Quadratics KEY **Study Guide**

TRINOMIAL FACTORING

How? ax^2+bx+c
-Use x-factor to find two numbers that multiply to ac and add to b.
-Replace bx with the two answers from your x-factor (multiplied by x).
-Take the GCF of the first two terms and the GCF of the second two terms.
-Write your answer:
(what's in parenthesis)(what's not)

#1 $x^2-13x+42$

$(x-6)(x-7)$

#3 $10x^2+7x-6$

$a\cdot c = -60$

swing method $12(-5)=-60$, $12-5=7$

$(x+\frac{12}{10})(x-\frac{5}{10})$

$(x+\frac{6}{5})(x-\frac{1}{2})$

$(5x+6)(2x-1)$

Ex. $5x^2+19x+12$

AC Method

60
15 4
19

$5x^2+15x+4x+12$
$5x(x+3)+4(x+3)$
$(5x+4)(x+3)$

#2 $9x^2-25$

difference of squares

$\sqrt{9x^2}=3x$

$\sqrt{25}=5$

$(3x-5)(3x+5)$

#4 Find the length and width of a rectangle with an area of $A=3x^2-x-10$.

$A=L\cdot W$

$(x-\frac{6}{3})(x+\frac{5}{3})$ $a\cdot c=-30$

$(x-2)(3x+5)$ $-6\cdot 5=-30$, $-6+5=-1$

SOLVE BY FACTORING

How? $y=ax^2+bx+c$
-Get all terms on the same side.
-Factor.
-Set each factor equal to zero and solve.

#1 $a^2+5a+6=0$

$(x+2)(x+3)=0$

$(x+2)=0$ $(x+3)=0$

$x=-2$ $x=-3$

#3 $2x^2-x=1$

$2x^2-x-1=0$ $a\cdot c=-2$, $-2+1=-1$

$(x-\frac{2}{2})(x+\frac{1}{2})=0$

$(x-1)(2x+1)=0$ $x=1$, $x=-\frac{1}{2}$

Ex. $x^2-11x+19=-5$

$+5 \quad +5$

$x^2-11x+24=0$

$(x-8)(x-3)=0$

$x-8=0 \quad x-3=0$

$+8+8+3+3$

$x=8x=3$

#2 $k^2-10k+22=-2$

$k^2-10k+24=0$

$(k-6)(k-4)=0$

$k-6=0$ $k-4=0$

$k=6$ $k=4$

#4 The product of two consecutive negative integers is 1122. What are the numbers?

$-n(n+1)=1122$

$n^2-n=1122$

$n^2-n-1122=0$

$(n-34)(n+33)=0$

-33, -34

SOLVE BY GRAPHING

How? $y=ax^2+bx+c$
-Get the quadratic in standard form.
-Graph the equation in y1 on your calculator.
-Graph y2 = 0
-Press second ->trace -> intersect
- Enter -> Enter -> go to your guess -> enter
-Repeat for your second root.

or use 2nd trace zero

#1 $a^2-a-6=0$

$x=-2$ & $x=3$

#4 Mr. Walsh's free throw is modeled by the equation $h(x)=-16x^2+20x+6$, where $h(x)$ represents the height of the ball and x represents the time in seconds after the ball is shot. When does it land?

after 1.5 seconds

SOLVE BY QUADRATIC FORMULA

How? $y = ax^2 + bx + c$

-Get in standard form.

-Plug into quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

-Simplify

Ex. $2x^2 + 3x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{9 + 18}}{4}$$

$$x = \frac{-3 \pm \sqrt{27}}{4}$$

$$x = \frac{-3 \pm 3\sqrt{3}}{4}$$

#1 $3x^2 - 8x = 11$, $3x^2 - 8x - 11 = 0$

$\frac{8 \pm \sqrt{(-8)^2 - 4(3)(-11)}}{2(3)} = x$

$\frac{8 \pm 14}{6} = x$ $x = -1$, $x = 11/3$

#2 $2x^2 - x = 1$, $2x^2 - x - 1 = 0$

$\frac{1 \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)} = x$

$\frac{1 \pm 3}{4} = x$, $x = 1$, $x = -1/2$

#3 $4k^2 + 25k - 21 = 0$

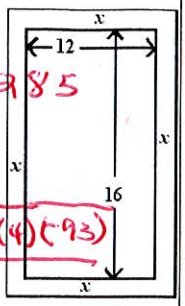
$\frac{-25 \pm \sqrt{(25)^2 - 4(4)(-21)}}{2(4)} = x$

$\frac{-25 \pm 31}{8} = x$

$x = -7$

$x = 3/4$

#4 A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around it, increasing the total area to 285 square meters. What will be the width of the pathway?



$(2x + 12)(2x + 16) = 285$

$4x^2 + 56x - 93 = 0$

$x = \frac{-56 \pm \sqrt{56^2 - 4(4)(-93)}}{2(4)}$

$x = \frac{12}{8}$, $\frac{-124}{8}$ 1.5 meters

COMPLEX ROOTS

Complex Numbers

Format: $a + bi$ where a is the real part and bi is the imaginary part.

$$i = \sqrt{-1}$$
$$i^2 = -1$$

Ex. $4x^2 - 2x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{2 \pm \sqrt{4 - 48}}{8} \quad x = \frac{2 \pm \sqrt{-44}}{8}$$

$$x = \frac{2 \pm 2i\sqrt{11}}{8}$$

$$x = \frac{1 \pm i\sqrt{11}}{4}$$

#1 $k^2 + 2k + 5 = 0$

$\frac{-2 \pm \sqrt{4 - 4(1)(5)}}{2(1)} = x$

$\frac{-2 \pm \sqrt{-16}}{2}$, $\frac{-2 \pm 4i}{2} =$

$-1 \pm 2i$

#2 $2k^2 - 5k + 7 = 0$

$\frac{5 \pm \sqrt{25 - 4(2)(7)}}{2(2)}$

$\frac{5 \pm \sqrt{-31}}{4}$, $\frac{5 \pm i\sqrt{31}}{4}$

#3 $k^2 - 3x + 5 = 0$

$\frac{3 \pm \sqrt{9 - 4(1)(5)}}{2(1)}$

$\frac{3 \pm \sqrt{-11}}{2}$, $\frac{3 \pm i\sqrt{11}}{2}$

#4 $2k^2 + 7x - 4 = 0$

$\frac{-7 \pm \sqrt{49 - 4(2)(-4)}}{2(2)}$

$\frac{-7 \pm \sqrt{81}}{4} = \frac{-7 \pm 9}{4}$

$x = -4, 1/2$

COMPLETE THE SQUARE TO FIND VERTEX FORM

How? $y = ax^2 + bx + c$
-Get in $y - c = ax^2 + bx$ form.
-Divide each term by a.
-add $\left(\frac{b}{2}\right)^2$ to both sides
-factor the right to $\left(x + \frac{b}{2}\right)^2$
-solve for y

Ex. $y = 2x^2 - 4x + 4$

$-4 \quad -4$

$y - 4 = 2x^2 - 4x$

$\div 2 \quad \div 2 \quad \div 2$

$\frac{y}{2} - 2 = x^2 - 2x$

$+1 \quad +1$

$\frac{y}{2} - 1 = x^2 - 2x + 1$

$\frac{y}{2} - 1 = (x-1)^2$

$+1 \quad +1$

$\frac{y}{2} = (x-1)^2 + 1$

$\cdot 2 \quad \cdot 2 \quad \cdot 2$

$y = 2(x-1)^2 + 2$

#1 $y = x^2 - 4x + 6$

$y - 6 = x^2 - 4x$

$y - 6 + 4 = x^2 - 4x + 4$

$y - 2 = (x-2)^2$

$y = (x-2)^2 + 2$

#2 $y = 2k^2 + 10k + 8$

$y - 8 = 2(k^2 + 5k)$

$y - 8 + \frac{50}{4} = 2(k^2 + 5k + \frac{25}{4})$

$y - \frac{18}{4} = 2(k + \frac{5}{2})^2$

$y = 2(k + \frac{5}{2})^2 + \frac{9}{2}$

note: $\frac{18}{4} = \frac{9}{2}$

#3 $2x^2 - 5x + y = 3$

$y - 3 = -2x^2 + 5x$

$y - 3 - \frac{50}{16} = -2(x^2 - \frac{5}{2}x + \frac{25}{16})$

$y - \frac{98}{16} = -2(x - \frac{5}{4})^2$

$y = -2(x - \frac{5}{4})^2 + 6\frac{1}{8}$

Suppose there is an arch that follows the equation

$F(x) = -\frac{1}{70}x^2 + 6x$.

How far apart are the ends of the arch?

graph + find zeros

420 ft apart

SOLVING RADICAL EQUATIONS THAT HAVE EXTRANEOUS SOLUTIONS

How? $\sqrt[a]{(x+b)^c} = x + d$
-Get rid of the radicals by raising to the reciprocal power (don't forget to distribute if squaring (x+d))
-solve for x by any method
-check for extraneous solutions by plugging in your answers to the original equation (if it doesn't work, its extraneous)

Ex. $\sqrt{x-1} = x - 7$

$\sqrt{x-1}^2 = (x-7)^2$

$x - 1 = (x-7)(x-7)$

$x - 1 = x^2 - 14x + 49$

$-x \quad +1 \qquad -x \quad +1$

$0 = x^2 - 15x + 50$

$0 = (x-5)(x-10)$

$x = 5, x = 10$

Now test each solution in the original equation.

$\sqrt{5-1} = 5 - 7$

$\sqrt{4} = -2$

$2 = -2$

FALSE! Extraneous

$\sqrt{10-1} = 10 - 7$

$\sqrt{9} = 3$

$3 = 3$ *true*

#1 $\sqrt{x-2} = 5$

$x - 2 = 25$

$x = 27$

#2 $\sqrt{c-5} = c + 1$

$c - 5 = c^2 + 2c + 1$

$0 = c^2 + c + 6$

imaginary solution

#3 $b - 6 = \sqrt{18 - 3b}$

$(b-6)^2 = 18 - 3b$

$b^2 - 12b + 36 = 18 - 3b$

$b^2 - 9b + 18 = 0$

$(b-3)(b-6) = 0$

① $3 - 6 = \sqrt{18 - 3(3)}$

$-3 = 3$ FALSE

$6 - 6 = \sqrt{18 - 3(6)}$

$0 = 0$ TRUE

WRITING EQUATIONS FROM ZEROES

How? $x = -3 \quad x = {}^{2}/_{3}$

-multiply then add or subtract to move everything to the left side.

-multiply the left sides together

Ex. $x = -3 \quad x = {}^{2}/_{3}$ +

3 ·3 ·3

$x + 3 = 0 \quad 3x = 2$

$-2 \quad -2$

$x + 3 = 0 \quad 3x - 2 = 0$

$(x + 3)(3x - 2)$

$3x^2 + 9x - 2x - 6$

$3x^2 + 7x - 6$

$y = 3x^2 + 7x - 6$

#1 $x = 2 \quad x = -4$

$(x-2)(x+4) = 0$

$x^2 + 4x - 8 = 0$

#2 $x = -1 \; x = {}^{3}/_{5} \; x = 1$

$(x+1)(x-\frac{3}{5})(x-1) = 0$

$(x-1)^2(5x-3) = 0$

#3 $x = \frac{1}{4} \quad x = -\frac{4}{3}$

$(x-\frac{1}{4})(x+\frac{4}{3}) = 0$

$(4x-1)(3x+4) = 0$

$12x^2 + 13x - 4 = 0$

#4 A person dives off of a board into the water. She goes under 2 seconds after diving and resurfaces 4.5 seconds after diving. Write an equation to represent the time that she was underwater (no decimals).

$x = 2, \quad x = 4\frac{1}{2} = {}^{9}/_{2}$

$(x-2) = 0 \quad (x-\frac{9}{2}) = 0$

$(x-2)(2x-9) = 0$

$2x^2 - 13x + 18 = 0$

QUADRATICS OF BEST FIT

How?

x-value	a	b	c
y-value	d	e	f

-stat -> edit -> type x-values in L1 and y-values in L2.

-stat->calc->quadreg->vars-> y-vars-> function-> y1 -> enter

-To find an x-value, type the given y-value into y2= and 2nd ->trace -> intersect ->enter ->enter->enter

-to find a y-value, 2nd -> trace -> value -> type given x-value -> enter

#1

x	-1	0	1	2	4
y	6	1	-2	2	21

What is x when y=3?

$x = 2.23$

What is y when x=-2?

$y = 16.66$

#3 $y = 14.5x^2 - 9.3x + 2.6$

time	0	1	3	4
height	0	13	100	200

What is the height after 12 seconds?

1979

At what time(s) would the object be 490 feet high?

6.127

FOCUS AND DIRECTRIX

How? $(y-k) = \frac{1}{4p}(x-h)^2$

-remember (h,k) is the vertex. P is the distance from the vertex to the focus.

-plug in your information or gather your information from this equation.

Ex. Write an equation for a parabola with a directrix of $y = -3$ and a focus of (5,7)

Vertex at (5,2) and $p = 7 - 2 = 5$

$(y-k) = \frac{1}{4p}(x-h)^2$

$(y-2) = \frac{1}{4(5)}(x-5)^2$

$(y-2) = \frac{1}{20}(x-5)^2$

#1 Find the focus, directrix, and vertex of

$(y-2) = \frac{1}{8}(x+3)^2$

$V = (-3, 2) \quad D: y = 0$

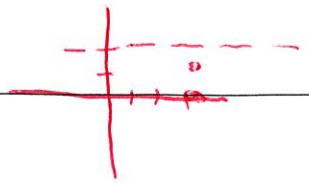
$F = (-3, 4)$

#2 Write an equation for a parabola with a directrix of $y = 2$ and a focus of (3,0)

$V: (3,1) \quad p = 1$

$a = \frac{1}{4(1)}$

$y = \frac{1}{4}(x-3)^2 + 1 = \frac{1}{4}$



#3 Find the focus, directrix, and vertex of

$y = 3(x+2)^2 - 5 \quad p = \frac{1}{12}$

$V = (-2, -5)$

$F = (-2, -4\frac{11}{12}) \quad D \; y = 5\frac{1}{12}$

#4 Find the focus of a quadratic with a vertex of (3,-5) and a directrix of $y = -9$

$|p| = 4$

focus $(3, -1)$