

KEY



1 Which expression is equivalent to $(x + 3)^3 - 9x(x + 3)$?

- A $x^3 + 27$
- B $x^3 - 27$
- C $x^3 - 9x^2 - 27x + 27$
- D $x^3 - 9x^2 + 27x + 27$

$(x+3)(x+3)(x+3) - 9x^2 - 27x$

$x^3 + 6x^2 + 9x + 3x^2 + 18x + 27 - 9x^2 - 27x$

2 Suppose $p(x) = x^3 - 2x^2 + 13x + k$. The remainder of the division of $p(x)$ by $(x + 1)$ is -8 . What is the remainder of the division of $p(x)$ by $(x - 1)$?

- A -8
- B 8
- C 16
- D 20

$k - 16 = -8$
 $k = 8$

$p(1) = \text{remainder} = 20$

3 What is the approximate solution to the equation $3^{x-1} = 4^{2x+5}$?

- A 3.875
- B 1.262
- C -2.354
- D -4.797

take log of both sides
 $\log 3^{x-1} = \log 4^{2x+5}$

$(x-1)\log 3 = (2x+5)\log 4$
 $(x-1)1.097 = (2x+5)1.386$
 distribute & solve for x



4

Samantha invested \$10,000 in each of two different financial plans in 2013. The predicted value of each plan is modeled below.

- Plan M: a rate of 7.5%, compounded continuously. $Pert \Rightarrow 10000e^{0.075x}$
- Plan N: The value is determined by the function $y = 5x^3 - 50x^2 + 4x + 10,000$, where x is the number of years after 2013.

Plan N has a greater predicted value than Plan M during which years?

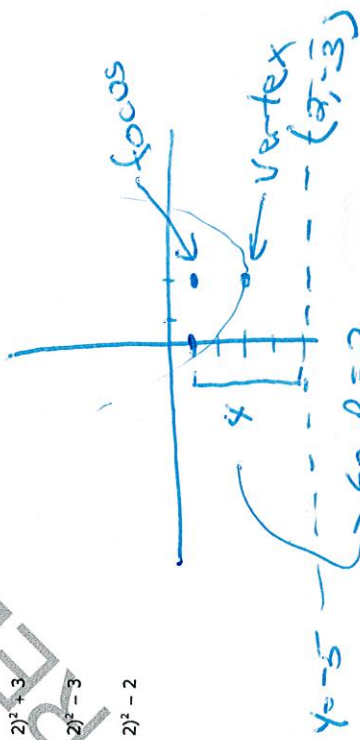
- A from 2014 to 2041
- B from 2028 to 2055
- C from 2042 to 2073
- D Plan N never has a greater value than Plan M.

Put in calculator & compare y values

5

Which is an equation of a parabola that has a directrix of $y = -5$ and a focus at $(2, -1)$?

- A $y = \frac{1}{2}(x + 2)^2 + 2$
- B $y = \frac{1}{8}(x + 2)^2 + 3$
- C $y = \frac{1}{8}(x - 2)^2 - 3$
- D $y = \frac{1}{2}(x - 2)^2 - 2$

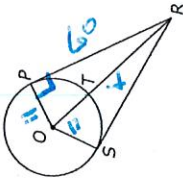


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6 In the figure below, \overline{PR} and \overline{SR} are tangent to circle O .



If $OT = 11$ cm and $PR = 60$ cm, what is the length of \overline{OR} ?

- A 61 cm
- B 59 cm
- C 50 cm
- D 48 cm

$$11^2 + 60^2 = OR^2$$

$$11^2 + 60^2 = (11+x)^2$$

$$121 + 3600 = 121 + 22x + x^2$$

$$0 = x^2 + 22x - 3600$$

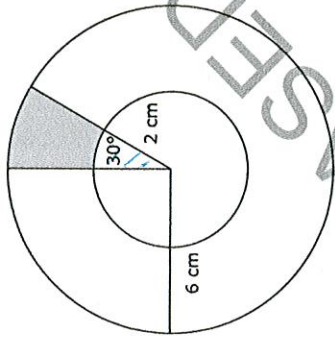
Solve using graphing or quad formula.

2 solutions, $-x \rightarrow 50$
 $50 + 11 = 61$

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7 In the figure below, the larger circle has a radius of 6 cm, and the smaller circle has a radius of 2 cm.



What is the approximate area of the shaded region?

- A 2.1 cm²
- B 3.4 cm²
- C 4.2 cm²
- D 8.4 cm²

$$\text{Area of Circle} = \pi r^2$$

Small Circle
 $A = \pi(2)^2 = 4\pi$

Sector area $= \frac{30}{360} (4\pi) = \frac{\pi}{3}$

$$3\pi - \frac{\pi}{3} = \frac{8\pi}{3} \approx 8.4$$

Large circle
 $A = \pi(6)^2 = 36\pi$
 $= 36\pi$

Sector Area $= \frac{30}{360} (36\pi)$
 $= 3\pi$

Go to the next page.



8 Which choice shows the solutions to the equation $8x^2 + 3x = -7$?

- A $\frac{-3 \pm \sqrt{215}}{16}$
- B $\frac{3 \pm \sqrt{215}}{16}$
- C $\frac{-3 \pm \sqrt{233}}{16}$
- D $\frac{3 \pm \sqrt{233}}{16}$

14

$8x^2 + 3x + 7 = 0$

now put in quadratic formula

9 A system of equations is shown below.

$$y = |x - 3|$$

$$y = \frac{1}{2}x$$

What is the distance between the points of intersection of the system?

- A $\sqrt{6}$
- B $\sqrt{20}$
- C $\sqrt{48}$
- D $\sqrt{80}$

put each function in $y_1 = y_2$ find intersection points.

$$(2, 1) = (0, 3)$$

distance formula

$$d = \sqrt{(2-0)^2 + (1-3)^2}$$

$$= \sqrt{16+4}$$

$$= \sqrt{20}$$



10 Fred drives an average of 15,000 miles per year, and his car gets 20 miles per gallon of gasoline.

- The average cost of gasoline is \$3.25 per gallon.
- He buys a new car.
- In his new car, Fred continues to average 15,000 miles per year, and the average cost of gasoline remains the same.

Approximately how many more miles per gallon does the new car get if Fred has a savings of \$650 per year on gasoline?

- A 5.8 mpg
- B 7.3 mpg
- C 8.8 mpg
- D 10.3 mpg

$$1) \frac{15,000 (3.25)}{20} = 2,437.5$$

$$2) 2,437.5 - 650 = 1,787.5$$

$$3) 1,787.5 = \frac{15,000 (3.25)}{x} \dots x = 7.3$$

11 A student wants to determine the most liked professor at her college. Which type of study would be the most practical to obtain this information?

- A a simulation
- B an experiment
- C a survey
- D an observation



- 12 A principal wants to survey 150 students to determine which electives to offer during the next school year. There are 1,800 students in the school. Which procedure could the principal use to select a sample using a systematic random sample?
- A Obtain a list of all students. Start with the eighth student, and select every twelfth student until 150 students have been selected.
 - B Select the first 150 students who enter the school.
 - C Choose the fifth student to come into the cafeteria, and then select every third student who comes into the cafeteria until 150 students have been selected.
 - D Place students' names on slips of paper and select 150 slips.

13 What value of h is needed to complete the square for the equation $x^2 + 10x - 8 = (x - h)^2 - 33$?

- A -25
- B -5
- C 5
- D 25

Handwritten work for Question 13:

$$x^2 + 10x - 8$$

$$\left(\frac{10}{2}\right)^2 = 25$$

$$x^2 + 10x + 25 - 8 - 25$$

$$(x + 5)^2 - 33$$

$$h = -5$$

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- 14 A shipping company is designing boxes to meet specific requirements.
- Each box must be a completely closed rectangular prism with no overlapping material.
 - The boxes must hold 24 cans in two layers of 12 cans each.
 - The cans are 3 inches in diameter and 5 inches in height.

What is the smallest amount of cardboard needed to meet the specifications?

- A 1,080 in.²
- B 840 in.²
- C 636 in.²
- D 540 in.²



15 Which expression is equivalent to $\frac{\cos(\theta)}{1 - \sin(\theta)} \tan(\theta)$?

- A $\sec(\theta)$
- B $\sin(\theta)$
- C $\cos(\theta)$
- D $\csc(\theta)$

Handwritten work for Question 15:

$$\frac{\cos \theta}{1 - \sin \theta} \cdot \frac{\sin \theta}{\cos \theta}$$

$$\frac{\cos \theta \cos \theta - \sin \theta (\sin \theta)}{\cos \theta (1 - \sin \theta)}$$

$$\frac{\cos^2 \theta - (\sin \theta + \sin^2 \theta)}{\cos \theta (1 - \sin \theta)}$$

Handwritten work for Question 15 (continued):

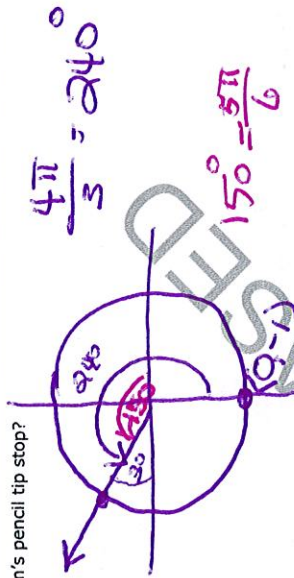
$$\frac{(1 - \sin \theta)}{\cos \theta (1 - \sin \theta)} = \frac{1}{\cos \theta} = \sec \theta$$

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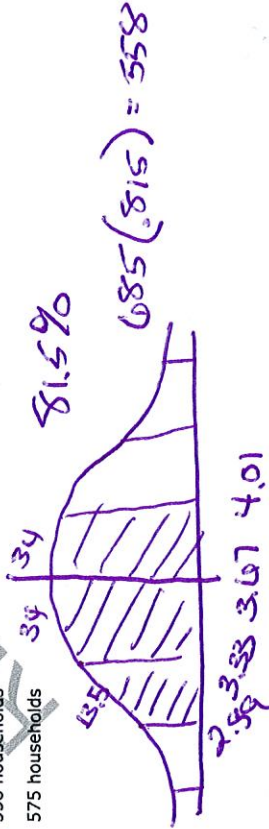
- 16 William put the tip of his pencil on the outer edge of a graph of the unit circle at the point $(0, -1)$. He moved his pencil tip through an angle of $\frac{4\pi}{3}$ radians in the counterclockwise direction along the edge of the circle. At what angle of the unit circle did William's pencil tip stop?

- A $\frac{\pi}{3}$
 B $\frac{5\pi}{6}$
 C $\frac{7\pi}{6}$
 D $\frac{5\pi}{3}$



- 17 A town has 685 households. The number of people per household is normally distributed with a mean, μ , of 3.67 and a standard deviation, σ , of 0.34. **Approximately** how many households have between 2.99 and 4.01 people?

- A 493 households
 B 520 households
 C 558 households
 D 575 households



- 18 The graph of the function $f(x) = x^3$ will be shifted down 2 units and to the right 3 units. Which is the function that corresponds to the resulting graph?

- A $g(x) = (x + 3)^3 + 2$
 B $g(x) = (x + 3)^3 - 2$
 C $g(x) = (x - 3)^3 + 2$
 D $g(x) = (x - 3)^3 - 2$

- 19 Which is the inverse of $f(x) = 1.5^x + 4$?

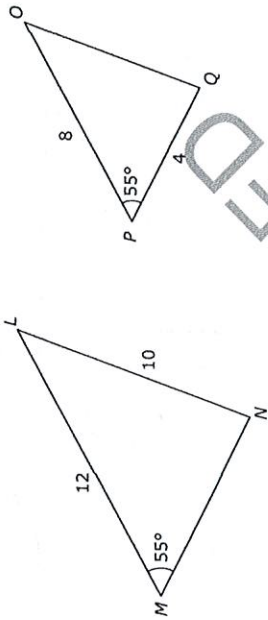
- A $f^{-1}(x) = \frac{x - 4}{1.5}$
 B $f^{-1}(x) = \frac{\log(x) - 4}{1.5}$
 C $f^{-1}(x) = \frac{\log(x - 4)}{\log(1.5)}$
 D $f^{-1}(x) = \frac{4 - \log(x)}{\log(1.5)}$

Handwritten solutions for question 19:

- $x = 1.5^y + 4$
- $(x - 4) = 1.5^y$
- $\log_{1.5}(x - 4) = y$
- change of base
- $\frac{\log(x - 4)}{\log 1.5} = y$



20 Triangles LMN and OPQ are shown below.



What additional information is sufficient to show that $\triangle LMN$ can be transformed and mapped onto $\triangle OPQ$?

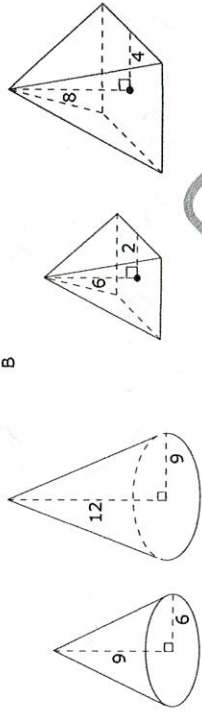
- A $OQ = 6$
- B $MN = 9$
- C $\angle LMN \cong \angle QOP$
- D $\angle NLM \cong \angle QOP$

9/10 ≠ 2/3
4/9 ≠ 3/10
 not corresponding angles

RATIO MB $\frac{8}{12} = \frac{2}{3}$



21 Which choice shows a pair of similar figures?



R

D

$\frac{9}{8} = \frac{3}{4}$

$\frac{5}{3} = \frac{5}{3}$

22 What is the approximate value of the sum:

$$8 - \frac{8}{7} + \frac{8}{49} - \dots + 8 \cdot \left(-\frac{1}{7}\right)^{2,500} ?$$

(Note: The sum of a series can be calculated using the formula $S_n = \frac{a_1(1-r^n)}{1-r}$, where $r \neq 1$.)

- A 1
- B 7
- C 8
- D 9

h

$$\frac{8 \left(1 + \left(-\frac{1}{7}\right)^{2500}\right)}{1 - \left(-\frac{1}{7}\right)} \approx 7$$



23 The volume of a rectangular prism is represented by the expression $(x^3 - 2x^2 - 20x - 24)$. If the length is $(x - 6)$ and the height and width are equal, what is the width of the prism?

A $x + 2$
 B $x - 2$
 C $x + 4$
 D $x - 4$

$$\begin{array}{r}
 6 \cancel{1} - 2 - 20 - 24 \\
 \underline{1 \quad 4 \quad + \quad 4} \\
 x^2 + 4x + 4 = (x+2)(x+2)
 \end{array}$$

24 A right triangle is shown below.



Which expression would result in an irrational number?

- A $x^2 + y^2$
 B $\frac{1}{2}xy$
 C $x + y + z$
 D $x^2 - z^2$
- $16 + 4 = 20$
 $\frac{1}{2}(2)(4) = 4$
 $2 + 4 + \sqrt{20} = 6 + \sqrt{20}$ (the only rational number)
 $16 - 20 = -4$

$$\begin{aligned}
 2^2 + 4^2 &= 2^2 \\
 \sqrt{20} &= 2
 \end{aligned}$$



25 Which expression is equivalent to $(4 - 3i)^2 + (6 + i)^2$?

- A 30
 B $42 - 12i$
 C 50
 D $62 - 12i$

$$\begin{aligned}
 (4 - 3i)(4 - 3i) &= 16 - 24i + 9i^2 \\
 &= 16 - 24i - 9 \\
 &= 7 - 24i \\
 (6 + i)(6 + i) &= 36 + 12i + i^2 \\
 &= 35 + 12i
 \end{aligned}$$

This is the end of the multiple-choice portion of the test.

$$\begin{array}{r}
 (4 - 3i)(4 - 3i) \quad (6 + i)(6 + i) \\
 \downarrow \qquad \qquad \qquad \downarrow \\
 -24i + 7 \qquad \qquad \qquad 12i + 35 \\
 \hline
 42 - 12i
 \end{array}$$



The questions you read next will require you to answer in writing.

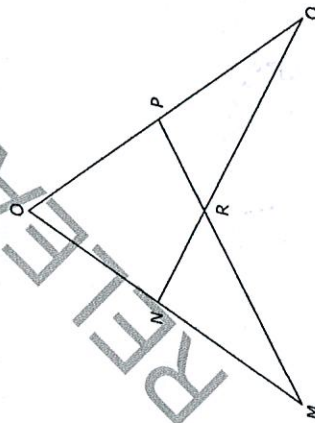
1. Write your answers on separate paper.
2. Be sure to write your name on each page.

1 The function f is defined as $f(x) = 6x^4 + x^3 + 4x^2 + x - 2$.

- Using the Remainder Theorem, determine if $\frac{1}{2}$ is a root of $f(x)$. Explain.
- If $\frac{1}{2}$ is also a root, what are the other two roots?

$f(\frac{1}{2}) = 0$ so $\frac{1}{2}$ is a root
 -i has to be a root
 use calculator
 to find other
 real root

2 In the diagram of $\triangle OMP$ and $\triangle OQN$, $\angle M \cong \angle Q$ and $\overline{MO} \cong \overline{OQ}$.



Based on the diagram, write a proof showing $\overline{MN} \cong \overline{QP}$.



3 Given the function:

$$g(x) = \frac{(x-2)(3x+2)}{(x+4)(x-2)(x-6)}$$

- What are the equations of the asymptotes of the function? Explain why or why not.
- Determine if there are any points of discontinuity. Explain why or why not.
- Describe the end behavior as x approaches $-\infty$ and as x approaches $+\infty$.

VA $x = -4$ & 6

hole @ $x = 2$

graph in your calculator
 and look @ far ends
 of graph



1 A board is made up of 9 squares. A certain number of pennies is placed in each square, following a geometric sequence. The first square has 1 penny, the second has 2 pennies, the third has 4 pennies, etc. When every square is filled, how many pennies will be used in total?

$1 + 2 + 4 + 8 + \dots$
 $n = 9$
 $r = 2$
 $S_n = 1 \left(\frac{1-r^n}{1-r} \right) = \frac{1-2^9}{1-2} = 511$

2 Let $f(x) = 14x^2 + 28x - 46$ and $g(x) = 2x + 7$. Which is the solution set to the equation $\frac{1}{12}f(x) = g(x)$?

- A $\{-3, 0, 1\}$
- B $\{-3, -1, 2\}$
- C $\{-2, 1, 3\}$
- D $\{1, 5, 11\}$

$\frac{1}{12}(14x^2 + 28x - 46) = 2x + 7$
 $14x^2 + 28x - 46 = 24x + 84$
 $14x^2 - 4x - 130 = 0$
 $7x^2 - 2x - 65 = 0$
 Simplify = combine like terms
 Graph to find roots

3 The equation $2x^2 - 5x - 12$ is rewritten in the form of $2(x - p)^2 + q = 0$. What is the value of q ?

- A $\frac{167}{16}$
- B $\frac{71}{8}$
- C $\frac{25}{8}$
- D $\frac{25}{16}$

$2x^2 - 5x - 12 = 0$
 $h = \frac{-b}{2a} = \frac{5}{4}$
 then sub $\frac{5}{4}$ back into



4 A box with an open top will be constructed from a rectangular piece of cardboard. The piece of cardboard is 8 inches wide and 12 inches long. The box will be constructed by cutting out equal squares of side x at each corner and then folding up the sides.



- A $0 < x < 4$
- B $0 < x < 6$
- C $0 < x < 8$
- D $0 < x < 12$

What is the entire domain for the function $V(x)$ that gives the volume of the box as a function of x ?

$V = (12-2x)(8-2x)$
 graph & look at first quadrant

$f(x) = \begin{cases} -x^2 + 2x & \text{for } x \leq 3 \\ 2x - 5 & \text{for } -3 < x < 4 \\ \frac{2x-5}{x-7} & \text{for } x \geq 4 \end{cases}$

What is the value of the expression $f(-3) + 2f(-1) - f(4)$?

- A $\frac{101}{36}$
- B $\frac{32}{9}$
- C 4
- D 22

$f(-3) = -(-3)^2 + 2(-3) = -15$
 $f(-1) = 2(2(-1) - 5) = 36$
 $f(4) = \frac{2(4) - 5}{4 - 7} = \frac{3}{-3} = -1$

$-15 + 36 + 1 = 22$



6 Which function goes to positive ∞ most quickly as x increases?

- A $y = \log(x) + 100$
- B $y = e^{x-9} - 3$
- C $y = x^2 + 5x + 6$
- D $y = 3x^5 + 4x^3 - 11x - 6$

Graph

but you can also remember that in general, exponential functions grow fastest

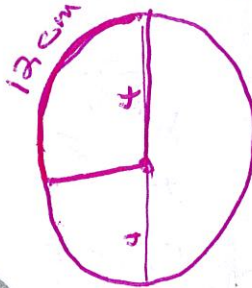
7 Which expression is equivalent to $\frac{\sin^2(\theta) - \cos^4(\theta)}{\sin^2(\theta) - \cos^2(\theta)}$, where $\sin^2(\theta) \neq \cos^2(\theta)$?

- A $\sin^2(\theta) - \cos^2(\theta)$
- B $\cos^2(\theta) - \sin^2(\theta)$
- C 2
- D 1

$(\sin^2 + \cos^2)(\sin^2 - \cos^2)$
 $(\sin^2 - \cos^2)$

8 The diameter of a circle is 8 centimeters. A central angle of the circle intercepts an arc of 12 centimeters. What is the radian measure of the angle?

- A $\frac{3}{2}$
- B 3
- C 4
- D 8π



$12 = \frac{x}{360} (2\pi \cdot 4)$

$\frac{4320}{8\pi} = x$
 $\frac{540}{\pi}$

$\frac{540}{\pi} \left(\frac{\pi}{180} \right) = 3$

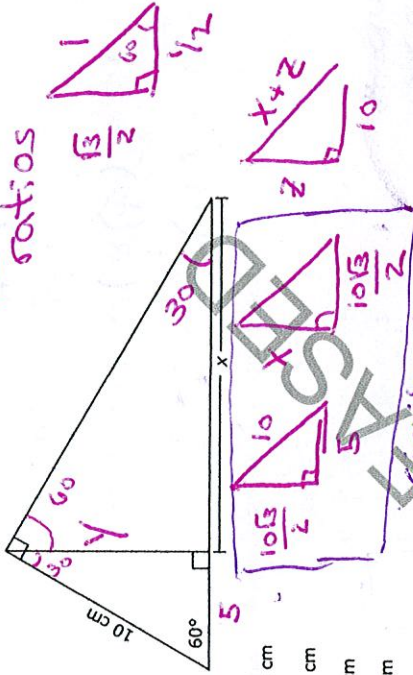
→ convert to radians

$\frac{x}{\frac{10\sqrt{3}}{2}} = \frac{\frac{10\sqrt{3}}{2}}{5}$
 $5x = \left(\frac{10\sqrt{3}}{2} \right) \cdot 2$
 $5x = 75$
 $x = 15$



9 What is the value of x in the triangle below?

remember 30 60 90 ratios



- A $\frac{5\sqrt{3}}{2}$ cm
- B $5\sqrt{3}$ cm
- C 10 cm
- D 15 cm

similar triangles

10 To completely cover a spherical ball, a ball company uses a total area of 36 square inches of material. What is the maximum volume the ball can have?

(Note: Surface area of a sphere = $4\pi r^2$. Volume of a sphere = $\frac{4}{3}\pi r^3$.)

- A 27π cubic inches
- B $36\sqrt{\pi}$ cubic inches
- C $\frac{36}{\sqrt{\pi}}$ cubic inches
- D $\frac{27}{\pi}$ cubic inches

$36 = 4\pi r^2$

$r = \sqrt{\frac{36}{4\pi}}$
 $= \frac{6}{2\sqrt{\pi}}$
 $= \frac{3}{\sqrt{\pi}}$
 $V = \frac{4}{3}\pi \left(\frac{3}{\sqrt{\pi}} \right)^3 = \frac{4\pi \cdot 27}{3 \cdot 8\pi\sqrt{\pi}} = \frac{36}{\sqrt{\pi}}$

MATH III — RELEASED ITEMS



- 11 A farmer wants to buy between 90 and 100 acres of land.
- He is interested in a rectangular piece of land that is 1,500 yards long and 300 yards wide.
 - The piece of land is being sold as one complete unit for \$87,000.
- If the farmer does not want to spend more than \$900 an acre, does the land meet all of his requirements? (1 acre \approx 43,560 ft²)
- A Yes, the amount of land satisfies his needs, and the price is low enough.
 B No, the price is low enough, but there is too much land.
 C No, the price is low enough, but there is not enough land.
 D No, the amount of land satisfies what he needs, but the price is too high.

- 12 A reporter wants to know the percentage of voters in the state who support building a new highway. What is the reporter's population?
- A the number of people who live in the state
 B the people who were interviewed in the state
 C all voters over 25 years old in the state
 D all eligible voters in the state

- 13 In a set of test scores that are normally distributed, a test score of 76 is 3 standard deviations below the mean. A score of 88 is 1 standard deviation above the mean. What is the mean of the data?

- A 79
 B 82
 C 84
 D 85

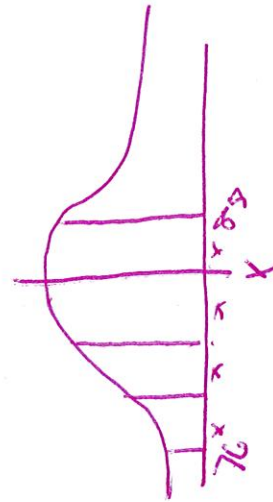
\$87,000

405,000 square ft.

300 yards
 1500 yards
 450,000 ft²

divide by 43560 to get approx 93 acres

$87,000 \div 93 = 935.48$



$76 + 4x = 88$
 $4x = 12$
 $x = 3$

$88 - 3 = 85$

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