I. Function Characteristics

Domain: *Interval* of possible x values for a given function. (Left,Right)

Range: *Interval* of possible y values for a given function. (down, up)

End Behavior: What is happening at the far ends of the graph?

For each side	Left side	Right side
	$x \to -\infty$,	$x \to \infty$
Pick one of these	Points Down	Points Up
	$y \to -\infty$	$y \to \infty$

Increasing Intervals: *Interval* of x values for which the corresponding y values are increasing.

Decreasing Intervals: *Interval* of x values for which the corresponding y values are decreasing.

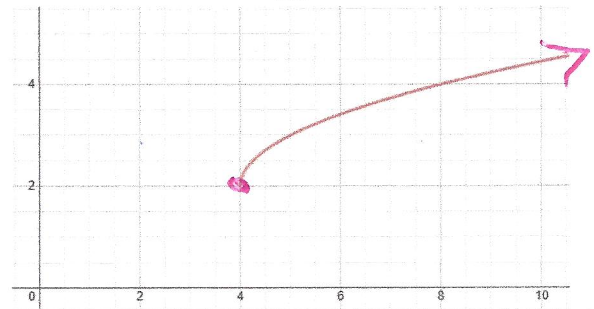
x-Intercepts: *points* where the graph crosses the x axis. $(x, 0)$

y-Intercepts: *points* where the graph crosses the y axis. $(0, y)$

Maximums: *points* where the graph changes from increasing to decreasing. Peaks in the graph.

Minimums: *points* where the graph changes from decreasing to increasing. Valleys in the graph.

E1

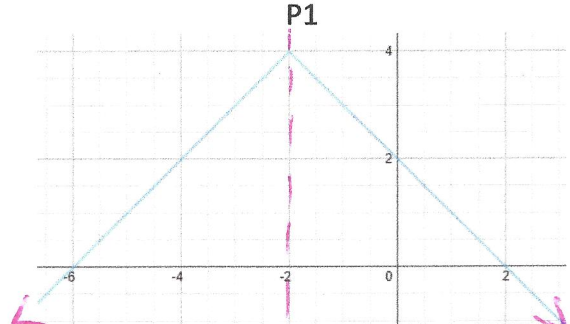


Domain: $[4, \infty)$
Range: $[2, \infty)$

Increasing Intervals: $[4, \infty)$
Decreasing Intervals: *None*

End Behavior:
As $x \to -\infty, y \to 2$
As $x \to \infty, y \to \infty$
x-Intercepts: *None*
y-Intercepts: *None*
Maximums: *None*
Minimums: *None*

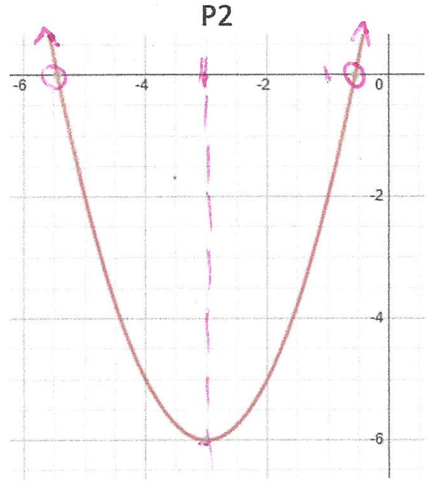
P1



Domain: $(-\infty, \infty)$
Range: $(-\infty, 4]$
Increasing Intervals: $(-\infty, -2)$
Decreasing Intervals: $(-2, \infty)$

End Behavior:
As $x \to -\infty$, $y \to -\infty$
As $x \to \infty, y \to -\infty$
x-Intercepts: $(-6, 0), (2, 0)$
y-Intercepts: $(0, 2)$
Maximums: $(-2, 4)$
Minimums: none

P2



Domain: $(-\infty, \infty)$
Range: $[-6, \infty)$
Increasing Intervals: $(-3, \infty)$
Decreasing Intervals: $(-\infty, -3)$
End Behavior:
As $x \to -\infty$, $y \to \infty$
As $x \to \infty$, $y \to \infty$
x-Intercepts: $(-5.5, 0), (-.5, 0)$
y-Intercepts: not shown
Maximums: none
Minimums: $(-3, -6)$

Graphing a function from an equation

1. Identify the parent function to determine a general shape.

absolute value

2. Think about where the vertex or critical points are usually found for the parent function.

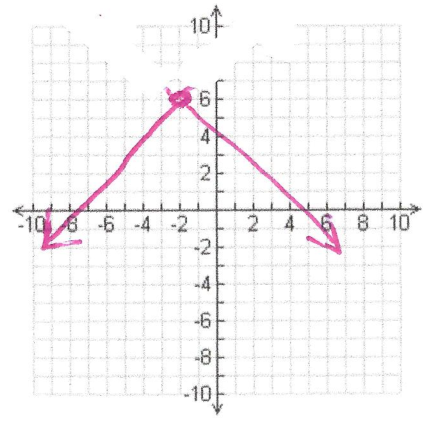
vertex at $(0,0)$

3. Where are the critical points of the new function given the transformations in the equation? L2, U6

$(-2, 6)$ · flip

4. Plot your critical points and sketch in the graph.

P6. Graph $f(x) = -|x+2| + 6$



Writing Function Equation from a description of the transformations

How do translations effect the function equation?

$f(x) = -a(x-h) + k$

"-" flip over x axis

a compression or stretch

h horizontal shift in the opposite direction of the sign

k vertical shift in the same direction of the sign

E6

Write the equation for a quadratic function with a vertical shift down 3, left 7 and a vertical stretch by a factor of 4.

Quadratic : x^2

Down 3: subtract 3 on the "outside"

Left 7: add 7 to x (inside)

V. stretch by 4: multiply the "x part" by 4

$$y = 4(x+7)^2 - 3$$

P7. Write the equation for an absolute value function that has been shifted down three units and left 17 units.

$f(x) = |x+17| - 3$

P8. Write the equation for a Quadratic function that has been flipped vertically, shifted up 5 units, and shifted right 2 units.

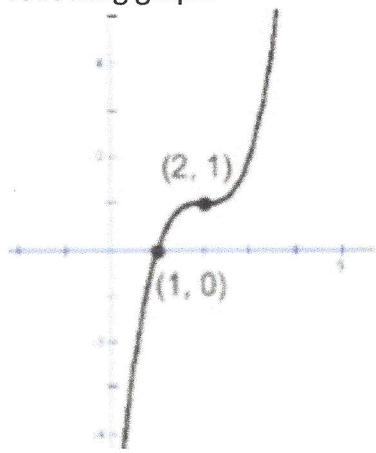
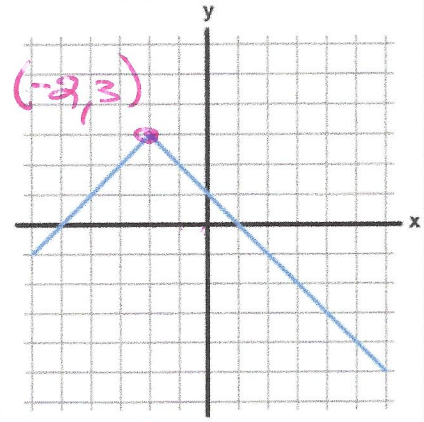
$f(x) = -(x-2) + 5$

P9. Write the equation for a square root function that has been shifted down 11 units, shifted left 5 units, and stretched by a factor of 2.

$f(x) = 2\sqrt{x+5} - 11$

P10. Write the equation for an absolute value function that has been compressed by a factor of ~~2~~ $\frac{1}{2}$ and shifted down three units.

$f(x) = \frac{1}{2}|x| - 3$

Steps for Determining Equation from Graph	**E7.** Write the equation for the following graph.	**P9.** Write the equation for the following graph
What's the parent function? *Has the same shape as a cubic function* Where's the vertex or critical point of the <u>parent</u> function? (0,0) Where's the vertex or critical point of <u>this</u> function? (2,1) How did we get from the parent function critical point to the critical point of this function? *Right 2 and up 1* How do I translate those changes into an equation? *Horizontal changes go with the x* *Vertical changes go outside the x*	 $y = (x-1)^3 + 2$	 parent: absolute value vertex went from (0,0) to (-2,3) ⇒ L2 U3 is also flipped $y = -\|x+2\|+3$

Shifts of Shifts	
In these types of problems, you start with a function that has already been shifted around and now you're going to apply some more shifts. **E8.** $f(x) = (x+1)^2 - 1$, If this function is shifted left 3 units, up 2 units, flipped vertically and stretched by a factor of 4 what is the resulting function equation?	**P10.** $f(x) = 2(x)^3 + 4$ If this function is shifted <u>up 2</u>, <u>right 1</u> and ~~compressed~~ stretched by a factor of <u>6</u> what is the resulting equation? $f(x) = \boxed{2}\boxed{(x\ )^3} + \boxed{4}$ $\boxed{\times 6}\ \boxed{-1}\ \ +\ \boxed{2}$ $= 12(x-1)^3 + 6$ **P11.** $f(x) = -\|x-5\|$ If this function is <u>shifted up 4</u>, <u>left 3</u>, <u>stretched</u> by a factor <u>of 2</u>, and <u>flipped</u> vertically, what is the resulting equation? $f(x) = \boxed{-1}\|x-5\|$ multiply $\boxed{-2}$ -3 $+4$ $= 2\|x-8\|+4$

Transformation what to do

Left 3	add 3 to the number "with x"	$1 + 3 = 4$
Up 2	add 2 to the number "outside" of x	$-1 + 2 = 1$
Flipped vertically	flip the sign in front of the equation	change to −
Stretched by 4	Multiply the number in front by 4	$1(4) = 4$

Resulting function: $g(x) = -4(x+4)^2 + 1$

for compressions, divide

II. Function Transformations

General form: $g(x) = a\,f(x-h)+k$

f(x) parent function
g(x) transformed function

a if negative, flip vertically

$0 < |a| < 1$ vertical compression
$|a| > 1$ vertical stretch

h if negative, horizontal shift right
if positive, horizontal shift left

k if negative, vertical shift down
if positive, vertical shift up

Examples

$E1.\; g(x) = x^2 + 2$
Parent Function:
quadratic
Transformations:
shift up 2 units

E2. $g(x) = -(x-4)^3 - 1$
Parent Function:
cubic
Transformations:
flip vertically
shift right 4 units
shift down 1 unit

E3. $g(x) = 3\sqrt{x+1} - 7$
Parent Function:
Radical (square root)
Transformations:
Stretch by a factor of 3
Shift left 1 unit
Shift down 7 units

E4. $g(x) = -\frac{1}{2}(x-3)^2 + 1$
Parent Function:
quadratic
Transformations:
Flip vertically
Compression by a factor of $\frac{1}{2}$
Shift Right 3 units
Shift up 1 unit

P3. $g(x) = 2^{x-3} + 5$
Parent Function:
exponential
Transformations:
R 3
U 5

P4. $g(x) = -(x+7)^2$
Parent Function:
quadratic
Transformations:
flip
L 7

P5. $g(x) = 2\log(x-2) - 1$
Parent Function:
log
Transformations:
R 2
D 1
Stretch factor 2

III. Graphing a function from an equation - Example

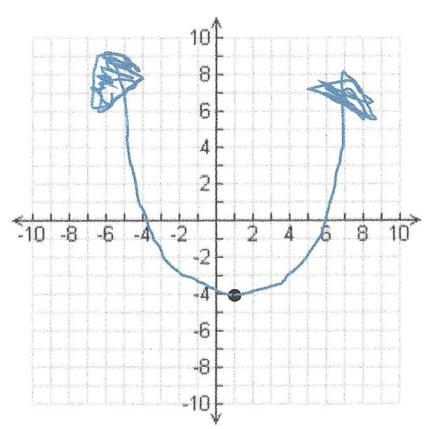
1. Identify the parent function to determine a general shape.
<u>*Quadratic*</u>

2. Think about where the vertex or critical points are usually found for the parent function.
<u>*Centered at the origin. Shaped like the letter U.*</u>

3. Where are the critical points of the new function given the transformations in the equation? <u>*Since there is a horizontal shift right 1 unit and a vertical shift down four units, the vertex is at the point (1,-4).*</u>

4. Use the location of the critical points to sketch the new graph.

E5. Graph $f(x) = (x-1)^2 - 4$



Shifts of Shifts part 2

In this type of problem you have to identify the transformations that would change one function equation to another.

E9. What transformations would change the function equation $f(x) = -3\sqrt{x-4} + 1$ to $g(x) = 27\sqrt{x+5} + 7$

	Original	New	How to get there
Horizontal	-4	$+5$	$+5 - (-4) = 9$ Left 8
Vertical	$+1$	$+7$	$+7 - (+1) = 6$ Up 6
Flip	$-$	$+$	Signs Changed Vertical Flip
Compression or Stretch	3	27	$27 \div 3 = 9$ Stretch factor of 9

P12. What transformations would change the function equation $f(x) = -3(x-1)^2 - 3$ to $g(x) = -(x+4)^2 - 5$

	Original	New	How to get there
H	-1	+4	4-(-1)=5
V	-3	-5	-5-(-3)=2
F	–	–	no change
C/S	3	1	1÷3 = 1/3

L5, U2 compress by factor of ~~3~~ 1/3

P13. What transformations would change the function equation $f(x) = |x+2| - 3$ to $g(x) = -2|x+1| + 2$

	Original	New	How to get there
H	+2	+1	1-2 = -1
V	-3	+2	2-(-3)=5
F	+	–	sign change Vertical
C/S	1	2	2÷1=2

R1
U5
flip
stretch factor of 2