I. Function Characteristics

Domain: *Interval* of possible x values for a given function. (Left,Right)

Range: *Interval* of possible y values for a given function. (down, up)

End Behavior: What is happening at the far ends of the graph?

	Left side	Right side
For each side	$x \to -\infty,$	$x \to \infty$
Pick one of these	Points Down $y \to -\infty$	Points Up $y \to \infty$

Increasing Intervals: *Interval* of x values for which the corresponding y values are increasing.

Decreasing Intervals: *Interval* of x values for which the corresponding y values are decreasing.

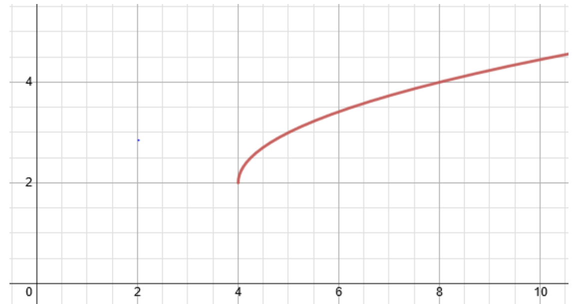
x-Intercepts: *points* where the graph crosses the x axis. $(x, 0)$

y-Intercepts: *points* where the graph crosses the y axis. $(0, y)$

Maximums: *points* where the graph changes from increasing to decreasing. Peaks in the graph.

Minimums: *points* where the graph changes from decreasing to increasing. Valleys in the graph.

E1

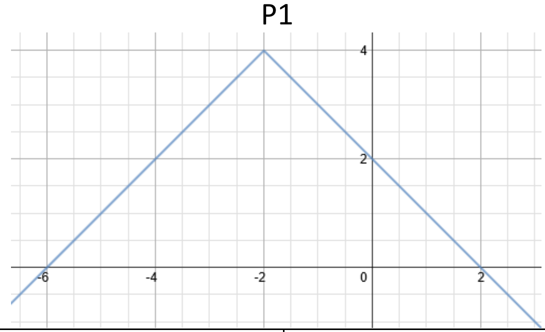


Domain: $[4, \infty)$
Range: $[2, \infty)$

Increasing Intervals:
$[4, \infty)$
Decreasing Intervals:
$None$

End Behavior:
As $x \to -\infty, y \to 2$
As $x \to \infty, y \to \infty$
x-Intercepts: $None$
y-Intercepts: $None$
Maximums: $None$
Minimums: $None$

P1

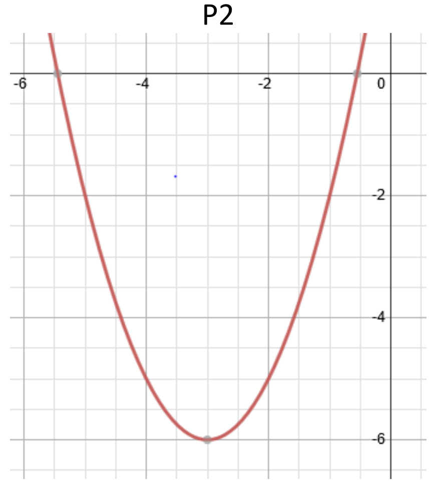


Domain:
Range:
Increasing Intervals:

Decreasing Intervals:

End Behavior:
As $x \to -\infty,\ y \to$
As $x \to \infty, y \to$
x-Intercepts:
y-Intercepts:
Maximums:
Minimums:

P2



Domain:
Range:
Increasing Intervals:

Decreasing Intervals:

End Behavior:
As $x \to -\infty,$
As $x \to \infty,$
x-Intercepts:
y-Intercepts:
Maximums:
Minimums:

II. Function Transformations

General form: $g(x) = a\,f(x-h) + k$

f(x) parent function
g(x) transformed function

a if negative, flip vertically

$0 < |a| < 1$ vertical compression
$|a| > 1$ vertical stretch

h if negative, horizontal shift right
if positive, horizontal shift left

k if negative, vertical shift down
if positive, vertical shift up

Examples

$E1.\ g(x) = x^2 + 2$
Parent Function:
quadratic
Transformations:
shift up 2 units

E2. $g(x) = -(x-4)^3 - 1$
Parent Function:
cubic
Transformations:
flip vertically
shift right 4 units
shift down 1 unit

E3. $g(x) = 3\sqrt{x+1} - 7$
Parent Function:
Radical (square root)
Transformations:
Stretch by a factor of 3
Shift left 1 unit
Shift down 7 units

E4. $g(x) = -\frac{1}{2}(x-3)^2 + 1$
Parent Function:
quadratic
Transformations:
Flip vertically
Compression by a factor of $\frac{1}{2}$
Shift Right 3 units
Shift up 1 unit

P3. $g(x) = 2^{x-3} + 5$
Parent Function:

Transformations:

P4. $g(x) = -(x+7)^2$
Parent Function:

Transformations:

P5. $g(x) = 2\log(x-2) - 1$
Parent Function:

Transformations:

III. Graphing a function from an equation - Example

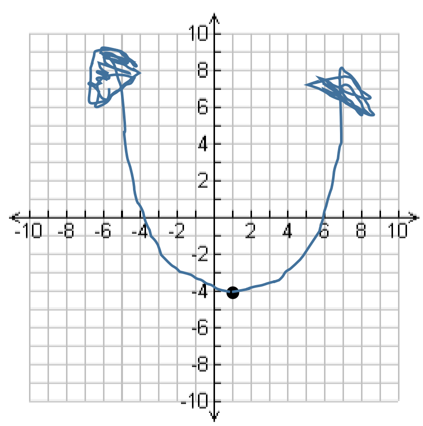
1. Identify the parent function to determine a general shape.
Quadratic

2. Think about where the vertex or critical points are usually found for the parent function.
Centered at the origin. Shaped like the letter U.

3. Where are the critical points of the new function given the transformations in the equation? *Since there is a horizontal shift right 1 unit and a vertical shift down four units, the vertex is at the point (1,-4).*

4. Use the location of the critical points to sketch the new graph.

E5. Graph $f(x) = (x-1)^2 - 4$



Graphing a function from an equation

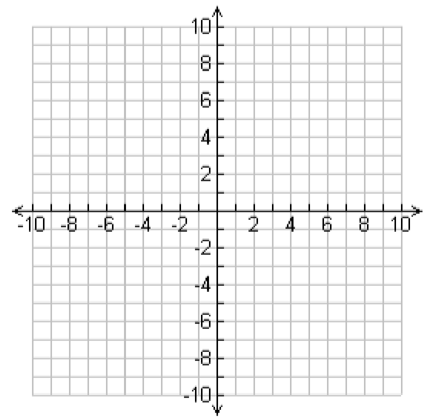
1. Identify the parent function to determine a general shape.

2. Think about where the vertex or critical points are usually found for the parent function.

3. Where are the critical points of the new function given the transformations in the equation?

4. Plot your critical points and sketch in the graph.

P6. Graph $f(x) = -|x+2| + 6$



Writing Function Equation from a description of the transformations

How do translations effect the function equation?

$f(x) = -a(x-h) + k$

"-" flip over x axis

a compression or stretch

h horizontal shift in the opposite direction of the sign

k vertical shift in the same direction of the sign

E6

Write the equation for a quadratic function with a vertical shift down 3, left 7 and a vertical stretch by a factor of 4.

Quadratic : x^2

Down 3: subtract 3 on the "outside"

Left 7: add 7 to x (inside)

V. stretch by 4: multiply the "x part" by 4

$$y = 4(x+7)^2 - 3$$

P7. Write the equation for an absolute value function that has been shifted down three units and left 17 units.

P8. Write the equation for a Quadratic function that has been flipped vertically, shifted up 5 units, and shifted right 2 units.

P9. Write the equation for a square root function that has been shifted down 11 units, shifted left 5 units, and stretched by a factor of 2.

P10. Write the equation for an absolute value function that has been compressed by a factor of 2 and shifted down three units.

Steps for Determining Equation from Graph

What's the parent function?

Has the same shape as a cubic function

Where's the vertex or critical point of the <u>parent</u> function?

(0,0)

Where's the vertex or critical point of <u>this</u> function?

(2,1)

How did we get from the parent function critical point to the critical point of this function?

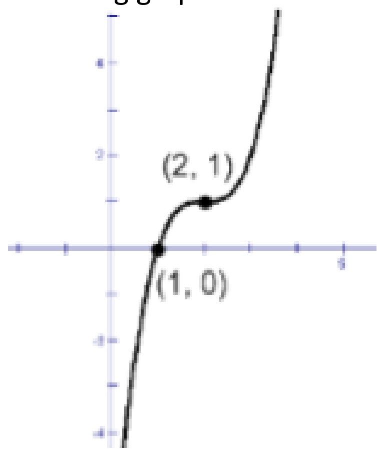
Right 2 and up 1

How do I translate those changes into an equation?

Horizontal changes go with the x

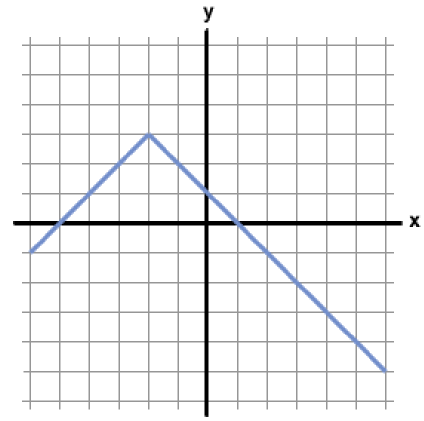
Vertical changes go outside the x

E7. Write the equation for the following graph.



$y = (x-1)^3 + 2$

P9. Write the equation for the following graph



Shifts of Shifts

In these types of problems, you start with a function that has already been shifted around and now you're going to apply some more shifts.

E8. $f(x) = (x+1)^2 - 1$,

If this function is shifted left 3 units, up 2 units, flipped vertically and stretched by a factor of 4 what is the resulting function equation?

Left 3	add 3 to the number "with x"	$1 + 3 = 4$
Up 2	add 2 to the number "outside" of x	$-1 + 2 = 1$
Flipped vertically	flip the sign in front of the equation	change to $-$
Stretched by 4	Multiply the number in front by 4	$1(4) = 4$

Resulting function: $g(x) = -4(x+4)^2 + 1$

P10. $f(x) = 2(x)^3 + 4$

If this function is shifted up 2, right 1 and compressed by a factor of 6 what is the resulting equation?

P11. $f(x) = -|x-5|$

If this function is shifted up 4, left 3, stretched by a factor of 2, and flipped vertically, what is the resulting equation?

Shifts of Shifts part 2

In this type of problem you have to identify the transformations that would change one function equation to another.

E9. What transformations would change the function equation
$f(x) = -3\sqrt{x-4} + 1$ to $g(x) = 27\sqrt{x+5} + 7$

	Original	New	How to get there
Horizontal	-4	$+5$	$+5-(-4)=9$ Left 8
Vertical	$+1$	$+7$	$+7-(+1)=6$ Up 6
Flip	$-$	$+$	Signs Changed Vertical Flip
Compression or **S**tretch	3	27	$27 \div 3 = 9$ Stretch factor of 9

P12. What transformations would change the function equation $f(x) = -3(x-1)^2 - 3$ to $g(x) = -(x+4)^2 - 5$

	Original	New	How to get there
H			
V			
F			
C/S			

P13. What transformations would change the function equation $f(x) = |x+2| - 3$ to $g(x) = -2|x+1| + 2$

	Original	New	How to get there
H			
V			
F			
C/S			