Name_______________________ Class__________ Date__________

2-5 Reteaching

Reasoning in Algebra and Geometry

When you solve equations you use the Properties of Equality.

Property	Words	Example
Addition Property	You can add the same number to each side of an equation.	If $x = 2$, then $x + 2 = 4$.
Subtraction Property	You can subtract the same number from each side of an equation.	If $y = 8$, then $y - 3 = 5$.
Multiplication Property	You can multiply by the same number on each side of an equation.	If $z = 2$, then $5z = 10$.
Division Property	You can divide each side an equation by the same number.	If $6m = 12$, then $m = 2$.
Substitution Property	You can exchange a part of an expression with an equivalent value.	If $3x + 5 = 3$ and $x = 2$, then $3(2) + 5 = 3$.

Exercises

Support each conclusion with a reason.

1. Given: $6x + 2 = 12$
Conclusion: $6x = 10$

2. Given: $m\angle 1 + m\angle 2 = 90$
Conclusion: $m\angle 1 = 90 - m\angle 2$

3. Given: $x = m\angle C$
Conclusion: $2x = m\angle C + x$

4. Given: $q - x = r$
Conclusion: $4(q - x) = 4r$

5. Given: $m\angle Q - m\angle R = 90$,
$m\angle Q = 4m\angle R$
Conclusion: $4m\angle R - m\angle R = 90$

6. Given: $CD = AF - 2CD$
Conclusion: $3CD = AF$

7. Given: $5(y - x) = 20$
Conclusion: $5y - 5x = 20$

8. Given: $m\angle AOX = 2m\angle XOB$
$2m\angle XOB = 140$
Conclusion: $m\angle AOX = 140$

9. Order the steps below to complete the proof.
Given: $m\angle P + m\angle Q = 90$, $m\angle Q = 5m\angle P$
Prove: $m\angle Q = 75$
a) $6m\angle P = 90$ by the Distributive Property
b) $m\angle Q = 5 \cdot 15 = 75$ by the Substitution and Multiplication Properties
c) $m\angle P = 15$ by the Division Property
d) $m\angle P + 5m\angle P = 90$ by the Substitution Property

2-8 even
9

Name_______________________ Class__________ Date__________

2-5 Reteaching (continued)

Reasoning in Algebra and Geometry

Several other important properties are also needed to write proofs.

Example	Words
Reflexive Property of Equality $AB = AB$	Any value is equal to itself.
Reflexive Property of Congruence $\angle Z \cong \angle Z$	Any geometric object is congruent to itself.
Symmetric Property of Equality If $XY = ZA$, then $ZA = XY$.	You can change the order of an equality.
Symmetric Property of Congruence If $\angle Q \cong \angle R$, then $\angle R \cong \angle Q$.	You can change the order of a congruence statement.
Transitive Property of Equality If $KL = MN$ and $MN = TR$, then $KL = TR$.	If two values are equal to a third value, Then they are equal to each other.
Transitive Property of Congruence If $\angle 1 \cong \angle 2$, and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.	If two values are congruent to a third value, then they are congruent to each other.

Exercises

Match the property to the appropriate statement.

10. $\overline{RT} \cong \overline{RT}$

11. If $\angle YER \cong \angle IOP$
and $\angle IOP \cong \angle WXZ$
then $\angle YER \cong \angle WXZ$

12. If $\overline{PQ} \cong \overline{MN}$
then $\overline{MN} \cong \overline{PQ}$

13. If $XT = YZ$
and $YZ = UP$
then $XT = UP$

14. $m\angle 1 = m\angle 1$

15. If $m\angle RQS = m\angle TEF$
then $m\angle TEF = m\angle RQS$

a) Reflexive Property of Equality
b) Reflexive Property of Congruence
c) Symmetric Property of Equality
d) Symmetric Property of Congruence
e) Transitive Property of Equality
f) Transitive Property of Congruence

16. Writing Write six new mathematical statements that represent each of the properties given above.

10 - 15

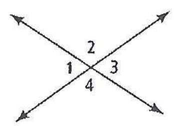
Name_______________ Class_______ Date_______

2-6 Reteaching
Proving Angles Congruent

A *theorem* is a conjecture or statement that you prove true using deductive reasoning. You prove each step using any of the following: given information, definitions, properties, postulates, and previously proven theorems.

The proof is a chain of logic. Each step is justified, and then the Laws of Detachment and Syllogism connect the steps to prove the theorem.

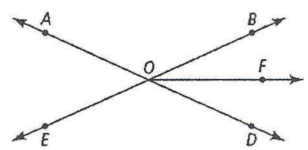
Vertical angles are angles on opposite sides of two intersecting lines. In the figure at the right, $\angle 1$ and $\angle 3$ are vertical angles. $\angle 2$ and $\angle 4$ are also vertical angles. The Vertical Angles Theorem states that vertical angles are always congruent. The symbol $\cong$ means *is congruent to.*



Problem

Given: $m\angle BOF = m\angle FOD$

Prove: $2m\angle BOF = m\angle AOE$



Statements	Reasons
1) $m\angle BOF = m\angle FOD$	1) Given
2) $m\angle BOF + m\angle FOD = m\angle BOD$	2) Angle Addition Postulate
3) $\angle BOF + m\angle BOF = m\angle BOD$	3) Substitution Property
4) $2(m\angle BOF) = m\angle BOD$	4) Combine like terms.
5) $\angle AOE = \angle BOD$	5) Vertical Angles are $\cong$.
6) $m\angle AOE = m\angle BOD$	6) Definition of Congruence
7) $2m\angle BOF = m\angle AOE$	7) Substitution Property

Exercises

Write a paragrap0068 proof.

1. **Given:** $\angle AOB$ and $\angle XOZ$ are vertical angles.

   $m\angle AOB = 80$

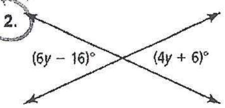
   $m\angle XOZ = 6x + 5$

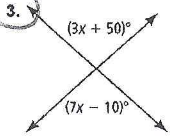
   Prove: $x = 12.5$

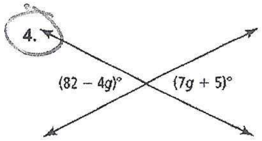
Name_______________ Class_______ Date_______

2-6 Reteaching (continued)
Proving Angles Congruent

Find the value of each variable.

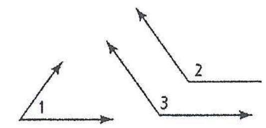
2. 

3. 

4. 

You can use numbers to help understand theorems that may seem confusing.

Congruent Supplements Theorem: If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.



Think about it: Suppose $m\angle 1 = 50$. Any angle supplementary to $\angle 1$ must have a measure of 130. So, supplements of $\angle 1$ must be congruent.

If $\angle 2$ and $\angle 3$ are both supplementary to $\angle 1$, then $\angle 2 \cong \angle 3$.

Congruent Complements Theorem: If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.

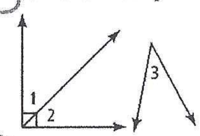
If $\angle 4$ and $\angle 5$ are both complementary to $\angle 6$, then $\angle 4 \cong \angle 5$.

Think about it: Suppose $m\angle 6 = 30$. Any complement of $\angle 6$ has a measure of 60. So, all complements of $\angle 6$ must be congruent.

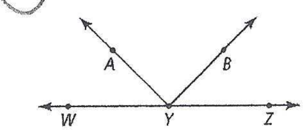
Exercises

Name a pair of congruent angles in each figure. Justify your answer.

5. **Given:** $\angle 2$ is complementary to $\angle 3$.



6. **Given:** $\angle AYZ \cong \angle BYW$



7. **Reasoning** Explain why the following statement is true. Use numbers in your explanation. "If $\angle 1$ is supplementary to $\angle 2$, $\angle 2$ is supplementary to $\angle 3$, $\angle 3$ is supplementary to $\angle 4$, and $\angle 4$ is supplementary to $\angle 5$, then $\angle 1 \cong \angle 5$."

Name_____________________ Class __________ Date __________

3-1 Reteaching
Lines and Angles

Not all lines and planes intersect.

- Planes that do not intersect are *parallel planes.*
- Lines that are in the same plane and do not intersect are *parallel.*
- The symbol || shows that lines or planes are parallel: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ means "Line AB is parallel to line CD."
- Lines that are not in the same plane and do not intersect are *skew.*

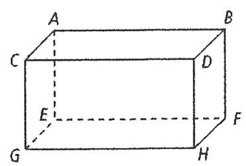
Parallel planes: plane $ABDC$ || plane $EFHG$

plane $BFHD$ || plane $AEGC$

plane $CDHG$ || plane $ABFE$

Examples of parallel lines: $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB} \parallel \overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$

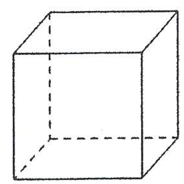
Examples of skew lines: $\overleftrightarrow{CD}$ is skew to $\overleftrightarrow{BF}$, $\overleftrightarrow{AE}$, $\overleftrightarrow{EG}$, and $\overleftrightarrow{FH}$.



Exercises

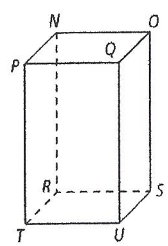
In Exercises 1–3, use the figure at the right.

1. Shade one set of parallel planes.
2. Trace one set of parallel lines with a solid line.
3. Trace one set of skew lines with a dashed line.



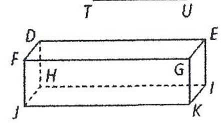
In Exercises 4–7, use the diagram to name each of the following.

4. a line that is parallel to $\overleftrightarrow{RS}$
5. a line that is skew to $\overleftrightarrow{QU}$
6. a plane that is parallel to $NRTP$
7. three lines that are parallel to $\overleftrightarrow{OQ}$



In Exercises 8–11, describe the statement as *true* or *false*. If *false*, explain.

8. plane $HIKJ$ □ plane $IEGK$
9. $\overleftrightarrow{DH} \parallel \overleftrightarrow{GK}$
10. $\overleftrightarrow{HJ}$ and $\overleftrightarrow{HD}$ are skew lines.
11. $\overleftrightarrow{FG} \parallel \overleftrightarrow{KI}$



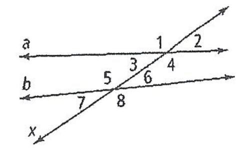
Name_____________________ Class __________ Date __________

3-1 Reteaching (continued)
Lines and Angles

The diagram shows lines a and b intersected by line x.

Line x is a transversal. A transversal is a line that intersects two or more lines found in the same plane.

The angles formed are either interior angles or exterior angles.



Interior Angles

between the lines cut by the transversal

$\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ in diagram above

Exterior Angles

outside the lines cut by the transversal

$\angle 1$, $\angle 2$, $\angle 7$, and $\angle 8$ in diagram above

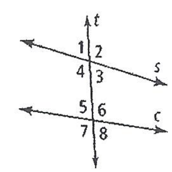
Four types of special angle pairs are also formed.

Angle Pair	Definition	Examples
alternate interior	inside angles on opposite sides of the transversal, not a linear pair	$\angle 3$ and $\angle 6$ $\angle 4$ and $\angle 5$
alternate exterior	outside angles on opposite sides of the transversal, not a linear pair	$\angle 1$ and $\angle 8$ $\angle 2$ and $\angle 7$
Same-side interior	inside angles on the same side of the transversal	$\angle 3$ and $\angle 5$ $\angle 4$ and $\angle 6$
Corresponding	in matching positions above or below the transversal, but on the same side of the transversal	$\angle 1$ and $\angle 5$ $\angle 3$ and $\angle 7$ $\angle 2$ and $\angle 6$ $\angle 4$ and $\angle 8$

Exercises

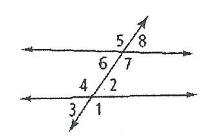
Use the diagram at the right to answer Exercises 12–15.

12. Name all pairs of corresponding angles.
13. Name all pairs of alternate interior angles.
14. Name all pairs of same-side interior angles.
15. Name all pairs of alternate exterior angles.



Use the diagram at the right to answer Exercises 16 and 17. Decide whether the angles are *alternate interior angles, same-side interior angles, corresponding,* or *alternate exterior angles.*

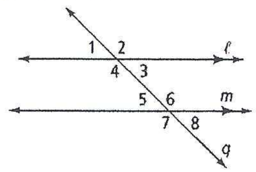
16. $\angle 1$ and $\angle 5$
17. $\angle 4$ and $\angle 6$



3-2 Reteaching
Properties of Parallel Lines

When a transversal intersects parallel lines, special congruent and supplementary angle pairs are formed.

Congruent angles formed by a transversal intersecting parallel lines:



- corresponding angles (Postulate 3-1)

  $\angle 1 \cong \angle 5$ $\quad$ $\angle 2 \cong \angle 6$

  $\angle 4 \cong \angle 7$ $\quad$ $\angle 3 \cong \angle 8$

- alternate interior angles (Theorem 3-1)

  $\angle 4 \cong \angle 6$ $\quad$ $\angle 3 \cong \angle 5$

- alternate exterior angles (Theorem 3-3)

  $\angle 1 \cong \angle 8$ $\quad$ $\angle 2 \cong \angle 7$

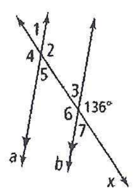
Supplementary angles formed by a transversal intersecting parallel lines:

- same-side interior angles (Theorem 3-2)

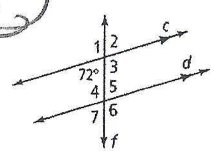
  $m\angle 4 + m\angle 5 = 180$ $\quad$ $m\angle 3 + m\angle 6 = 180$

Identify all the numbered angles congruent to the given angle. Explain.

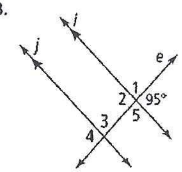
1.



2.



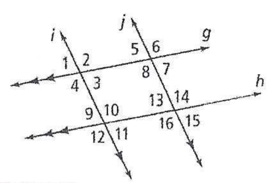
3.



4. Supply the missing reasons in the two-column proof.

Given: $g \parallel h$, $i \parallel j$

Prove: $\angle 1$ is supplementary to $\angle 16$.

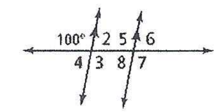


Statements	Reasons
1) $\angle 1 \cong \angle 3$	1) ?
2) $g \parallel h$; $i \parallel j$	2) Given
3) $\angle 3 \cong \angle 11$	3) ?
4) $\angle 11$ and $\angle 16$ are supplementary.	4) ?
5) $\angle 1$ and $\angle 16$ are supplementary.	5) ?

3-2 Reteaching (continued)
Properties of Parallel Lines

You can use the special angle pairs formed by parallel lines and a transversal to find missing angle measures.

Problem

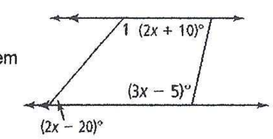


If $m\angle 1 = 100$, what are the measures of $\angle 2$ through $\angle 8$?

Supplementary angles:	$m\angle 2 = 180 - 100$	$m\angle 2 = 80$
	$m\angle 4 = 180 - 100$	$m\angle 4 = 80$
Vertical angles:	$m\angle 1 = m\angle 3$	$m\angle 3 = 100$
Alternate exterior angles:	$m\angle 1 = m\angle 7$	$m\angle 7 = 100$
Alternate interior angles:	$m\angle 3 = m\angle 5$	$m\angle 5 = 100$
Corresponding angles:	$m\angle 2 = m\angle 6$	$m\angle 6 = 80$
Same-side interior angles:	$m\angle 3 + m\angle 8 = 180$	$m\angle 8 = 80$

Problem

What are the measures of the angles in the figure?



$(2x + 10) + (3x - 5) = 180$	Same-Side Interior Angles Theorem
$5x + 5 = 180$	Combine like terms.
$5x = 175$	Subtract 5 from each side.
$x = 35$	Divide each side by 5.

Find the measure of these angles by substitution.

$2x + 10 = 2(35) + 10 = 80$ $\qquad$ $3x - 5 = 3(35) - 5 = 100$

$2x - 20 = 2(35) - 20 = 50$

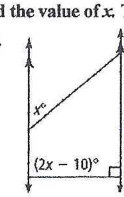
To find $m\angle 1$, use the Same-Side Interior Angles Theorem:

$50 + m\angle 1 = 180$, so $m\angle 1 = 130$

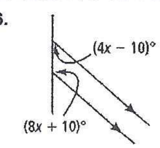
Exercises

Find the value of x. Then find the measure of each labeled angle.

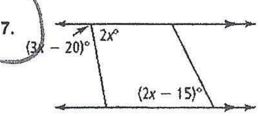
5.



6.



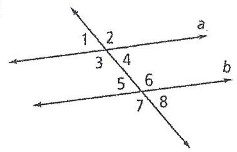
7.



3-3 Reteaching

Proving Lines Parallel

Special angle pairs result when a set of parallel lines is intersected by a transversal. The converses of the theorems and postulates in Lesson 3-2 can be used to prove that lines are parallel.



Postulate 3-2: Converse of Corresponding Angles Postulate

If $\angle 1 \cong \angle 5$, then $a \parallel b$.

Theorem 3-4: Converse of the Alternate Interior Angles Theorem If $\angle 3 \cong \angle 6$, then $a \parallel b$.

Theorem 3-5: Converse of the Same-Side Interior Angles Theorem

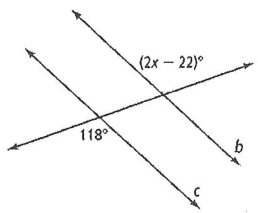
If $\angle 3$ is supplementary to $\angle 5$, then $a \parallel b$.

Theorem 3-6: Converse of the Alternate Exterior Angles Theorem

If $\angle 2 \cong \angle 7$, then $a \parallel b$.

Problem

For what value of x is $b \parallel c$?



The given angles are alternate exterior angles. If they are congruent, then $b \parallel c$.

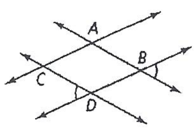
$2x - 22 = 118$

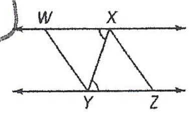
$2x = 140$

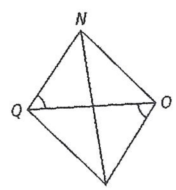
$x = 70$

Exercises

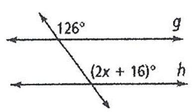
Which lines or line segments are parallel? Justify your answers.

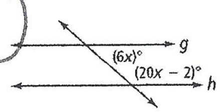
1. 

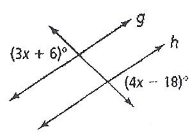
2. 

3. 

Find the value of x for which $g \parallel h$. Then find the measure of each labeled angle.

4. 

5. 

6. 

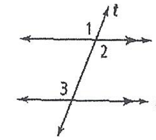
3-3 Reteaching (continued)

Proving Lines Parallel

A flow proof is a way of writing a proof and a type of graphic organizer. Statements appear in boxes with the reasons written below. Arrows show the logical connection between the statements.

Problem

Write a flow proof for Theorem 3-1: If a transversal intersects two parallel lines, then alternate interior angles are congruent.



Given: $\ell \parallel m$

Prove: $\angle 2 \cong \angle 3$

$\boxed{\ell \parallel m}$ → $\boxed{\angle 1 \cong \angle 3}$ → $\boxed{\angle 2 \cong \angle 3}$

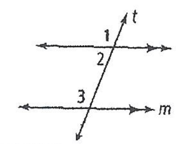
Given; If $\parallel$ lines, then corresponding $\angle$s are $\cong$.; Transitive Property of $\cong$

$\boxed{\angle 1 \cong \angle 2}$ → $\boxed{\angle 2 \cong \angle 3}$

Vertical angles are $\cong$.

Exercises

Complete a flow proof for each.

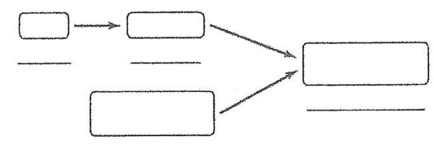


7. Complete the flow proof for Theorem 3-2 using the following steps. Then write the reasons for each step.
   a. $\angle 2$ and $\angle 3$ are supplementary. b. $\angle 1 \cong \angle 3$
   c. $\ell \parallel m$ d. $\angle 1$ and $\angle 2$ are supplementary.

   Theorem 3-2: If a transversal intersects two parallel lines, then same side interior angles are supplementary.

   Given: $\ell \parallel m$

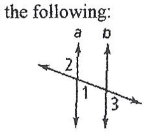
   Prove: $\angle 2$ and $\angle 3$ are supplementary.



8. Write a flow proof for the following:

   Given: $\angle 2 \cong \angle 3$

   Prove: $a \parallel b$



Name________________________ Class____________ Date____________

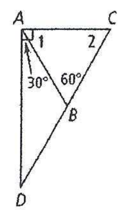
3-5 Reteaching
Parallel Lines and Triangles

Triangle Angle-Sum Theorem:

The measures of the angles in a triangle add up to 180.

Problem

In the diagram at the right, ΔACD is a right triangle. What are $m\angle 1$ and $m\angle 2$?



Step 1

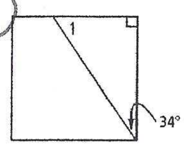
$m\angle 1 + m\angle DAB = 90$	Angle Addition Postulate
$m\angle 1 + 30 = 90$	Substitution Property
$m\angle 1 = 60$	Subtraction Property of Equality

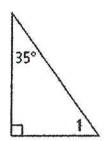
Step 2

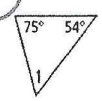
$m\angle 1 + m\angle 2 + m\angle ABC = 180$	Triangle Angle-Sum Theorem
$60 + m\angle 2 + 60 = 180$	Substitution Property
$m\angle 2 + 120 = 180$	Addition Property of Equality
$m\angle 2 = 60$	Subtraction Property of Equality

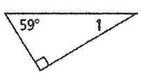
Exercises

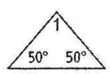
Find $m\angle 1$.

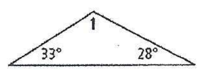
1. 

2. 

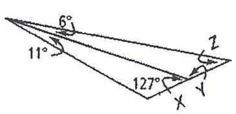
3. 

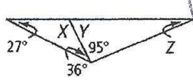
4. 

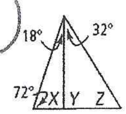
5. 

6. 

Algebra Find the value of each variable.

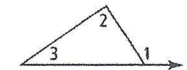
7. 

8. 

9. 

Name________________________ Class____________ Date____________

3-5 Reteaching (continued)
Parallel Lines and Triangles



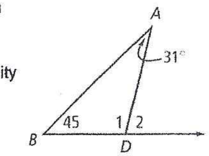
In the diagram at the right, $\angle 1$ is an exterior angle of the triangle. An exterior angle is an angle formed by one side of a polygon and an extension of an adjacent side.

For each exterior angle of a triangle, the two interior angles that are not next to it are its remote interior angles. In the diagram, $\angle 2$ and $\angle 3$ are remote interior angles to $\angle 1$.

The *Exterior Angle Theorem* states that the measure of an exterior angle is equal to the sum of its remote interior angles. So, $m\angle 1 = m\angle 2 + m\angle 3$.

Problem

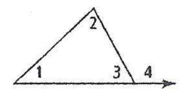
What are the measures of the unknown angles?



$m\angle ABD + m\angle BDA + m\angle BAD = 180$	Triangle Angle-Sum Theorem
$45 + m\angle 1 + 31 = 180$	Substitution Property
$m\angle 1 = 104$	Subtraction Property of Equality
$m\angle ABD + m\angle BAD = m\angle 2$	Exterior Angle Theorem
$45 + 31 = m\angle 2$	Substitution Property
$76 = m\angle 2$	Subtraction Property of Equality

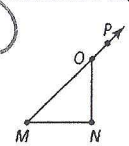
Exercises

What are the exterior angle and the remote interior angles for each triangle?

10. 

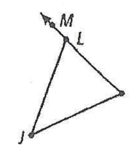
exterior:

interior:

11. 

exterior:

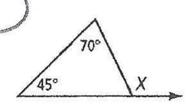
interior:

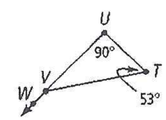
12. 

exterior:

interior:

Find the measure of the exterior angle.

13. 

14. 

15.