

# 11-7 Reteaching

## Samples and Surveys

When doing a survey, it usually is not practical to get the opinion of every member of a population. You can get a fairly accurate picture of the opinion of a population by surveying a *sample* of the population. A sample is a smaller group that represents the whole population. There are several ways to choose a sample:

**Convenience** choosing any people easily available

**Self-selection** having people volunteer to participate in the survey

**Systematic** ordering the population and choosing participants at regular intervals (such as choosing every fifth person from the telephone book)

**Random** all members of the population have an equal chance of being asked to participate

The way you choose the sample can introduce *bias*, or systematic error, into the survey. When a survey is biased, the results are inaccurate.

### Problem

An athletic shoe company wants to learn which brand of athletic shoes is worn most often by local high-school students. The company sets up a booth in a local mall and offers a coupon for a free pair of their athletic shoes to anyone who answers the question, "What is your favorite brand of athletic shoes?"

- a. What is the sampling method used? There may be more than one.  
b. Is there any bias in the company's sampling method?

- a. People in the mall are readily available to the booth. Also, people must volunteer to participate. The sample is a convenience sample and is self-selected.  
b. The survey is biased in several ways:
- People who do not shop at the mall are excluded.
  - Only people who choose to walk up to the booth participate in the survey.
  - People who are not high-school students may participate in the survey.
  - People may be more likely to say this company makes their favorite shoes when they are offered a free pair.

### Exercises

A politician wants to know what issues are most important to the voters in his district. Identify the sampling method and any bias in the method.

1. The politician spends 9:00 A.M. to 4:00 P.M. on Tuesday talking to people as they enter a grocery store.
2. The politician sets up a questionnaire on his website.

# 11-7 Reteaching (continued)

## Samples and Surveys

You can introduce bias into a survey by using poorly written questions. Survey questions should NOT be:

*Confusing* by asking about more than one issue or by using double negatives

*Ambiguous* by offering answer choices that overlap

*Loaded* by using words that might provoke strong reactions in all or some people

*Leading* by suggesting that one particular answer is correct

Usually simple questions make the best survey questions.

### Problem

An athletic shoe company wants to learn which shoe features are important to local high-school students. Is there any bias in any of these survey questions? Explain.

- a. Choose the feature that is most important to you in a shoe.  
A fit      B style      C color      D appearance
- b. Isn't fit the most important feature in any shoe?  
c. How important is it to you that shoe materials have not been tested on laboratory animals?

- a. Style and color both contribute to the appearance of a shoe. The question is ambiguous by offering overlapping answer choices and leaves out other choices.  
b. This question implies that fit is the most important feature. It is a leading question.  
c. The question introduces a second issue, laboratory-animal testing. This is a loaded question.

### Exercises

A politician wants to know what issues are most important to the voters in his district. Identify the type of bias in each survey question.

3. How do you feel about the toxic pollution being released into the air by the local manufacturing plants?
4. Isn't a school bond not the right way to raise money for local education?
5. Which city service is most important to you?  
A road maintenance      B fire      C public works      D police
6. Do you think your local library should offer videos and Internet access?

Name \_\_\_\_\_ Class \_\_\_\_\_ Date \_\_\_\_\_

# 11-7 Practice

Samples and Surveys

Form K

Identify the sampling methods used in each of the following situations.

Then state whether the sampling method has any bias.

1. A television station invites viewers to call in and name their favorite game show.
2. A school principal gathers an alphabetical list of all the students at her school. Then she selects every 15th student to take a survey about the cafeteria's lunch menu.
3. A reporter asks people leaving a movie theater to take a survey about their television viewing habits.
4. A psychologist uses a computer program to randomly select names from a list of students at a university. The members of the sample will take a survey about student housing at the university.
5. **Writing** A group of television producers plans to survey 10-year-olds to determine their opinions about a new cartoon. Describe a sampling method that could be used to gather a biased sample in this situation. Then describe a method to gather an unbiased sample.
6. **Multiple Choice** A school psychologist sits in a school cafeteria and takes notes on students' behavior while they eat lunch. Which of the following types of studies is the researcher conducting?  
 A controlled experiment     B observational study     C survey
7. **Open-Ended** Your classmate is randomly selecting a sample of students at his high school to take a survey. You say that your classmate's sample is biased because it only contains high-school students. In what case might you be wrong?

Name \_\_\_\_\_ Class \_\_\_\_\_ Date \_\_\_\_\_

# 11-7 Practice

Samples and Surveys

Form K

Identify and describe the bias in the following survey questions.

8. Isn't summer a much more pleasant season than winter?
  9. Are college students better at studying useful subjects such as math or impractical subjects such as art history?
  10. Do you believe that this year's class field trip was fun and educational?
  11. Do you agree that Mrs. Regis's class is more interesting than Mr. Wright's class?
- Rewrite the following survey questions so that they are no longer biased.
12. Do you prefer the excitement of rock and roll or the tediousness of classical music?
  13. Would you agree that dogs make better pets than cats?
  14. Do you believe that Mayor Johnson is friendly and effective?
  15. **Writing** A supervisor wants to determine what percent of people in his office building believe it is important to have an Internet connection at home. What sampling method can he use to gather an unbiased sample? What is an example of a survey question that is likely to yield unbiased information?

# 11-6 Reteaching

## Standard Deviation

The mean tells you what the center of a set of data values looks like. But two very different data sets can have the same mean. For example, each of these data sets has a mean of 25.



Notice that the data values on the number line for set B are much more spread out from the mean than the data values for set A. *Variance* and *standard deviation* are measures of how widely data values differ from the mean.

The lowercase Greek letter sigma,  $\sigma$ , is the symbol for standard deviation. Variance is the square of the standard deviation, and is written as  $\sigma^2$ . For a set of  $n$  data values:

$$\sigma^2 = \frac{\sum (x - \bar{x})^2}{n}$$

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

### Problem

What are the variance and standard deviation for the data set {100 158 170 192}?

**Step 1** Find the mean of the values.

$$\bar{x} = \frac{100 + 158 + 170 + 192}{4} = 155.$$

**Step 2** Subtract the mean from each value in the data set. Then square each difference.

$$\begin{aligned} (100 - 155)^2 &= 3025 & (158 - 155)^2 &= 9 \\ (170 - 155)^2 &= 225 & (192 - 155)^2 &= 1369 \end{aligned}$$

**Step 3** Find the mean of the squared differences. This is the variance.

$$\sigma^2 = \frac{3025 + 9 + 225 + 1369}{4} = 1157$$

**Step 4** Find the square root of the variance. This is the standard deviation.

$$\sigma = \sqrt{1157} \approx 34$$

The variance for the data set is 1157 and the standard deviation is about 34.

### Exercises

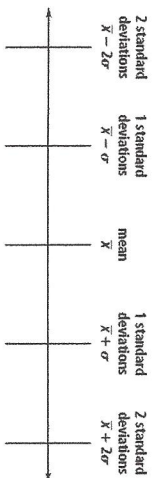
Find the variance and standard deviation for each data set.

- 6.5 7.0 9.0 8.0 7.5
- 5.6 5.8 5.9 6.1
- 201 203 208 210 211
- 12 14 15 17 19

# 11-6 Reteaching (continued)

## Standard Deviation

You can describe the spread of a set of data values by counting the number of standard deviations from the mean that it takes to include some or all of the values.



### Problem

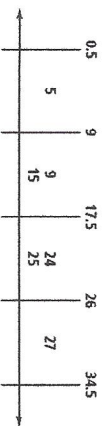
The data set {3 9 15 24 25 27} has a mean of 17.5 and a standard deviation of about 8.5. Within how many standard deviations of the mean do all the values fall?

**Step 1** Draw five lines to represent the mean and two standard deviations on either side of the mean.

**Step 2** Substitute the values for  $\bar{x}$  and  $\sigma$ . Simplify each expression.

$$\begin{aligned} \bar{x} - 2\sigma &= 17.5 - 2(8.5) = 17.5 - 17.0 = 0.5 \\ \bar{x} - \sigma &= 17.5 - 8.5 = 9 \\ \bar{x} &= 17.5 \\ \bar{x} + \sigma &= 17.5 + 8.5 = 26 \\ \bar{x} + 2\sigma &= 17.5 + 17.0 = 34.5 \end{aligned}$$

**Step 3** Label each line with the appropriate value. Write each data value in the appropriate section. If a value falls on a line between two sections, write it in the section closest to the mean.



The drawing shows that all the values fall within two standard deviations of the mean.

### Exercises

- A family buys groceries weekly. Over four weeks their grocery costs are \$72.42, \$91.50, \$58.99, and \$69.02.
- Within how many standard deviations of the mean do all the costs fall?
  - How many costs fall within one standard deviation of the mean?
  - Within how many standard deviations of the mean would a cost of \$102.00 fall?

# 11-6 Practice

Standard Deviation Form K

Find the mean, variance, and standard deviation for each data set.

1. 6, 13, 12, 9, 10

Mean  
 $\bar{x} = 50 \div 5 = 10$

Variance				
x	$\bar{x}$	$x - \bar{x}$	$(x - \bar{x})^2$	
6	10	-4	16	
13	10	3	9	
12	10	2	4	
9	10	-1	1	
10	10	0	0	

Standard Deviation  
 $\sigma = \sqrt{\sigma^2} = \sqrt{6} \approx 2.4$

$$\sigma^2 = \frac{\sum(x - \bar{x})^2}{n} = \frac{30}{5} = 6$$

2. 8, 16, 12, 15, 4

3. 25, 18, 20, 19, 22, 16

4. 27, 34, 45, 30, 26, 42

Use a graphing calculator to solve the following problems.

5. The most recent test scores for a math class are displayed in the table below. What are the mean and the standard deviation for this data set?

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Score	77	86	79	94	65	82	76	97	65	77	89	78	84	79	88

6. Your sister's bowling scores for the last 12 games are displayed in the table below. What are the mean and standard deviation for this data?

Game	1	2	3	4	5	6	7	8	9	10	11	12
Score	212	187	176	205	193	229	201	175	203	216	227	235

# 11-6 Practice (continued)

Standard Deviation Form K

Determine the whole number of standard deviations that include all of the following data values.

7. You brother is buying his textbooks for his first semester of college. The price of each of his books is shown in the table below. The mean of the data set is \$65.85, and the standard deviation is about 36. Within how many standard deviations of the mean do all of the prices fall?

Book	1	2	3	4	5	6
Price	\$25.60	\$57.00	\$38.25	\$128.40	\$84.00	\$53.85

8. The table below shows the weights of the five starting players on a basketball team. Within how many standard deviations of the mean do all of the weights fall?

Player	1	2	3	4	5
Weight (lb)	146	189	246	178	203

9. **Open-Ended** Describe an example of how it can be useful to know the standard deviation of a data set.

10. **Writing** How is standard deviation similar to range and interquartile range?

11. **Error Analysis** Your classmate calculated the standard deviation of the data set shown below and got 46.53. What error did she make? What is the correct standard deviation?

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
High Temperature (°F)	76°	82°	63°	69°	79°	84°	75°

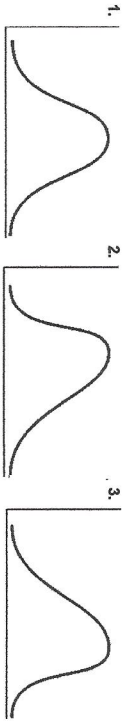


# 11-9 Practice

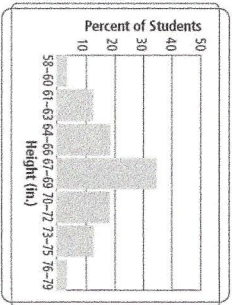
Normal Distributions

Form K

Identify each of the following distributions as *possibly skewed*, *negatively skewed*, or *normally distributed*.



The bar graph below displays the heights of the students at a high school. Use the graph to answer the following questions.



- Approximately what percent of students are between 64 in. and 75 in. tall?
- Approximately what percent of students are between 61 in. and 67 in. tall?
- Approximately what percent of students are between 64 in. and 75 in. tall?
- Approximately what percent of students are between 70 in. and 79 in. tall?
- Reasoning** Your mother has a rose garden. Every day, she sprays fertilizer on the roses in one section of the garden. Do you expect that the heights of the rose bushes in her garden are normally distributed? Explain why or why not.

# 11-9 Practice (continued)

Normal Distributions

Form K

Sketch a normal curve to represent each of the following normal distributions.

- The average weight of a tomato in a tomato garden is 10 oz. The standard deviation is 1.6 oz. Sketch a normal curve showing the tomato weights at one, two, and three standard deviations from the mean.

- The average score on a math test is 76. The standard deviation is 6.2. Sketch a normal curve showing the test scores at one, two, and three standard deviations from the mean.

Draw a normal curve to solve the following problems.

- A local bakery makes chocolate chip cookies. The number of chocolate chips in the cookies is approximately normally distributed, with mean 11.4 and standard deviation 1.3. What percent of the cookies have between 8.8 and 14 chocolate chips?
- The bakery described in Exercise 10 sold 200 chocolate chip cookies. How many of the cookies had less than 8.8 chocolate chips?
- Reasoning** One of the cookies sold by the bakery had 18 chocolate chips. Would this be considered an outlier? Explain why or why not.

# 11-9 Reteaching

## Normal Distributions

If a data set has a *normal distribution*:

- 2.35% of the values will be between 2 and 3 standard deviations below the mean.
- 13.5% of the values will be between 1 and 2 standard deviations below the mean.
- 34% of the values will be within 1 standard deviation below the mean.
- 34% of the values will be within 1 standard deviation above the mean.
- 13.5% of the values will be between 1 and 2 standard deviations above the mean.
- 2.35% of the values will be between 2 and 3 standard deviations above the mean.

The graph of a normal distribution is a *normal curve*.

- A normal curve is shaped like a bell, with the highest point at the mean and tapering down evenly on either side of the bell.

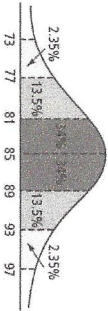
### Problem

The weight in pounds of newborn calves on a farm is distributed normally, with a mean of 85 and a standard deviation of 4. What percent of newborn calves on the farm weigh between 77 lb and 89 lb?

**Step 1** Draw a normal curve. Label the mean.

**Step 2** Divide the graph into 6 equal sections.

Each section should be one standard deviation wide, which is 4 lb in this problem. Label each section with the appropriate percent for a normal distribution.



**Step 3** Add the percents for the sections with weights 77 lb–81 lb, 81 lb–85 lb, and 85 lb–89 lb.  
 $13.5 + 34 + 34 = 81.5$

About 82% of newborn calves will weigh 77 lb–89 lb.

### Exercises

Use the graph above to find the percent of calf weights within each interval.

1. from 73 lb to 81 lb
2. greater than 81 lb
3. from 77 lb to 97 lb
4. less than 85 lb
5. at most 89 lb
6. at least 93 lb

# 11-9 Reteaching (continued)

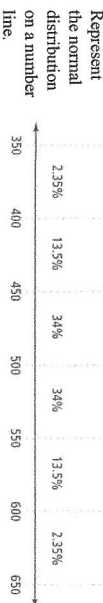
## Normal Distributions

You can use the percents associated with a normal distribution to make predictions.

### Problem

The number of hours a certain type of battery will last is distributed normally with a mean of 500. The standard deviation is 50. Out of 250 batteries tested, how many batteries would you expect to be still working after 550 h?

**Step 1** Represent the normal distribution on a number line.



**Step 2** Find the percent of batteries that last 550 h or longer. Which sections of the distribution contain values of 550 or greater?  $13.5 + 2.35 = 15.85 \approx 16\%$

**Step 3** Find 16% of 250 batteries.  
 $x = 16(0.16)(250)$   
 $x = 40$

You can expect about 40 of the 250 batteries to be still working after 550 h.

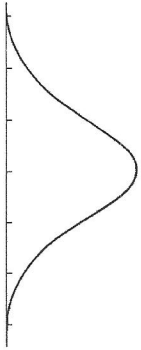
### Exercises

Sketch the normal distribution for the following data. Make a prediction based on your sketch.

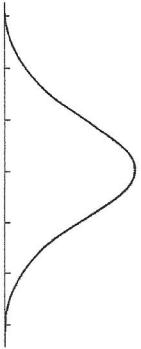
7. A certain type of light bulb lasts an average of 219 h. Out of 1000 bulbs, how many would you expect to last less than 79 h if the standard deviation is 70 h?
8. The 26 students in a math class can finish 100 problems in a mean time of 4 min. The standard deviation is 1 min. How many students in the class will still be working after 5 min?
9. A group of 71 frogs hops a mean distance of 66 in. with a standard deviation of 3 in. How many frogs would you expect to hop more than 72 in.?

For each question, construct a normal distribution curve and label the horizontal axis. Then answer each question.

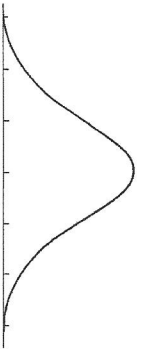
- The mean life of a tire is 30,000 km. The standard deviation is 2000 km.
  - 68% of all tires will have a life between \_\_\_\_\_ km and \_\_\_\_\_ km.
  - 95% of all tires will have a life between \_\_\_\_\_ km and \_\_\_\_\_ km.
  - What percent of the tires will have a life that exceeds 26,000 km? \_\_\_\_\_
  - If a company purchased 2000 tires, how many tires would you expect to last more than 28,000 km? \_\_\_\_\_



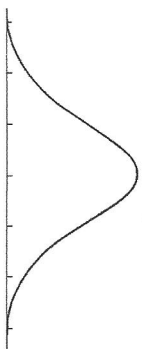
- The shelf life of a particular dairy product is normally distributed with a mean of 12 days and a standard deviation of 3 days.
  - About what percent of the products last between 9 and 15 days? \_\_\_\_\_
  - About what percent of the products last between 12 and 15 days? \_\_\_\_\_
  - About what percent of the products last 6 days or less? \_\_\_\_\_
  - About what percent of the products last 15 or more days? \_\_\_\_\_



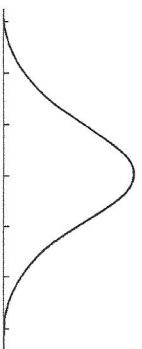
- A line up for tickets to a local concert had an average (mean) waiting time of 20 minutes with a standard deviation of 4 minutes.
  - What percentage of the people in line waited for more than 28 minutes? \_\_\_\_\_
  - If 2000 ticket buyers were in line, how many of them would expect to wait for less than 16 minutes? \_\_\_\_\_



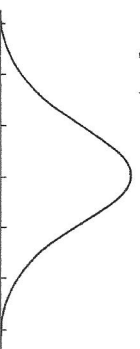
- On a recent math test, the mean score was 75 and the standard deviation was 5. Mike made 93. Would his mark be considered an outlier if the marks were normally distributed? Explain. \_\_\_\_\_



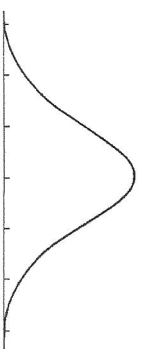
- In an Oreo factory, the mean mass of a cookie is given as 40 g. For quality control, the standard deviation is 2 g.
  - If 10,000 cookies were produced, how many cookies are within 2 g of the mean? \_\_\_\_\_
  - Cookies are rejected if they weigh more than 44 g or less than 36 g. How many cookies would you expect to be rejected in a sample of 10,000 cookies? \_\_\_\_\_



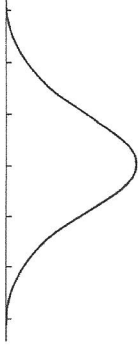
- The speeds of cars on the highway have a mean of 95 km/h with a standard deviation of 5 km/h.
  - What percentage of cars averaged less than 85 km/h? \_\_\_\_\_
  - If a police car stopped cars that were going more than 105 km/h, how many cars would they stop if there were 8000 cars on the highway? \_\_\_\_\_



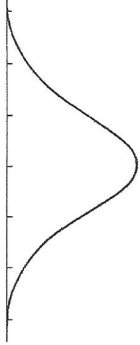
- The Floppy Disk Company makes 3.5 inch floppy disks for disk drives that are 3.7 inches wide. The size of a manufacturer's 1000 disks every hour.
  - What % of the disks would you expect to be greater than 3.5 inches? \_\_\_\_\_
  - In one hour, how many disks would you expect to be between 3.4 inches and 3.7 inches? \_\_\_\_\_
  - About how many disks will be unable to fit in the disk drive (3.7 inch won't fit)? \_\_\_\_\_



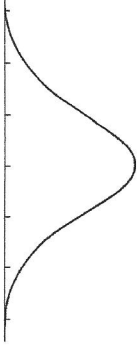
8. The mean life of a battery is 50 hours with a standard deviation of 6 hours. The manufacturer advertises that they will replace all batteries that last less than 38 hours. If 50,000 batteries were produced, how many would they expect to replace?
- 



9. A bottle of fruit punch contains at least 473 ml. The machine that fills the bottles is set so that the mean volume is 477 ml. The volumes in the bottles are normally distributed.
- a) What percent of the bottles are underfilled if the standard deviation is 2 ml? \_\_\_\_\_
- b) What percent of the bottles are underfilled if the standard deviation is 4 ml? \_\_\_\_\_



10. A grading scale is set up for 1000 students' test scores. It is assumed that the scores are normally distributed with a mean score of 75 and a standard deviation of 15
- a) How many students will have scores between 45 and 75? \_\_\_\_\_
- b) If 60 is the lowest passing score, how many students are expected to pass the test? \_\_\_\_\_



11. The monthly income of 5,000 workers at the Microsoft plant are distributed normally. Suppose the mean monthly income is \$1,250 and the standard deviation is \$250.
- a) How many workers earn more than \$1500 per month? \_\_\_\_\_
- b) How many workers earn less than \$750 per month? \_\_\_\_\_
- c) What percentage of the workers earn between \$750 and \$1500 per month? \_\_\_\_\_
- d) What percentage of the workers earn less than \$1750 per month? \_\_\_\_\_

