-r}\right)$$

TI-84 Graphing Calculator

[2nd][STAT] Math [5] [STAT] Ops [5] *Expression* [,] *Variable*[,] *start*[,] *end*

Or for newer operating systems

[MATH] [0]