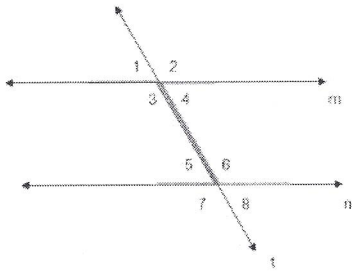
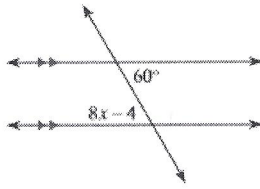


Lines, Transversals and Angles

Parallel

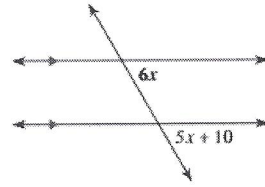


Find the angle measures. Create an equation setting appropriate angles equal to each other.



Use $8x - 4 = 60$ and solve for x.

1.



$$5x + 10 = 6x$$

$$10 = x$$

Congruent Angles

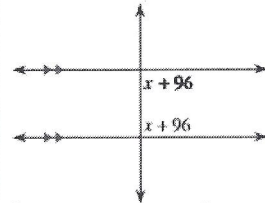
Vertical: 1 and 4, 2 and 3, 5 and 8, 6 and 7

Alternate Interior: 3 and 6, 4 and 5

Alternate Exterior: 2 and 7, 1 and 8

Corresponding: 2 and 6, 4 and 8, 1 and 5, 3 and 7.

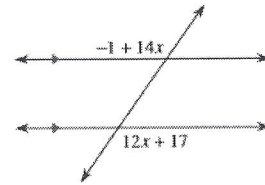
2.



$$(x + 96) + (x + 96) = 180$$

$$x = -6$$

3.



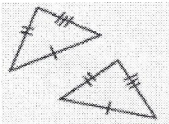
$$-1 + 14x = 12x + 17$$

$$2x = 18, x = 9$$

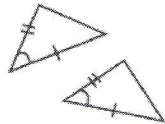
Congruent Triangles \cong

Triangles can be declared congruent if one of the following conditions are true.

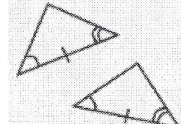
SSS



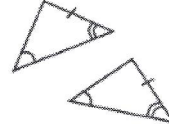
SAS



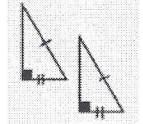
ASA



AAS

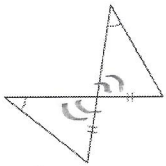


HL



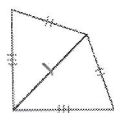
State which congruency theorem applies to the triangle pictured.

4.



AAS

5.



SSS

6.



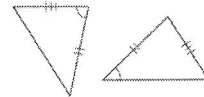
SAS

7.



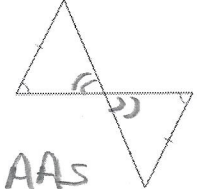
ASA

8.



NONE

9.

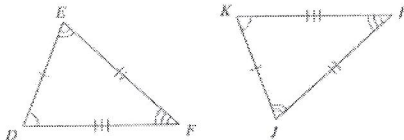


AAS

Write the congruency statement for the triangle pairs pictured below.

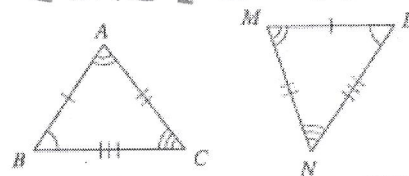
10.

$\triangle DEF \cong \triangle KJL$



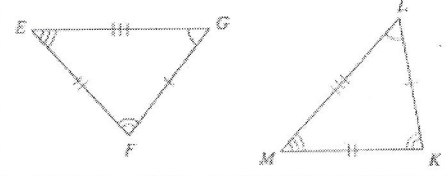
11.

$\triangle DAC \cong \triangle LMN$



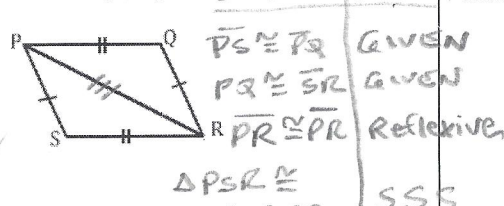
13.

$\triangle GFE \cong \triangle LKM$



14.

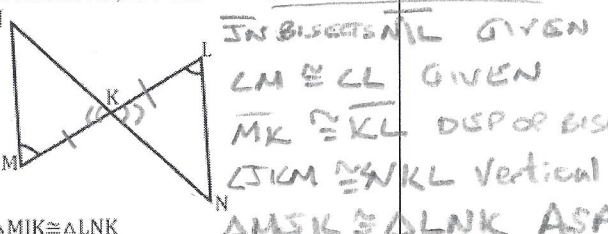
Given: $\overline{PS} \cong \overline{QR}$, $\overline{PQ} \cong \overline{SR}$ stmt reason



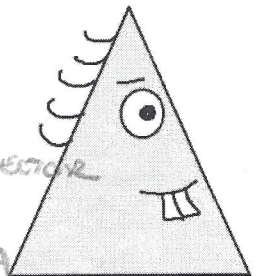
Prove: $\triangle PRS \cong \triangle RPQ$

15.

Given: \overline{JN} bisects \overline{ML} , $\angle M \cong \angle L$ stmt reason



Prove: $\triangle MJK \cong \triangle LNK$



Similar Triangles ~

Triangles are similar if their corresponding (matching) angles are congruent and the ratio of their corresponding sides are in proportion. The following Similarity Theorems can be used to prove similarity.

SSS, SAS and AAA (or AA)

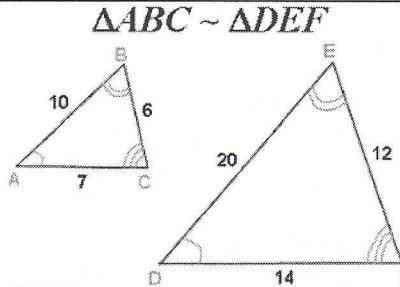
Facts about similar triangles:

$$\angle A \cong \angle D$$

$$\angle B \cong \angle E$$

$$\angle C \cong \angle F$$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

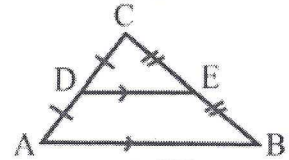


$$\frac{6}{12} = \frac{7}{14} = \frac{10}{20}$$

1 The ratio of the corresponding sides is called the ratio of similitude or scale factor.
2

similar

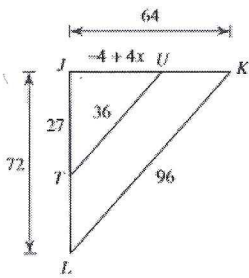
Mid-segment



D is the midpoint of \overline{AC} , E is the midpoint of \overline{BC} . The mid-segment \overline{DE} ; $\overline{DE} \parallel \overline{AB}$; $DE = \frac{1}{2}AB$.

Given the following sets of triangles are ~~congruent~~, find the value of x.

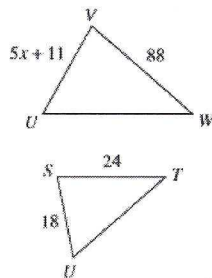
16.



$$\frac{-4 + 4x}{64} = \frac{27}{72}$$

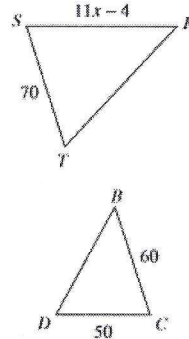
$$72(-4 + 4x) = 27(64)$$

17.



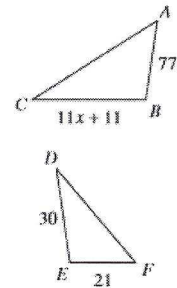
$$\frac{18}{5x + 11} = \frac{24}{88}$$

18.



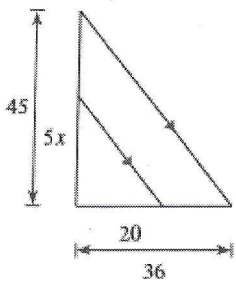
$$\frac{70}{50} = \frac{11x - 4}{60}$$

19.



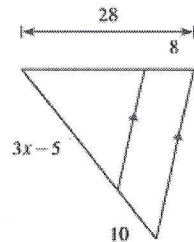
$$\frac{21}{77} = \frac{30}{11x + 11}$$

20.



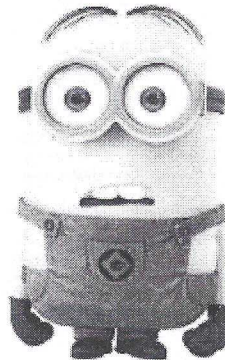
$$\frac{5x}{45} = \frac{20}{36}$$

21.

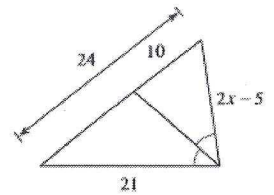


$$\frac{3x - 5}{(3x - 5) + 10} = \frac{10}{28}$$

22.



23.



$$\frac{2x - 5}{21} = \frac{10}{14}$$

CIRCLES: Angle Measures

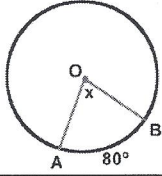
Central Angle = Intercepted Arc

$$m\angle AOB = m\widehat{AB}$$

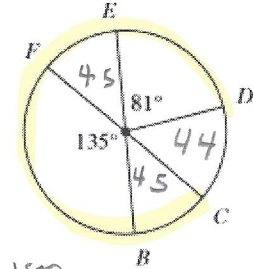
$\angle AOB$ is a central angle.

Its *intercepted arc* is the minor arc from A to B .

$$m\angle AOB = 80^\circ$$



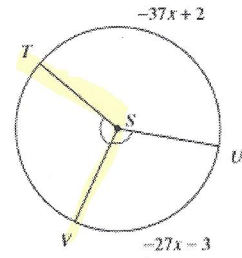
24. Find $m\widehat{CFD}$



$$\begin{array}{r} 180 \\ -135 \\ \hline 45 \end{array}$$

$$\begin{array}{r} 360 \\ -316 \\ \hline 44 \\ 2135 \\ 90 \\ \hline 81 \\ \hline 306 \end{array}$$

25.. Find $m\angle VST$



$$2(-27x-3) - 37x + 2 = 360$$

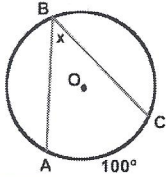
Inscribed Angle = $\frac{1}{2}$ Intercepted Arc

$$m\angle ABC = \frac{1}{2} m\widehat{AC}$$

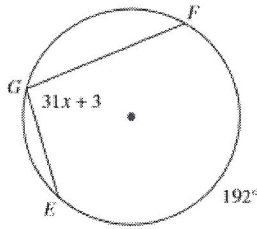
$\angle ABC$ is an inscribed angle.

Its *intercepted arc* is the minor arc from A to C .

$$m\angle ABC = 50^\circ$$

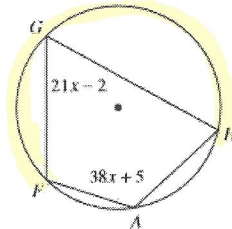


26. Find x



$$31x + 3 = \frac{1}{2}(192)$$

27. Find $m\widehat{FGH}$



$$2(38x+5) + 2(21x-2) = 360$$

Tangent Chord Angle =

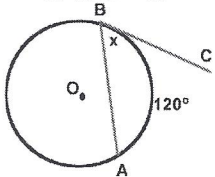
$\frac{1}{2}$ Intercepted Arc

$$m\angle ABC = \frac{1}{2} m\widehat{AB}$$

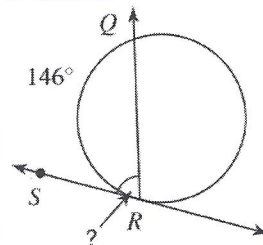
$\angle ABC$ is an angle formed by a tangent and chord.

Its *intercepted arc* is the minor arc from A to B .

$$m\angle ABC = 60^\circ$$

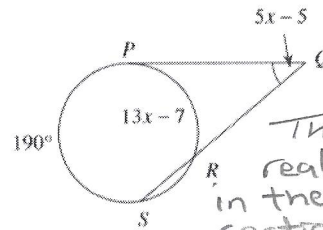


28. Find the indicated angle measure.



$$\angle SRQ = \frac{1}{2}(146)$$

29. Solve for x .



This problem really belongs in the next section.

$$5x - 5 = \frac{1}{2}(190 - 13x - 7)$$

Angle Formed Inside by Two Chords =

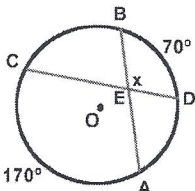
$\frac{1}{2}$ Sum of Intercepted Arcs

$$m\angle BED = \frac{1}{2}(m\widehat{AC} + m\widehat{BD})$$

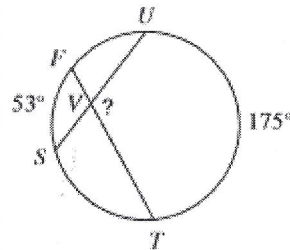
$$m\angle BED = \frac{1}{2}(70 + 170) = \frac{1}{2}(240) = 120^\circ$$

also, $m\angle CEA = 120^\circ$ (vertical angle)

$m\angle BEC$ and $m\angle DEA = 60^\circ$ by straight line.

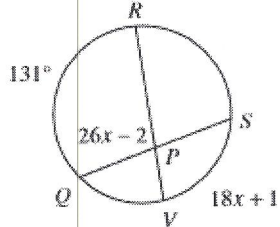


30. Find the indicated angle.



$$m\angle UVT = \frac{1}{2}(175 + 53)$$

31. Find $m\angle QPR$



$$20x - 2 = \frac{1}{2}(131 + 18x + 1)$$

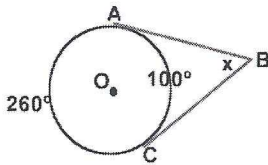
The formulas for all THREE of these situations are the same:
Angle Formed Outside = $\frac{1}{2}$ Difference of Intercepted Arcs
 (When subtracting, start with the larger arc.)

Two Tangents:

$\angle ABC$ is formed by two tangents intersecting outside of circle O .

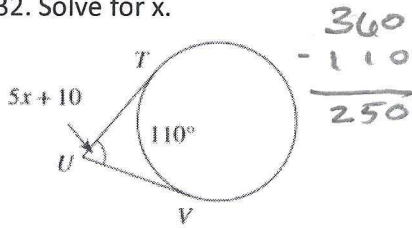
The *intercepted arcs* are minor arc \widehat{AC} and major arc \widehat{AC} . These two arcs together comprise the entire circle.

$$m\angle ABC = \frac{1}{2}(260 - 100) = 80^\circ$$



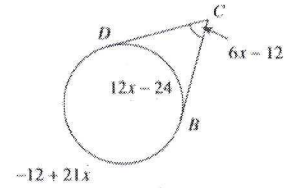
$$m\angle ABC = \frac{1}{2}(m\widehat{AC}_{\text{major}} - m\widehat{AC}_{\text{minor}})$$

32. Solve for x .



$$5x + 10 = \frac{1}{2}(250 - 110)$$

33. Find $m\angle BD$



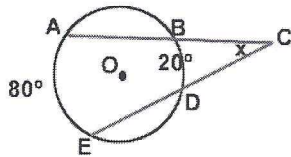
$$6x - 12 = \frac{1}{2}(-12 + 21x - 12x + 24)$$

Two Secants:

$\angle ACE$ is formed by two secants intersecting outside of circle O .

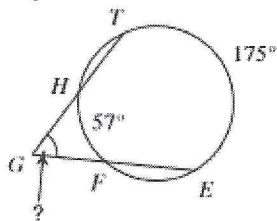
The *intercepted arcs* are minor arcs \widehat{BD} and \widehat{AE} .

$$m\angle ACE = \frac{1}{2}(80 - 20) = 30^\circ$$



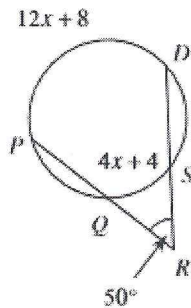
$$m\angle ACE = \frac{1}{2}(m\widehat{AE} - m\widehat{BD})$$

34. Find the measure of the indicated angle.



$$x = \frac{1}{2}(175 - 57)$$

35. Solve for x .



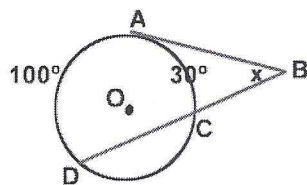
$$50 = \frac{1}{2}(12x + 8 - 4x - 4)$$

a Tangent and a Secant:

$\angle ABD$ is formed by a tangent and a secant intersecting outside of circle O .

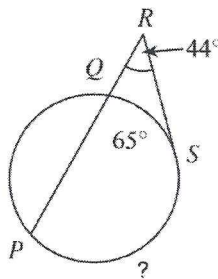
The *intercepted arcs* are minor arcs \widehat{AC} and \widehat{AD} .

$$m\angle ABD = \frac{1}{2}(100 - 30) = 35^\circ$$



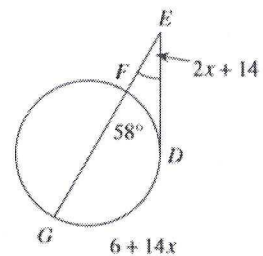
$$m\angle ABD = \frac{1}{2}(m\widehat{AD} - m\widehat{AC})$$

36. Find $m\widehat{PS}$



$$44 = \frac{1}{2}(x - 65)$$

37. Find $m\angle DEG$

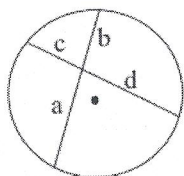


$$2x + 14 = 6 + 14x - 58$$

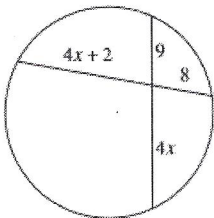
CIRCLES: Chord and Segment Lengths

Intersecting Chords Rule:
 (segment piece) × (segment piece) =
 (segment piece) × (segment piece)

$$a \cdot b = c \cdot d$$

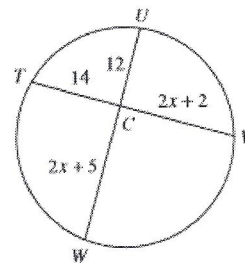


38. Solve for x.



$$9(4x) = 8(4x+2)$$

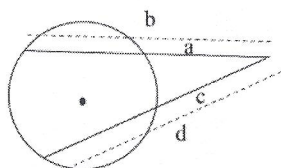
39. Find UW



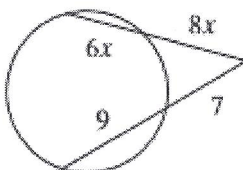
$$14(2x+5) = 12(2x+2)$$

Secant-Secant Rule:
 (whole secant) × (external part) =
 (whole secant) × (external part)

$$a \cdot b = c \cdot d$$

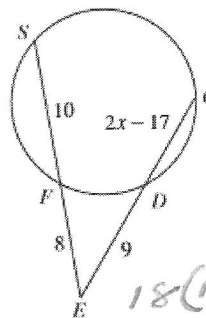


40. Solve for x.



$$(6x+8x)8x = 16(7)$$

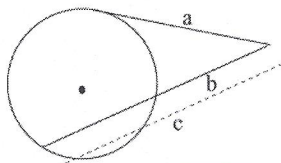
41. Find CE



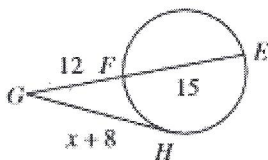
$$18(10) = (2x-17+9)(9)$$

Secant-Tangent Rule:
 (whole secant) × (external part) =
 (tangent)²

$$b \cdot c = a^2$$

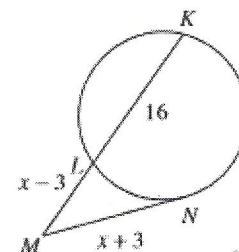


42. Find HG



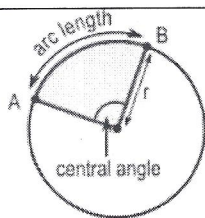
$$(x+8)^2 = (27)(12)$$

43. Find NM



$$(x+3)^2 = (x-3+16)(x-3)$$

Parts of a Circle



Two circle formulas you should already know:

$$\text{Circumference} = 2\pi r$$

$$\text{Area} = \pi r^2$$

An **ARC** is a portion of the circumference of a circle. It can be referred to by its length or measure. Arcs with a measure less than 180° are minor arcs. Arcs with a measure greater than 180° are considered the major arc.

ARC MEASURE is equal to the central angle that intercepts the ARC.

ARC LENGTH is a portion of the circumference of a circle.

$$L = \frac{n}{360} 2\pi r$$

AREA OF A SECTOR: Portion of the area of a circle.

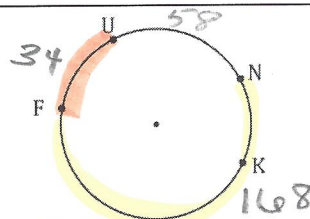


$$A = \frac{n}{360} \pi r^2$$

where n is the number of degrees in the central angle of the sector.

44. Given $m\widehat{FKN} = 168^\circ$, $m\widehat{UF} = 34^\circ$.
 Find $m\widehat{UN}$

$$360 - 168 - 34 = 58$$



45. Find the area of a sector with a central angle of 60° and a radius of 10.

$$\frac{60}{360} (\pi 10^2)$$

46. Find the area of a sector with an arc length of 75. 1.74 or radius

$$4. \dots$$

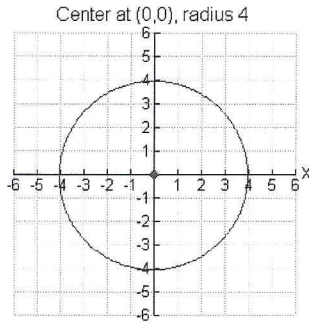
Equations of Circles

Circle whose center is at the origin

Equation: $x^2 + y^2 = r^2$

Example: Circle with center (0,0), radius 4
 $x^2 + y^2 = 16$

Graph:

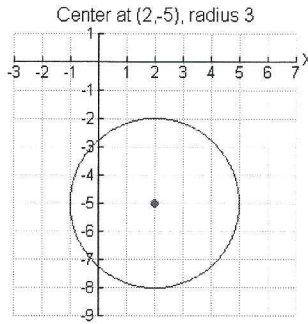


Circle whose center is at (h,k)
(This will be referred to as the "center-radius form". It may also be referred to as "standard form".)

Equation: $(x - h)^2 + (y - k)^2 = r^2$

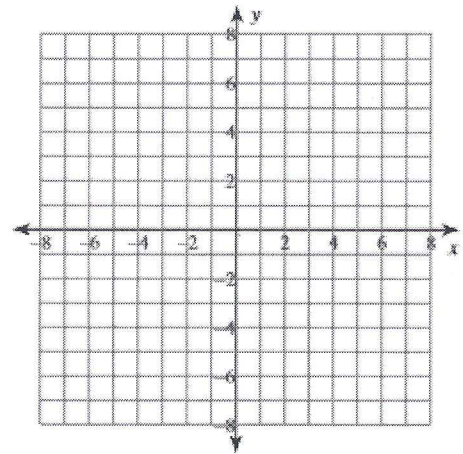
Example: Circle with center (2,-5), radius 3
 $(x - 2)^2 + (y + 5)^2 = 9$

Graph:



47. Find the center and radius of this circle. Then graph.

$$(x - 1)^2 + (y + 4)^2 = 9$$



Finding Equations of Circles

Writing the equation of a circle from the endpoint of the diameter; find the center by using the midpoint formula, find the radius by using the distance formula. Substitute this information into the formula for a circle.

Example: Find the equation of the circle whose diameter has endpoints (-6,7) and (4,-1)

Find the center by using the midpoint formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-6 + 4}{2}, \frac{7 + (-1)}{2} \right)$$

$$= (-1, 3)$$

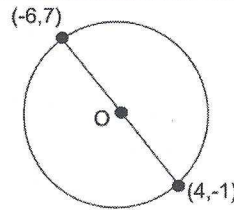
Find the radius by using the distance formula.

Points (-6,7) and (-1,3) were used here. (d = distance, or radius)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

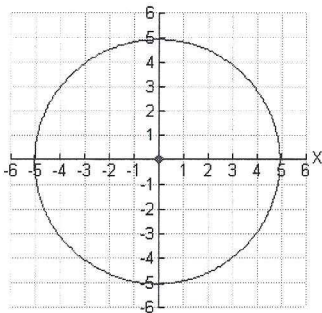
$$= \sqrt{(-6 - (-1))^2 + (7 - 3)^2}$$

$$= \sqrt{(-5)^2 + (4)^2} = \sqrt{25 + 16} = \sqrt{41}$$



48. Write the equation of a circle whose endpoints of a diameter are (18, -13) and (4,-3).

Example: Write the equation for the circle shown below if it shifted 3 units to the right and 4 units up.



A shift of 3 units to the right and 4 units up places the center at the point (3, 4). The radius of the circle can be seen from the graph to be 5.

Equation: $(x - 3)^2 + (y - 4)^2 = 25$

49. Write the equation of the circle pictured at the left if it is shifted left 7 and down 4.

50. Write the equation of a circle whose center is the point (-13, -16) with a point on the circle at (-10, -16).