Name______________________________________ Class_____________ Date____________

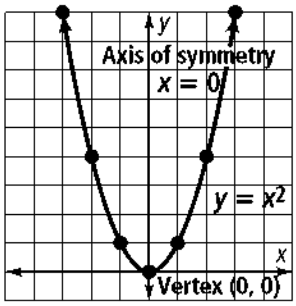
4-1 Reteaching

Quadratic Functions and Transformations

Parent Quadratic Function

The parent quadratic function is $y = x^2$.

Substitute 0 for x in the function to get $y = 0$. The vertex of the parent quadratic function is (0, 0).



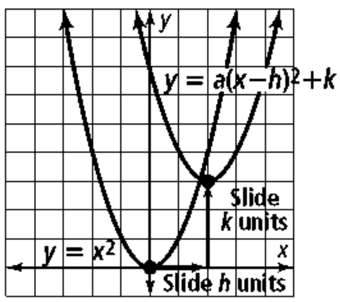
A few points near the vertex are:

x	–3	–2	–1	1	2	3
y	9	4	1	1	4	9

The graph is symmetrical about the line $x = 0$. This line is the axis of symmetry.

Vertex Form of a Quadratic Function

The vertex form of a quadratic function is $y = a(x - h)^2 + k$. The graph of this function is a transformation of the graph of the parent quadratic function $y = x^2$. The vertex of the graph is (h, k). If $a = 1$, you can graph the function by sliding the graph of the parent function h units along the x-axis and k units along the y-axis.



Problem

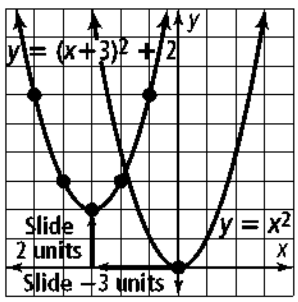
What is the graph of $y = (x + 3)^2 + 2$? What are the vertex and axis of symmetry of the function?

Step 1 Write the function in vertex form: $y = 1[x - (-3)]^2 + 2$

Step 2 Find the vertex: $h = -3$, $k = 2$. The vertex is (–3, 2).

Step 3 Find the axis of symmetry. Since the vertex is (–3, 2), the graph is symmetrical about the line $x = -3$. The axis of symmetry is $x = -3$.

Step 4 Because $a = 1$, you can graph this function by sliding the graph of the parent function –3 units along the x-axis and 2 units along the y-axis. Plot a few points near the vertex to help you sketch the graph.



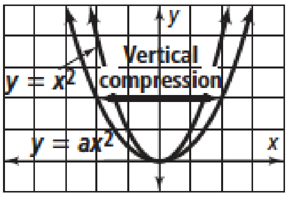
x	–5	–4	–3	–2	–1
y	6	3	2	3	6

4-1 Reteaching (continued)

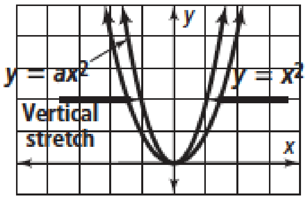
Quadratic Functions and Transformations

If $a \neq 1$, the graph is a stretch or compression of the parent function by a factor of $|a|$.

$0 < |a| < 1$
The graph is a vertical compression of the parent function.

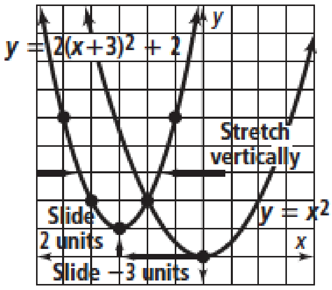


$|a| > 1$
The graph is a vertical stretch of the parent function



What is the graph of $y = 2(x + 3)^2 + 2$?

Problem



Step 1 Write the function in vertex form: $y = 2[x - (-3)]^2 + 2$

Step 2 The vertex is $(-3, 2)$.

Step 3 The axis of symmetry is $x = -3$.

Step 4 Because $a = 2$, the graph of this function is a vertical stretch by 2 of the parent function. In addition to sliding the graph of the parent function 3 units left and 2 units up, you must change the shape of the graph. Plot a few points near the vertex to help you sketch the graph.

x	–5	–4	–3	–2	–1
y	10	4	2	4	10

Exercises

Graph each function. Identify the vertex and axis of symmetry.

1. $y = (x - 1)^2 + 3$

2. $y = (x + 4)^2 - 2$

3. $y = (x + 2)^2 + 1$

4. $y = 2(x - 1)^2 + 3$

5. $y = \frac{1}{2}(x + 4)^2 - 2$

6. $y = 0.9(x + 2)^2 + 1$

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4-2 Reteaching

Standard Form of a Quadratic Function

- The graph of a quadratic function, $y = ax^2 + bx + c$, where $a \neq 0$, is a parabola.
- The axis of symmetry is the line $x = -\frac{b}{2a}$.
- The x-coordinate of the vertex is $-\frac{b}{2a}$. The y-coordinate of the vertex is $y = f\left(-\frac{b}{2a}\right)$, or the y-value when $x = -\frac{b}{2a}$.
- The y-intercept is $(0, c)$.

Problem

What is the graph of $y = 2x^2 - 8x + 5$?

$x = -\frac{b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$ — Find the equation of the axis of symmetry.

x-coordinate of vertex: 2 — $-\frac{b}{2a}$

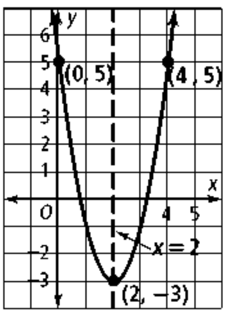
$f\left(-\frac{b}{2a}\right) = f(2)^2 - 8(2) + 5$ — Find the y-value when $x = 2$.

$= 8 - 16 + 5$

$= -3$

y-coordinate of vertex: –3 — The vertex is (2, –3).

y-intercept: (0, 5) — The y-intercept is at $(0, c) = (0, 5)$.



Because a is positive, the graph opens upward, and the vertex is at the bottom of the graph. Plot the vertex and draw the axis of symmetry. Plot (0, 5) and its corresponding point on the other side of the axis of symmetry.

Exercises

Graph each parabola. Label the vertex and the axis of symmetry.

1. $y = -3x^2 + 6x - 9$

2. $y = -x^2 - 8x - 15$

3. $y = 2x^2 - 8x + 1$

4. $y = -2x^2 - 12x - 7$

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4-2 Reteaching (continued)

Standard Form of a Quadratic Function

- Standard form of a quadratic function is $y = ax^2 + bx + c$. Vertex form of a quadratic function is $y = a(x - h)^2 + k$.
- For a parabola in vertex form, the coordinates of the vertex are (h, k).

Problem

What is the vertex form of $y = 3x^2 - 24x + 50$?

$y = ax^2 + bx + c$	
$y = 3x^2 - 24x + 50$	Verify that the equation is in standard form.
$b = -24,\ a = 3$	Find b and a.
$x\text{-coordinate} = -\dfrac{b}{2a}$	For an equation in standard form, the x-coordinate of the vertex can be found by using $x = -\dfrac{b}{2a}$.
$= -\dfrac{-24}{2(3)}$	Substitute.
$= 4$	Simplify.
$y\text{-coordinate} = 3(4)^2 - 24(4) + 50$	Substitute 4 into the standard form to find the y-coordinate.
$= 2$	Simplify.
$y = 3(x - 4)^2 + 2$	Substitute 4 for h and 2 for k into the vertex form.

Once the conversion to vertex form is complete, check by multiplying.

$y = 3(x^2 - 8x + 16) + 2$
$y = 3x^2 - 24x + 50$
The result is the standard form of the equation.

Exercises

Write each function in vertex form. Check your answers.

5. $y = x^2 - 2x - 3$ **6.** $y = -x^2 + 4x + 6$ **7.** $y = x^2 + 3x - 10$

8. $y = x^2 - 9x$ **9.** $y = x^2 + x$ **10.** $y = x^2 + 5x + 4$

11. $y = 4x^2 + 8x - 3$ **12.** $y = \dfrac{3}{4}x^2 + 9x$ **13.** $y = -2x^2 + 2x + 1$

Write each function in standard form.

14. $y = (x - 3)^2 + 1$ **15.** $y = 2(x - 1)^2 - 3$ **16.** $y = -3(x + 4)^2 + 1$

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4-4 Reteaching

Factoring Quadratic Expressions

Problem

What is $6x^2 - 5x - 4$ in factored form?

$a = 6$, $b = -5$, and $c = -4$ — Find *a*, *b*, and *c*; they are the coefficients of each term.

$ac = -24$ and $b = -5$ — We are looking for factors with product *ac* and sum *b*.

Factors of –24	1, –24	–1,–24	2,–12	–2,12	3, –8	–3,8	4, –6	–4, 6
Sum of factors	–23	23	–10	10	–5	5	–2	2

The factors 3 and –8 are the combination whose sum is –5.

$\underbrace{6x^2 + 3x} - \underbrace{8x - 4}$ — Rewrite the middle term using the factors you found.

$3x(2x + 1) - 4(2x + 1)$ — Find common factors by grouping the terms in pairs.

$(3x - 4)(2x + 1)$ — Rewrite using the Distributive Property.

Check $(3x - 4)(2x + 1)$ You can check your answer by multiplying the factors together.

$6x^2 + 3x - 8x - 4$

$6x^2 - 5x - 4$

Remember that not all quadratic expressions are factorable.

Exercises

Factor each expression.

1. $x^2 + 6x + 8$

2. $x^2 - 4x + 3$

3. $2x^2 - 6x + 4$

4. $2x^2 - 11x + 5$

5. $2x^2 - 7x - 4$

6. $4x^2 + 16x + 15$

7. $x^2 - 5x - 14$

8. $7x^2 - 19x - 6$

9. $x^2 - x - 72$

10. $2x^2 + 9x + 7$

11. $x^2 + 12x + 32$

12. $4x^2 - 28x + 49$

13. $x^2 - 3x - 10$

14. $2x^2 + 9x + 4$

15. $9x^2 - 6x + 1$

16. $x^2 - 10x + 9$

17. $x^2 + 4x - 12$

18. $x^2 + 7x + 10$

19. $x^2 - 8x + 12$

20. $2x^2 - 5x - 3$

21. $x^2 - 6x + 5$

22. $3x^2 + 2x - 8$

4-4 Reteaching (continued)

Factoring Quadratic Expressions

- $a^2 + 2ab + b^2 = (a + b)^2$ Factoring perfect square trinomials
  $a^2 - 2ab + b^2 = (a - b)^2$
- $a^2 - b^2 = (a + b)(a - b)$ Factoring a difference of two squares

Problem

What is $25x^2 - 20x + 4$ in factored form?

There are three terms. Therefore, the expression may be a perfect square trinomial.

$a^2 = 25x^2$ and $b^2 = 4$ Find a^2 and b^2.

$a = 5x$ and $b = 2$ Take square roots to find a and b.

Check that the choice of a and b gives the correct middle term.

$2ab = 2 \cdot 5x \cdot 2 = 20x$

Write the factored form.

$a^2 - 2ab + b^2 = (a - b)^2$

$25x^2 - 20x + 4 = (5x - 2)^2$

Check $(5x - 2)^2$ You can check your answer by multiplying the factors together.

$(5x - 2)(5x - 2)$ Rewrite the square in expanded form.

$25x^2 - 10x - 10x + 4$ Distribute.

$25x^2 - 20x + 4$ Simplify.

Exercises

Factor each expression.

23. $x^2 - 12x + 36$

24. $x^2 + 30x + 225$

25. $9x^2 - 12x + 4$

26. $x^2 - 64$

27. $9x^2 - 42x + 49$

28. $25x^2 - 1$

29. $27x^2 - 12$

30. $49x^2 + 42x + 9$

31. $16x^2 - 32x + 16$

32. $9x^2 - 16$

33. $8x^2 - 18$

34. $81x^2 + 126x + 49$

35. $125x^2 - 100x + 20$

36. $-x^2 + 196$

37. $-16x^2 - 24x - 9$

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4-5 Reteaching

Quadratic Equations

There are several ways to solve quadratic equations. If you can factor the quadratic expression in a quadratic equation written in standard form, you can use the Zero-Product Property.

If $ab = 0$ then $a = 0$ or $b = 0$.

Problem

What are the solutions of the quadratic equation $2x^2 + x = 15$?

$2x^2 + x = 15$	Write the equation.
$2x^2 + x - 15 = 0$	Rewrite in standard form, $ax^2 + bx + c = 0$.
$(2x - 5)(x + 3) = 0$	Factor the quadratic expression (the nonzero side).
$2x - 5 = 0$ or $x + 3 = 0$	Use the Zero-Product Property.
$2x = 5$ or $x = -3$	Solve for x.
$x = \frac{5}{2}$ or $x = -3$	

Check the solutions:

$x = \frac{5}{2}$: $2\left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right) \stackrel{?}{=} 15$

$\frac{25}{2} + \frac{5}{2} \stackrel{?}{=} 15$

$15 = 15$

$x = -3$: $2(-3)^2 + (-3) \stackrel{?}{=} 15$

$18 - 3 \stackrel{?}{=} 15$

$15 = 15$

Both solutions check. T e solutions are $x = \frac{5}{2}$ and $x = -3$.

Exercises

Solve each equation by factoring. Check your answers.

1. $x^2 - 10x + 16 = 0$ **2.** $x^2 + 2x = 63$ **3.** $x^2 + 9x = 22$

4. $x^2 - 24x + 144 = 0$ **5.** $2x^2 = 7x + 4$ **6.** $2x^2 = -5x + 12$

7. $x^2 - 7x = -12$ **8.** $2x^2 + 10x = 0$ **9.** $x^2 + x = 2$

10. $3x^2 - 5x + 2 = 0$ **11.** $x^2 = -5x - 6$ **12.** $x^2 + x = 20$

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4-5 Reteaching (continued)

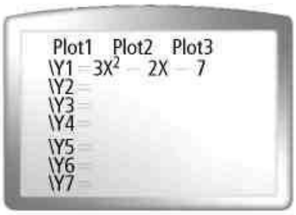
Quadratic Equations

Some quadratic equations are dif cult or impossible to solve by factoring. You can use a graphing calculator to find the points where the graph of a function intersects the x-axis. At these points $f(x) = 0$, so x is a zero of the function.

The values r_1 and r_2 are the zeros of the function $y = (x - r_1)(x - r_2)$. The graph of the function intersects the x-axis at $x = r_1$, or $(r_1, 0)$, and $x = r_2$, or $(r_2, 0)$.

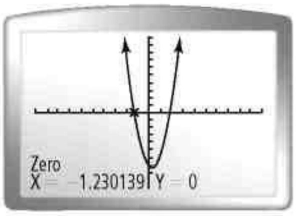
Problem

What are the solutions of the quadratic equation $3x^2 = 2x + 7$?



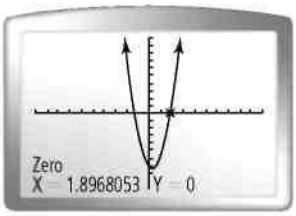
Step 1 Rewrite the equation in standard form, $ax^2 + bx + c = 0$.
$3x^2 - 2x - 7 = 0$

Step 2 Enter the equation as Y1 in your calculator.



Step 3 Graph Y1. Choose the standard window and see if the zeros of the function Y1 are visible on the screen. If they are not visible, zoom out and determine a better viewing window. In this case, the zeros are visible in the standard window.

Step 4 Use the ZERO option in the CALC feature. For the first zero, choose bounds of –2 and –1 and a guess of –1.5. The screen display gives the first zero as $x = -1.230139$.



Similarly, the screen display gives the second zero as $x = 1.8968053$.

The solutions to two decimal places are $x = -1.23$ and $x = 1.90$.

Exercises

Solve the equation by graphing. Give each answer to at most two decimal places.

13. $x^2 = 5$

14. $x^2 = 5x + 1$

15. $x^2 + 7x = 3$

16. $x^2 + x = 5$

17. $x^2 + 3x + 1 = 0$

18. $x^2 = 2x + 4$

19. $3x^2 - 5x + 9 = 8$

20. $4 = 2x^2 + 3x$

21. $x^2 - 6x = -7$

22. $-x^2 = 8x + 8$

4-7 Reteaching

The Quadratic Formula

You can solve some quadratic equations by factoring or completing the square. You can solve any quadratic equation $ax^2 + bx + c = 0$ by using the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Notice the ± symbol in the formula. Whenever $b^2 - 4ac$ is not zero, the Quadratic Formula will result in two solutions.

Problem

What are the solutions for $2x^2 + 3x = 4$? Use the Quadratic Formula.

$2x^2 + 3x - 4 = 0$ — Write the equation in standard form: $ax^2 + bx + c = 0$

$a = 2;\ b = 3;\ c = -4$ — a is the coefficient of x^2, b is the coefficient of x, c is the constant term.

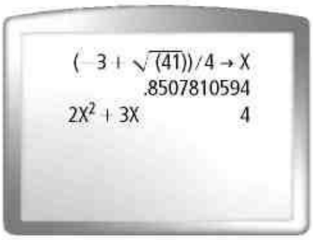
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Write the Quadratic Formula.

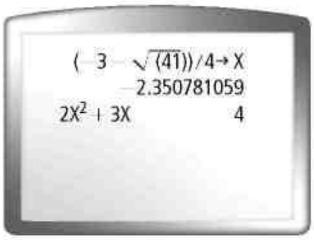
$= \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-4)}}{2(2)}$ — Substitute 2 for a, 3 for b, and −4 for c.

$= \frac{-3 \pm \sqrt{41}}{4}$ — Simplify.

$= \frac{-3 + \sqrt{41}}{4}$ or $= \frac{-3 - \sqrt{41}}{4}$ — Write the solutions separately.

Check your results on your calculator. Replace x in the original equation with $= \frac{-3+\sqrt{41}}{4}$ and $\frac{-3-\sqrt{41}}{4}$. Both values for x give a result of 4. The solutions check.





Exercises

What are the solutions for each equation? Use the Quadratic Formula.

1. $-x^2 + 7x - 3 = 0$

2. $x^2 + 6x = 10$

3. $2x^2 = 4x + 3$

4. $4x^2 + 81 = 36x$

5. $2x^2 + 1 = 5 - 7x$

6. $6x^2 - 10x + 3 = 0$

4-7 Reteaching (continued)

The Quadratic Formula

There are three possible outcomes when you take the square root of a real number *n*:

$$n \begin{cases} > 0 & \rightarrow \\ = 0 & \rightarrow \\ < 0 & \rightarrow \end{cases}$$

two real values (one positive and one negative)

one real value (0)

no real values

Now consider the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The value under the radical symbol determines the number of real solutions that exist for the equation $ax^2 + bx + c = 0$:

$$b^2 - 4ac \begin{cases} > 0 & \rightarrow & \text{two real solutions} \\ = 0 & \rightarrow & \text{one real solution} \\ < 0 & \rightarrow & \text{no real solutions} \end{cases}$$

The value under the radical, $b^2 - 4ac$, is called the **discriminant**.

Problem

What is the number of real solutions of $-3x^2 + 7x = 2$?

$-3x^2 + 7x = 2$	
$-3x^2 + 7x - 2 = 0$	Write in standard form.
$a = -3, b = 7, c = -2$	Find the values of *a*, *b*, and *c*.
$b^2 - 4ac$	Write the discriminant.
$(7)^2 - 4(-3)(-2)$	Substitute for *a*, *b*, and *c*.
$49 - 24$	Simplify.
25	

The discriminant, 25, is positive. The equation has two real roots.

Exercises

What is the value of the discriminant and what is the number of real solutions for each equation?

7. $x^2 + x - 42 = 0$ **8.** $-x^2 + 13x - 40 = 0$ **9.** $x^2 + 2x + 5 = 0$

10. $x^2 = 18x - 81$ **11.** $-x^2 + 7x + 44 = 0$ **12.** $\frac{1}{4}x^2 - 5x + 25 = 0$

13. $2x^2 + 7 = 5x$ **14.** $4x^2 + 25x = 21$ **15.** $x^2 + 5 = 3x$

16. $\frac{1}{9}x^2 = 4x - 36$ **17.** $\frac{1}{2}x^2 + 2x + 3 = 0$ **18.** $\frac{1}{6}x^2 = 2x + 18$

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4-8 Reteaching

Complex Numbers

- A *complex number* consists of a real part and an imaginary part. It is written in the form $a + bi$, where a and b are real numbers.
- $i = \sqrt{-1}$ and $i^2 = \left(\sqrt{-1}\right)\left(\sqrt{-1}\right) = -1$
- When adding or subtracting complex numbers, combine the real parts and then combine the imaginary parts.
- When multiplying complex numbers, use the Distributive Property or FOIL.

Problem

What is $(3 - i) + (2 + 3i)$?

$(3 - i) + (2 + 3i)$

$= \textcircled{3} - \boxed{i} + \textcircled{2} + \boxed{3i}$ Circle real parts. Put a square around imaginary parts.

$= (3 + 2) + (-1 + 3)i$ Combine.

$= 5 + 2i$ Simplify.

Problem

What is the product $(7 - 3i)(-4 + 9i)$?

Use FOIL to multiply:

$(7 - 3i)(-4 + 9i)$

First $= 7(-4)$

Outer $= 7(9i)$

Inner $= (-3i)(-4)$

Last $= (-3i)(9i)$

$(7 - 3i)(-4 + 9i) = 7(-4) + 7(9i) + (-3i)(-4) + (-3i)(9i)$

$= -28 + 63i + 12i - 27i^2$

$= -28 + 75i - 27i^2$

You can simplify the expression by substituting -1 for i^2.

$(7 - 3i)(-4 + 9i) = -28 + 75i - 27(-1)$

$= -1 + 75i$

Exercises

Simplify each expression.

1. $2i + (-4 - 2i)$

2. $(3 + i)(2 + i)$

3. $(4 + 3i)(1 + 2i)$

4. $3i(1 - 2i)$

5. $3i(4 - i)$

6. $3 - (-2 + 3i) + (-5 + i)$

7. $4i(6 - 2i)$

8. $(5 + 6i) + (-2 + 4i)$

9. $9(11 + 5i)$

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4-8 Reteaching (continued)

Complex Numbers

- The *complex conjugate* of a complex number $a + bi$ is the complex number $a - bi$.
- $(a + bi)(a - bi) = a^2 + b^2$
- To divide complex numbers, use complex conjugates to simplify the denominator.

Problem

What is the quotient $\frac{4+5i}{2-i}$?

$\frac{4+5i}{2-i}$ — The complex conjugate of $2 - i$ is $2 + i$.

$= \frac{4+5i}{2-i} \cdot \frac{2+i}{2+i}$ — Multiply both numerator and denominator $2 + i$.

$= \frac{8+4i+10i+5i^2}{(2-i)(2+i)}$ — Use FOIL to multiply the numerators.

$= \frac{8+4i+10i+5i^2}{2^2+1^2}$ — Simplify the denominator. $(a + bi)(a - bi) = a^2 + b^2$

$= \frac{8+14i+5(-1)}{4+1}$ — Substitute -1 for i^2.

$= \frac{3+14i}{5}$ — Simplify.

$= \frac{3}{5} + \frac{14}{5}i$ — Write as a complex number $a + bi$.

Exercises

Find the complex conjugate of each complex number.

10. $1 - 2i$ **11.** $3 + 5i$ **12.** i

13. $3 - i$ **14.** $2 + 3i$ **15.** $-5 - 2i$

Write each quotient as a complex number.

16. $\frac{3i}{1-2i}$ **17.** $\frac{6}{3+5i}$ **18.** $\frac{2+2i}{i}$

19. $\frac{2+5i}{3-i}$ **20.** $\frac{-4-i}{2+3i}$ **21.** $\frac{6+i}{-5-2i}$

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4-6 Reteaching

Completing the Square

Completing a perfect square trinomial allows you to factor the completed trinomial as the square of a binomial.

Start with the expression $x^2 + bx$. Add $\left(\frac{b}{2}\right)^2$. Now the expression is $x^2 + bx + \left(\frac{b}{2}\right)^2$,

which can be factored into the square of a binomial: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$.

To complete the square for an expression $ax^2 + abx$, first factor out a. Then find the value that completes the square for the factored expression.

Problem

What value completes the square for $-2x^2 + 10x$?

Think	Write
Write the expression in the form $a(x^2 + bx)$.	$-2x^2 + 10x = -2(x^2 - 5x)$
Find $\frac{b}{2}$.	$\frac{b}{2} = \frac{-5}{2} = -\frac{5}{2}$
Add $\left(\frac{b}{2}\right)^2$ to the inner expression to complete the square.	$-2\left[x^2 - 5x + \left(-\frac{5}{2}\right)^2\right] = -2\left(x^2 - 5x + \frac{25}{4}\right)$
Factor the perfect square trinomial.	$-2\left(x - \frac{5}{2}\right)^2$
Find the value that completes the square.	$-2\left(\frac{25}{4}\right) = -\frac{25}{2}$

Exercises

What value completes the square for each expression?

1. $x^2 + 2x$ **2.** $x^2 - 24x$ **3.** $x^2 + 12x$

4. $x^2 - 20x$ **5.** $x^2 + 5x$ **6.** $x^2 - 9x$

7. $2x^2 - 24x$ **8.** $3x^2 + 12x$ **9.** $-x^2 + 6x$

10. $5x^2 + 80x$ **11.** $-7x^2 + 14x$ **12.** $-3x^2 - 15x$

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4-6 Reteaching (continued)

Completing the Square

You can easily graph a quadratic function if you first write it in vertex form. Complete the square to change a function in standard form into a function in vertex form.

Problem

What is $y = x^2 - 6x + 14$ in vertex form?

Think	Write
Write an expression using the terms that contain x.	$x^2 - 6x$
Find $\frac{b}{2}$.	$\frac{b}{2} = \frac{-6}{2} = -3$
Add $\left(\frac{b}{2}\right)^2$ to the expression to Complete the square.	$x^2 - 6x + (-3)^2 = x^2 - 6x + 9$
Subtract 9 from the expression so that the equation is unchanged.	$y = x^2 - 6x + 9 + 14 - 9$
Factor the perfect square trinomial.	$y = (x - 3)^2 + 14 - 9$
Add the remaining constant terms.	$y = (x - 3)^2 + 5$

Exercises

Rewrite each equation in vertex form.

13. $y = x^2 + 4x + 3$

14. $y = x^2 - 6x + 13$

15. $y = 2x^2 + 4x - 10$

16. $y = x^2 - 2x - 3$

17. $y = x^2 + 8x + 13$

18. $y = -x^2 - 6x - 4$

19. $y = -x^2 + 10x - 18$

20. $y = x^2 + 2x - 8$

21. $y = 2x^2 + 4x - 3$

22. $y = 3x^2 - 12x + 8$

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4-3 Reteaching

Modeling With Quadratic Functions

Three non-collinear points, no two of which are in line vertically, are on the graph of exactly one quadratic function.

Problem

A parabola contains the points (0, –2), (–1, 5), and (2, 2). What is the equation of this parabola in standard form?

If the parabola $y = ax^2 + bx + c$ passes through the point (x, y), the coordinates of the point must satisfy the equation of the parabola. Substitute the (x, y) values into $y = ax^2 + bx + c$ to write a system of equations.

First, use the point (0, –2).	$y = ax^2 + bx + c$	Write the standard form.
	$-2 = a(0)^2 + b(0) + c$	Substitute.
	$-2 = c$	Simplify.
Use the point (–1, 5) next.	$5 = a(-1)^2 + b(-1) + c$	Substitute.
	$5 = a - b + c$	Simplify.
Finally, use the point (2, 2).	$2 = a(2)^2 + b(2) + c$	Substitute.
	$2 = 4a + 2b + c$	Simplify.

Because $c = -2$, the resulting system has two variables. Simplify the equations above.

$$a - b = 7$$
$$4a + 2b = 4$$

Use elimination to solve the system and obtain $a = 3$, $b = -4$, and $c = -2$. Substitute these values into the standard form $y = ax^2 + bx + c$.

The equation of the parabola that contains the given points is $y = 3x^2 - 4x - 2$.

Exercises

Find an equation in standard form of the parabola passing through the given points.

1. (0, –1), (1, 5), (–1, –5)

2. (0, 4), (–1, 9), (2, 0)

3. (0, 1), (1, 4), (3, 22)

4. (1, –1), (–2, 20), (2, 0)

5. (–1, –5), (0, –1), (2, 1)

6. (1, 3), (–2, –3), (–1, 3)

4-3 Reteaching (continued)

Modeling With Quadratic Functions

Problem

A soccer player kicks a ball of the top of a building. His friend records the height of the ball at each second. Some of her data appears in the table.

a. What is a quadratic model for these data?

b. Use the model to complete the table.

Time (s)	Height (ft)
0	112
1	192
2	
3	
4	
5	192
6	
7	

Use the points (0, 112), (1, 192), and (5, 192) to find the quadratic model. Substitute the (t, h) values into $h = at^2 + bt + c$ to write a system of equations.

(0, 112): $112 = a(0)^2 + b(0) + c \quad c = 112$

(1, 192): $192 = a(1)^2 + b(1) + c \quad a + b + c = 192$

(5, 192): $192 = a(5)^2 + b(5) + c \quad 25a + 5b + c = 192$

Use $c = 112$ and simplify the equations to obtain a system with just two variables.

$$a + b = 80$$

$$25a + 5b = 80$$

Use elimination to solve the system. The quadratic model for the data is

$$h = -16t^2 + 96t + 112$$

Now use this equation to complete the table for the t-values 2, 3, 4, 6, and 7.

$t = 2$: $h = -16(2)^2 + 96(2) + 112 = -64 + 192 + 112 = 240$

$t = 3$: $h = -16(3)^2 + 96(3) + 112 = -144 + 288 + 112 = 256$

$t = 4$: $h = -16(4)^2 + 96(4) + 112 = -256 + 384 + 112 = 240$

$t = 6$: $h = -16(6)^2 + 96(6) + 112 = -576 + 576 + 112 = 112$

$t = 7$: $h = -16(7)^2 + 96(7) + 112 = -784 + 672 + 112 = 0$

Time (s)	Height (ft)
0	112
1	192
2	240
3	256
4	240
5	192
6	112
7	0

Exercise

7. The number n of Brand X shoes in stock at the beginning of month t in a store follows a quadratic model. In January ($t = 1$), there are 36 pairs of shoes; in March ($t = 3$), there are 52 pairs; and in September, there are also 52 pairs.

a. What is the quadratic model for the number n of pairs of shoes at the beginning of month t?

b. How many pairs are in stock in June?