Definitions

A **RATIONAL FUNCTION** is a function that can be written as

$$f(x) = \frac{p(x)}{q(x)}$$

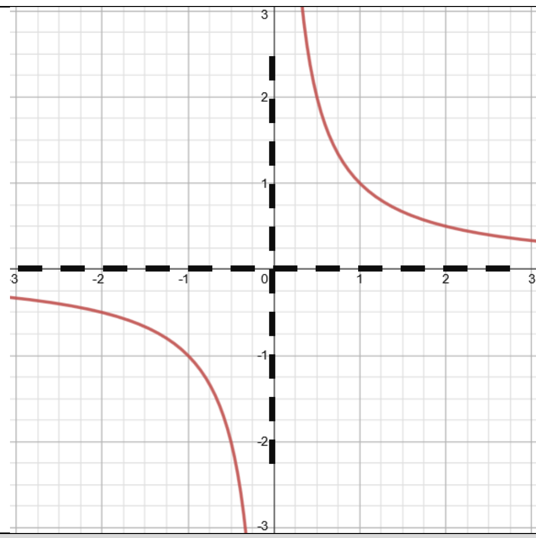
where p and q are polynomials.

Values of x that cause any factor of $q(x)$ to be equal to **ZERO** cause one of the following:

HOLES	**VERTICAL ASYMPTOTES**
Factors that are canceled out of both $p(x)$ and $q(x)$.	Invisible lines that the graph of $f(x)$ approaches but never reaches.

PARENT FUNCTION

$$\boldsymbol{f(x) = \frac{1}{x}}$$



Vertical Asymptote at the line $x = o$

Horizontal Asymptote at the line $y = 0$

Simplifying

Step 1: Factor the numerator and the denominator.
Step 2: Cancel out common factors of the numerator and denominator. Simplify.
Step 3: State any restrictions on the variable. (Values that would cause the original denominator to equal zero.)

Ex. 1 $\frac{v^2-7v-30}{v^2-5v-24}$ $= \frac{(v-10)(v+3)}{(v-8)(v+3)}$ $\frac{(v-10)\cancel{(v+3)}}{(v-8)\cancel{(v+3)}}$ $= \frac{(v-10)}{(v-8)}$ **Restrictions on** v. $v \neq 8 \text{ and } v \neq -3$	$\frac{x^2-16}{2x^2-9x+4}$	$\frac{a^2-8a}{3a^2-22a-16}$

Rationals **Study Guide**

Multiplying

Step 1: Factor all numerators and denominators.
Step 2: Cancel out common factors of the numerators and denominators.
Step 3: State any restrictions on the variable. (Values that would cause the original denominator to equal zero.)

Ex. 1 $\frac{v^2-7v-30}{v^2-5v-24} \times \frac{v+8}{4v-40}$

$$= \frac{(v-10)(v+3)}{(v-8)(v+3)} \times \frac{v+8}{4(v-10)}$$

$$= \frac{\cancel{(v-10)(v+3)}}{(v-8)\cancel{(v+3)}} \times \frac{v+8}{4\cancel{(v-10)}}$$

$$= \frac{1}{(v-8)} \times \frac{v+8}{4} = \frac{v+8}{4(v-8)}$$

$$\frac{v+8}{4v-32}$$

$\textit{Restrictions}$:
$v \neq 8, -3, 10$

$$\frac{c^2-6c-27}{5c^2+16c+3} \times \frac{2}{4c-36}$$

$$\frac{a-4}{a^2-16} \times \frac{a^2+5a+4}{a+1}$$

Dividing

Step 1: Flip the second fraction to change the division sign to multiplication.
Step 2: Factor all numerators and denominators.
Step 3: Cancel out common factors of the numerators and denominators. Simplify.
Step 4: State any restrictions on the variable. *Include those that were originally in the denominator before you flipped the fraction.*

Ex. 1 $\frac{b-5}{b^2-5b-6} \div \frac{b^2-25}{5b-30}$

$$= \frac{b-5}{b^2-5b-6} \times \frac{5b-30}{b^2-25}$$

$$= \frac{b-5}{(b+1)(b-6)} \times \frac{5(b-6)}{(b+5)(b-5)}$$

$$= \frac{\cancel{b-5}}{(b+1)\cancel{(b-6)}} \times \frac{5\cancel{(b-6)}}{(b+5)\cancel{(b-5)}}$$

$$= \frac{1}{(b+1)} \times \frac{5}{(b+5)}$$

$$= \frac{5}{(b+1)(b+5)} = \frac{5}{(b^2+6b+5)}$$

Restrictions:
$b \neq -1, -5, +5, 6$

$$\frac{x^2-1}{x+3} \div \frac{4x^2+7x+3}{16x^2+12x}$$

$$\frac{a-3}{6a} \div \frac{a^2-9}{a^2+5a+6}$$

Adding

Step 1: Factor all denominators.

Step 2: Find a common denominator. Multiply each term's numerator and denominator by the factors its denominator is missing from the other denominators.

Step 3: Simplify the numerators.

Step 4: Add the numerators. Simplify and create a single fraction.

Ex. 1 $\frac{b-5}{b^2-5b-6}+\frac{1}{5b-30}$

$$=\frac{b-5}{(b+1)(b-6)}+\frac{1}{5(b-6)}$$

$$=\frac{5(b-5)}{5(b+1)(b-6)}+\frac{1(b+1)}{5(b+1)(b-6)}$$

$$=\frac{5b-25}{5(b+1)(b-6)}+\frac{b+1}{5(b+1)(b-6)}$$

$$=\frac{6b-24}{5(b+1)(b-6)}$$

$$=\frac{6b-24}{5(b^2-5b-6)}$$

$$=\frac{6b-24}{5b^2-25b-30}$$

Restrictions: $b\neq-1 \quad b\neq6$

$$\frac{a-4}{a^2-1}+\frac{1}{a+1}$$

$$\frac{6a}{a+3}+\frac{3}{a^2+5a+6}$$

Subtracting

Same process as addition but be careful when you distribute the negative to the second fraction.

Ex. 1 $\frac{b-5}{b^2-5b-6}-\frac{1}{5b-30}$

$$=\frac{b-5}{(b+1)(b-6)}-\frac{1}{5(b-6)}$$

$$=\frac{5(b-5)}{5(b+1)(b-6)}-\frac{1(b+1)}{5(b+1)(b-6)}$$

$$=\frac{5b-25}{5(b+1)(b-6)}-\frac{b+1}{5(b+1)(b-6)}$$

$$=\frac{4b-26}{5(b+1)(b-6)}$$

$$=\frac{4b-26}{5(b^2-5b-6)}$$

$$=\frac{4b-26}{5b^2-25b-30}$$

$Restrictions$: $b\neq-1,\ b\neq6$

$$\frac{3}{x-2}-\frac{7x}{3x^2-5x-2}$$

$$\frac{3}{6a}-\frac{2a+1}{a^2+5a+6}$$

Solving

Step 1: Factor all denominators.

Step 2: Find a common denominator. Multiply each term's numerator and denominator by the factors its denominator is missing from the other denominators.

Step 3: Cancel the denominators and solve the remaining equation.

Ex. 1 $\frac{x}{x+2} - \frac{1}{x^2-4} = \frac{-3}{x-2}$

$$\frac{x}{x+2} - \frac{1}{(x+2)(x-2)} = \frac{-3}{x-2}$$

$$\frac{x(x-2)}{\cancel{x+2(x-2)}} - \frac{1}{\cancel{(x+2)(x-2)}} = \frac{-3(x+2)}{\cancel{x-2(x+2)}}$$

$$x(x-2) - 1 = -3(x+2)$$

$$x^2 - 2x - 1 = -3x - 6$$

$x^2 + x + 5 = 0$ **Graph: NO REAL SOLUTION**

$$\frac{3}{a} - \frac{1}{4a} = \frac{2}{3}$$

$$\frac{2}{x+2} - \frac{1}{x+3} = \frac{-3}{x^2+5x+6}$$

Work Problems

These types of problems involve situations such as two people working together to do something. You are usually told how long each person takes to complete the task individually. Then and you are asked how long it will take the two of them to complete the task working together. The TRICK: Think of the problem in terms of how much each person / machine / whatever does *in a given unit of time*.

Mr. Mealey can paint an entire house in twelve hours. Ms. Gerard can paint a house in eight hours. How long would it take the two painters together to paint the house?

Step 1: How much can each person do in a common amount of time?

Mr. Mealy can paint $\frac{1}{12}$ of a house in one hour.

Ms. Gerard can paint $\frac{1}{8}$ of a house in one hour.

Step 2: Find out how much they can do together in the common amount of time by adding the fractions created in step 1.

Together the painters can paint $\frac{1}{12} + \frac{1}{8}$ of a house in one hour.

Simplify:

$$\left(\frac{1}{3 \times 4} \times \frac{2}{2}\right) + \left(\frac{1}{2x4} \times \frac{3}{3}\right) = \frac{5}{24}$$

So together they can paint $\frac{5}{24}$ of a house in one hour.

Step 3: Multiply the ratio found in step 2 by the variable t and set that equal to the number of houses in question. Solving for t gives you the amount of time needed.

$$\frac{5}{24}t = 1$$

$$\left(\frac{24}{5}\right)\frac{5}{24}t = 1\left(\frac{24}{5}\right)$$

$$t = \frac{24}{5} = 4.8$$

So it would take 4.8 hours for them to paint one house working together. *(However, now that Mr. Mealey has someone to talk to it will probably take a lot longer. ☺)*

Phillip can mow a lawn in 2 hours. Jada can mow 2 lawns in 3 hours. How long would it take them to mow 10 lawns together?

I make $200 in the time you make $260. If you make $7 per hour more than me, how much money did we each make per hour?

Working alone, it takes Asanji 8 hours to dig a 10 ft by 10 ft hole. Brenda can dig the same home in 9 hours. How long would it take them if they work together.

Graphing Rational Functions

STEPS	EXAMPLE 1	EXAMPLE 2	EXAMPLE 3
	$f(x) = \dfrac{2x^2 - 6x + 4}{x^2 + x - 6}$	$f(x) = \dfrac{x^2 - 9}{(x+2)}$	$f(x) = \dfrac{x+3}{x^2 + 4x - 5}$
1. **SIMPLIFY** the function by factoring both numerator and denominator.	$\dfrac{2(x-1)(x-2)}{(x+3)(x-2)} = \dfrac{2(x+1)}{(x+3)}$	$\dfrac{(x+3)(x-3)}{(x+2)}$	
2. **HOLES** are identified by looking for factors that appear in both the numerator and denominator.	$x = 2$	$None$	$None$
3. **VERTICAL ASYMPTOTES (VA)** are found by looking at the denominator. Set the remaining factor(s) equal to zero and solve for x.	$x = 3$	$x = -2$	$x = 1, x = 0$
4**. HORIZONTAL ASYMPTOTES (HA)** are found by comparing the degree of the numerator and the denominator. *Top Heavy* – No HA *Equal* – divide the leading coefficient of the numerator by the leading coefficient of the denominator. *Bottom Heavy* – HA at the line $y = 0$	Equal $\dfrac{2}{1} = 2$ $y = 2$	Top Heavy, no HA	Bottom Heavy, HA $y = 0$
5. **ZEROS (X INTERCEPTS)** are determined by finding the x values that will cause the numerator to be zero. (If the numerator is equal to zero, the entire function will be zero.)	$(x-1) = 0$ $(x-2) = 0$ $(1{,}0), (2{,}0)$	$(x+3) = 0$ $(x-3) = 0$ $(-3{,}0), (3{,}0)$	$(x+3) = 0$ $(-3{,}0)$
6**. Y INTERCEPTS** are found by evaluating the function at the value $x = 0$.	$f(0) = \dfrac{4}{-6} = -\dfrac{2}{3}$ $(0, -\frac{2}{3})$	$f(0) = \dfrac{-9}{2} = -\dfrac{9}{2}$ $(0, -\frac{9}{2})$	$f(0) = \dfrac{3}{-5} = -\dfrac{3}{5}$ $(0, -\frac{3}{5})$
7. **SKETCH** the graph. Draw the vertical and horizontal asymptotes first. Plot the X and Y intercepts. Make a table of points. Be sure to include at least two points on each side of all vertical asymptotes. Use the table function on your calculator to find values. Connect the points and rough in your sketch.			

Graphing Rational Functions

STEPS	PROBLEM 1 $f(x) = \frac{3x^2 - 12}{x^2 + 7x + 10}$	PROBLEM 2 $f(x) = \frac{x^2 - 25}{(x + 2)}$	PROBLEM 3 $f(x) = \frac{x^2 + 7x + 10}{x^3 + 4x^2 - 5x}$
1. **SIMPLIFY** the function by factoring both numerator and denominator.			
2. **HOLES** are identified by looking for factors that appear in both the numerator and denominator.			
3. **VERTICAL ASYMPTOTES (VA)** are found by looking at the denominator. Set the remaining factor(s) equal to zero and solve for x.			
4**. HORIZONTAL ASYMPTOTES (HA)** are found by comparing the degree of the numerator and the denominator. *Top Heavy* – No HA *Equal* – divide the leading coefficient of the numerator by the leading coefficient of the denominator. *Bottom Heavy* – HA at the line $y = 0$			
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