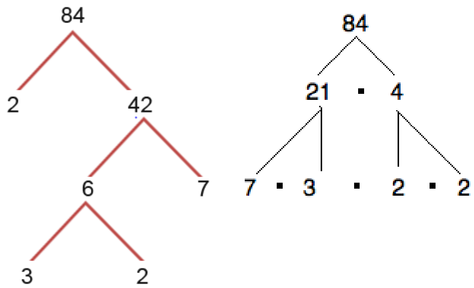
SIMPLIFYING RADICALS

EX: Simplify the expression $\sqrt{84x^4y^3}$

1.) Start by creating a factor tree for the constant. In this case 84. Keep factoring until all of your "nodes" are prime. Two factor trees are pictured below to show you that you will always end up with the same prime factors no matter how you begin your factoring.



2.) Identify any teams of 2. Each team can be brought out from under the radical. When you represent the team on the outside, only write the number once. All the other numbers must stay under the radical.

$$\sqrt{84} = 2\sqrt{7 \cdot 3} = 2\sqrt{21}$$

3.) The variables work the same way. We need teams of 2 to escape from the radical. $\sqrt{x^4y^3}$ can be written as $\sqrt{x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y}$. There are two teams of x and one team of y. So $\sqrt{x^4y^3} = x^2y\sqrt{y}$

4.) Bringing it all together our final answer is $2x^2y\sqrt{21y}$

1. $\sqrt{294}$

2. $\sqrt{36x^4}$

3. $\sqrt{162x^3y}$

4. $\sqrt{675x^6y^3z^7}$

COMPLEX NUMBERS

The imaginary number ***i*** is not actually imaginary. Remember that

$i = \sqrt{-1}$ and $i^2 = -1$

Imaginary numbers let you take the square root of a negative number. For example $\sqrt{-16} = 4i$ and $\sqrt{-24} = 2i\sqrt{6}$.

Complex numbers are written in the form $a + bi$ where a is the real part and bi is the imaginary part.

When dealing with complex numbers treat i like a variable when you combine like terms.

Don't forget to substitute -1 for i^2 when it pops up in your expression.

EX: Simplify the expression
$(-2 + 3i) + (5 - 2i)$

$$-2 + 3i + 5 - 2i = 3 + i$$

EX: Simplify the expression
$$(6 - 4i) - (4 + 5i)$$

$$6 - 4i - 4 - 5i = 2 - 9i$$

EX: Simplify the expression
$$(4 - 3i)(-5 + 4i)$$

Use FOIL or box method to multiply the binomials then gather like terms and simplify

$$\begin{aligned} -20 + 16i + 15i - 12i^2 & \\ &= -20 + 16i + 15i - 12(-1) \\ &= -20 + 16i + 15i + 12) \\ &= -8 + 31i \end{aligned}$$

1. $\sqrt{-49}$

2. $\sqrt{-108}$

3. $(1 + i) + (7 - 5i)$

4. $(1 - 6i) - (16 + 6i)$

5. $(2 - 3i)(1 + 2i)$

6. $3i(7 - 6i)$

Quadratics **Study Guide**

FACTORING A TRINOMIAL

EX: Factor the expression $6x^2 - 11x - 7$

STEP 1 Factor out the GCF if possible

STEP 2 Multiply a and c

a	c	$a \times c$
6	-7	-42

STEP 3 Find the two factors of $a \times c$ that add up to b.

Factors of $a \times c$		Sum of Factors
1	-42	-41
2	-21	-19
3	**-14**	**-11**
6	-7	-1

STEP 4 Construct two factors as follows where f_1 and f_2 are the two factors identified in step 3. If a is equal to 1 the two factors are unchanged.

$$(x + \frac{f_1}{a})(x + \frac{f_2}{a})$$

Then simplify the fractions if possible.

For our example the factors are

$$(x + \frac{3}{6})(x - \frac{14}{6})$$

Then simplified

$$(x + \frac{1}{2})(x - \frac{7}{3})$$

STEP 5 Swing the denominator of any remaining fractions in front of the x. This leaves us with

$$(2x + 1)(3x - 7)$$

CALCULATOR HINT: If you can't think of all the factors, enter the expression $a \times c/x$ into y_1. Now check the table. You now have a list of the factors of $a \times c$. You're only interested in the table entries in which x and y_1 are both integers.

1. $x^2 - 13x + 42$

2. $x^2 + 7x + 6$

3. $10x^2 + 7x - 6$

4. Find the length and width of a rectangle with an area of $A = 3x^2 - x - 10$.

DIFFERENCE OF SQUARES

A difference of squares is a quadratic of the form $(a^2 - b^2)$. This special case quadratic will always factor as follows:

$$(a^2 - b^2) = (a + b)(a - b)$$

EX: Factor the expression $(x^2 - 36)$
Since both x^2 and 36 are perfect squares the factored form is

$$(x^2 - 36) = (x + 6)(x - 6)$$

EX: Factor the expression $(9x^2 - 1)$

$$(9x^2 - 1) = (3x + 1)(3x - 1)$$

1. $9x^2 - 25$

2. $x^2 - 16y^2$

3. $1 - 9y^2$

4. $36x^2 - 18$

Quadratics **Study Guide**

SOLVE BY FACTORING

Put the quadratic equation in standard form and find the factors.

Set each factor equal to zero and solve.

Ex. $x^2 - 11x + 19 = -5$

$$+5 \quad +5$$

$$x^2 - 11x + 24 = 0$$

$$(x-8)(x-3) = 0$$

$$x - 8 = 0 \qquad x - 3 = 0$$

$$x = 8, \qquad x = 3$$

1. $a^2 + 5a + 6 = 0$

2. $k^2 - 10k + 22 = -2$

3. $2x^2 - x = 1$

4. The product of two consecutive negative integers is 1122. What are the numbers?

SOLVE BY GRAPHING

The graph of a quadratic equation crosses the x-axis at the real solutions for the equation. Use your calculator to find the exact values for the x-intercepts.

1.) Make sure the equation is in standard form $y = ax^2 + bx + c$
2.) Enter the equation in y_1
3.) Enter 0 in y_2
4.) Press [2nd] [TRACE] [5] – intersect
5.) Press [ENTER] [ENTER] move cursor near your guess and press [ENTER]
6.) Repeat from step 4 for your second root.

1. $a^2 - a - 6 = 0$

2. $x^2 + 4x - 1 = 0$

3. $2x^2 + 4x = 70$

4. Mr. Walsh's free throw is modeled by the equation

$$h(x) = -16x^2 + 20x + 6$$

where $h(x)$ represents the height of the ball and x represents the time in seconds after the ball is shot. When does it land?

SOLVE BY QUADRATIC FORMULA

Standard form first

$$y = ax^2 + bx + c$$

Plug into formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Simplify

Ex. $2x^2 + 3x - 4 = 0$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-4)}}{2(2)}$$

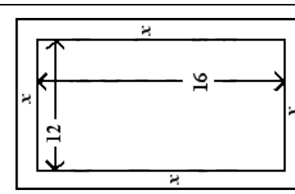
$$x = \frac{-3 \pm \sqrt{27}}{4}$$

$$x = \frac{-3 \pm 3\sqrt{3}}{4}$$

1. $3x^2 - 8x = 11$

2. $2x^2 - x = 1$

3. A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around it, increasing the total area to 285 square meters. What will be the width of the pathway?



4. $4k^2 + 25k - 21 = 0$

Quadratics **Study Guide**

COMPLEX ROOTS		
When the determinant (numbers under the radical) is negative, the solutions will be complex numbers. Ex. $4x^2 - 2x + 3 = 0$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(3)}}{2(4)}$ $x = \frac{2 \pm \sqrt{-44}}{8}$ $x = \frac{2 \pm 2i\sqrt{11}}{8}$ $x = \frac{1 \pm i\sqrt{11}}{4}$	1. $k^2 + 2k + 5 = 0$ 3. $k^2 - 3x + 5 = 0$	2. $2k^2 - 5k + 7 = 0$ 4. $2k^2 + 7x - 4 = 0$

SOLVING RADICAL EQUATIONS AND EXTRANEOUS SOLUTIONS		
Radical equations have a variable under the radical sign. To solve, isolate the radical term, square both sides to eliminate the radical and solve using any method. Check for extraneous solutions by plugging in your answers to the **original** equation. If it doesn't work, it's extraneous. Ex. $\sqrt{x-1} = x - 7$ $(\sqrt{x-1})^2 = (x-7)^2$ $x - 1 = (x-7)(x-7)$ $x - 1 = x^2 - 14x + 49$ $0 = x^2 - 15x + 50$ $0 = (x-5)(x-10)$ $x = 5, x = 10$ Now test each solution in the original equation. $\sqrt{5-1} = 5 - 7$ $\sqrt{4} = -2$ $2 = -2$ FALSE! Extraneous $\sqrt{10-1} = 10 - 7$ $\sqrt{9} = 3$ $3 = 3\ true$	1. $\sqrt{x-2} - 5 = 0$ 2. $\sqrt{c-5} = c + 1$	3. $b - 6 = \sqrt{18 - 3b}$

Quadratics **Study Guide**

WRITING EQUATIONS FROM ROOTS

Working backwards from the roots, create the equation of the quadratic function.

Start with the equations of the roots. If the root is a fraction, multiply both sides by the denominator.

Then add/subtract to get everything to the left side. Create factors from the left side and multiply together.

Ex. Find the equation for a parabola with the roots $x = -3, x = \frac{2}{3}$

$x = -3$ $\quad$ $x = \frac{2}{3}$

$x + 3 = 0$ $\quad$ $3x = 2$

$3x - 2 = 0$

$(x + 3)(3x - 2)$

$3x^2 + 9x - 2x - 6$

$3x^2 + 7x - 6$

$y = 3x^2 + 7x - 6$

1. $x = 2 \quad x = -4$

3. $x = \frac{1}{4} \quad x = -\frac{4}{3}$

2. $x = -1 \quad x = {}^{3}/_{5}$

4. A person dives off of a board into the water. She goes under 2 seconds after diving and resurfaces 4.5 seconds after diving. Write an equation to represent the time that she was underwater (no decimals).

QUADRATICS OF BEST FIT

Use data and the graphing calculator to create a quadratic equation that will model real world situations.

x-value	1	2	3
y-value	1	4	8

First enter your data
[STAT] [EDIT]
type x-values in L1
type y values in L2

Next create regression equation
[STAT][CALC][5:QuadReg] [VARS] [Y-VARS] [1:Function][Y1][ENTER]

To find an x-value,
Type the given y-value into $y_2 =$
Press [2nd] [TRACE] [5] – intersect
Press [ENTER] [ENTER] [ENTER]

To find a y-value
Press [2nd][TRACE][VALUE]
Type given x-value
Press [ENTER]

1. Use the table below to create a quadratic model. The answer the questions below.

x	-1	0	1	2	4
y	6	1	-2	2	21

What is the predicted value of x when $y = 3$?

What will y equal when $x = -2$?

2. A ball is thrown in the air. The table below represents the height of the ball after a number of seconds. Create a quadratic model from the data and answer the questions below.

time	0	1	3	4
height	0	13	100	200

What is the predicted height after 12 seconds?

At what time(s) would the object be 490 feet high?

Quadratics — Study Guide

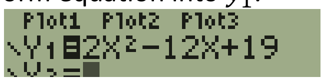
CONVERTING FROM STANDARD FORM TO VERTEX FORM

Standard Form: $ax^2 + bx + c = 0$
Vertex Form: $a(x-h)^2 + k = 0$, vertex is (h, k)

Method 1 – Use your calculator

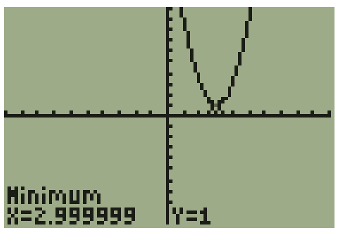
EX: Put the equation $y = 2x^2 - 12x + 19$ vertex form.

Put the standard form equation into y_1.



If a is negative the graph points down so you will find a **max**.
If a is positive the graph points up so you will find a **min**.

Use [2nd][TRACE][min/max]. Set a left bound and right bound. Press [ENTER]. The x/y values displayed are the coordinates of the vertex.



Note some calculators don't round up. If you see a value with a decimal portion of repeating 9's, round up to the next integer. In this case use (3,1) as the vertex

The value of a in the standard and vertex form and the same. Use the opposite of the x value for h and the y value for k.

Vertex form: $y = 2(x-3)^2 + 1$

Method 2 – Use the formula

EX: Put the equation $y = 2x^2 - 12x + 19$ vertex form.

The vertex of any quadratic (h, k), h can be found by the formula $h = \frac{-b}{2a}$. To find k plug in the h you calculate back in to the original equation.

$$h = \frac{-(-12)}{2(2)} = \frac{12}{4} = 3$$
$$k = 2(3^2) - 12(3) + 19 = 1$$

Using a from the original equation we get

$$y = 2(x-3)^2 + 1$$

Convert the following equations from standard form to vertex form. Use both methods.

1. $y = x^2 - 8x + 15$

2. $y = -2x^2 + 12x - 21$

3. $y = x^2 + 8x + 18$

Convert the following equations from vertex form to standard form.

1. $2(x-1)^2 - 3 = y$

2. $-(x+4)^2 + 1 = y$

Convert from Vertex Form to Standard Form

Use order of operations for expand the squared term, multiply the resulting terms by a and then add k.
EX: Convert $y = 2(x-3)^2 + 1$ to standard form.

$$y = 2[x^2 - 6x + 9] + 1$$
$$= 2x^2 - 12x + 18 + 1$$
$$= 2x^2 - 12x + 19$$