Polynomials Unit **Study Guide**

<table>
<tr><td colspan="3">DIVISION</td></tr>
<tr><td colspan="3">Long Division</td></tr>
<tr><td rowspan="2">Before beginning, look for skips in the power of each term. Add a place holder for missing powers.
Make a “house” and put the dividend inside of the house and the divisor outside.
Divide the first term of the dividend by the first term of the divisor.
Multiply each term of the divisor by the answer from the previous step. Write that polynomial beneath the dividend.
Subtract the appropriate terms form the dividend.
Repeat.

Ex: $(3x^3 - 5x^2 + 10x - 3) \div (3x + 1)$

$$
\begin{array}{rl}
 & x^2 \;\; - 2x \;\; + 4 \\ \cline{2-2}
3x+1 \,\big) & 3x^3 - 5x^2 + 10x - 3 \\
 & -(\underline{3x^3 + 1x^2}) \\
 & \quad - 6x^2 + 10x - 3 \\
 & \quad -(\underline{-6x^2 - \;\; 2x}) \\
 & \qquad\qquad 12x - 3 \\
 & \qquad\quad -(\underline{12x + 4}) \\
 & \qquad\qquad\quad -7
\end{array}
$$

Remainder: -7</td>
<td>1. $(x^2 - 7x - 11) \div (x - 8)$</td>
<td>2. $(5x^2 - 15) \div (x - 6)$</td></tr>
<tr><td>3. $(k^3 + 2k^2 - 20k + 4) \div (k + 7)$</td>
<td>4. A rectangular prism has a volume of $18x^3 + 27x^2 - 50x - 75$ and a length of $(2x + 3)$. Find the width and height in terms of x.</td></tr>
<tr><td colspan="3">Synthetic Division</td></tr>
<tr><td rowspan="2">Make your division pit. Place the zero form of the divisor outside the pit. Place the coefficients of the dividend (with place holders) in order inside the pit.
Bring down the first coefficient.
Multiply it by the root and place that number under the second coefficient in the pit. Add them and write the answer below. Repeat for remaining coefficients.
The answers are the coefficients for the new polynomial.

EX: $(3x^3 - 2x^2 + 3x - 4) \div (x - 3)$

$$
\begin{array}{r|rrrr}
3 & 3 & -2 & 3 & -4 \\
 & & 9 & 21 & 72 \\ \hline
 & 3 & 7 & 24 & 68
\end{array}
$$

$Answer$: $3x^2 + 7x + 24, R: 68$</td>
<td>5. $(x^2 + 10x + 18) \div (x + 5)$</td>
<td>6. $(x^5 - 3x^2 + 2) \div (x - 3)$</td></tr>
<tr><td>7. $(x^2 - 28) \div (x - 5)$</td><td></td></tr>
</table>

REMAINDER THEOREM

The remainder of a division problem is equal to the dividend evaluated at the remainder.

The divisor is a root of the dividend if the remainder of the division problem is zero.

EX: Is $(x+2)$ a factor of $x^3 - 8x + 6$?

Putting $(x+2)$ in factor form, we get $x = -2$.

$$(-2)^3 - 8(-2) + 6 = 30$$

Since 30 is not equal to zero, $(x+2)$ is not a factor of $x^3 - 8x + 6$

8. Is $(x-4)$ a factor of $3x^3 - 8x^2 + 12x - 1$?

9. Is $(x+3)$ a factor of $4x^3 - 36x$?

10. If $(x^2 - 3x + b) \div (x+1)$ has a remainder of 5 what is b?

SPECIAL CASES

Difference of squares

A difference of squares is a quadratic of the form $(a^2 - b^2)$. This special case quadratic will always factor as follows:

$$(a^2 - b^2) = (a+b)(a-b)$$

EX: Factor the expression $(x^2 - 36)$
Since both x^2 and 36 are perfect squares the factored form is

$$(x^2 - 36) = (x+6)(x-6)$$

EX: Factor the expression $(9x^2 - 1)$

$$(9x^2 - 1) = (3x+1)(3x-1)$$

12. $9x^2 - 25$

13. $1 - 9y^2$

14. $x^2 - 16y^2$

15. $32x^2 - 18$

Sum and difference of cubes

Sum and difference of cubes are of the form

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Use the pattern to identify a and b. Then substitute into the formula.

EX: Factor $8 - 125x^3$

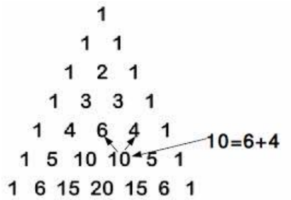
$$a = 2, b = 5x$$
$$(2 - 5x)(4 + 10x + 25x^2)$$

16. Factor $64c^3 + 1$

17. Factor $8y^3 + 27$

18. Factor $27x^3 - 64y^3$

19. Factor $x^6 - 64$

<table>
<tr><th colspan="3">POLYNOMIAL EXPANSION</th></tr>
<tr>
<td rowspan="2">Pascal's Triangle


Use Pascal's Triangle and the Binomial Theorem to construct each term in the expansion.
Expansions are in the form $(a+b)^n$
Each expansion will have $n+1$ terms.
The first term of each expansion will be a^n and the last term will be b^n.
Locate the appropriate row in Pascal's triangle for the coefficient for each term.
Exponents of a decrease from n and the exponents of b increase from 0.
EX: Write $(x-5)^4$ in standard form.
$(1)x^4 + 4(x^3)(-5) + 6(x^2)(-5)^2 + 4(x)(-5)^3 + 1(-5)^4$
Which simplifies to
$x^4 - 20x^3 + 150x^2 - 500x + 625$</td>
<td>20. Expand $(x+3)^5$</td>
<td>21. Find the second term of $(2x+3y)^6$</td>
</tr>
<tr>
<td>22. Find the coefficient of x^2 in $(x-2)^4$</td>
<td>23. Change the following from vertex form to standard form.
$f(x) = 2(x+5)^3 + 1$</td>
</tr>
<tr><th colspan="3">Finding All Roots</th></tr>
<tr>
<td>Graph the polynomial on your calculator. Use the roots you find to create the factors to divide out of the polynomial. Divide the first factor out of the polynomial. Repeat the process using the next root and the quotient from the division.
Once you are down to a quadratic quotient, use the quadratic formula to find the remaining roots.

EX: Find all roots for the polynomial
$y = 6x^4 - x^3 - 3x^2 - 2x - 30$
Roots from calculator, $x = -1.5 = -\frac{3}{2}$ and $x = 1.6666 = \frac{5}{3}$ which results in the factors $(2x+3)$ and $(3x-5)$.

Divide out the two roots.
$(6x^4 - x^3 - 3x^2 - 2x - 30) \div (2x+3)$ results in
$3x^3 - 5x^2 + 6x - 10$
Now divide using the second root.
$(3x^3 - 5x^2 + 6x - 10) \div (3x-5)$
This results in the quadratic $x^2 + 2$
Use the quadratic formula to find the remaining roots; $x = \pm 2i$.</td>
<td>24. Find all roots of the polynomial $y = 2x^4 + 11x^3 + 8x^2 - x + 60$.</td>
<td>25. Find all roots of the polynomial $y = x^4 - 4x^3 + 8x^2 - 16x + 16$.</td>
</tr>
</table>

WRITING POLYNOMIALS FROM ROOTS

In the previous unit we looked at writing quadratic equations from roots. Any polynomial can be written from its roots. There are just more of them. Take the roots and set them equal to zero to get them in factor form. Multiply all the roots together.

EX: Write an equation of a polynomial with the roots $x = 4, x = -3, x = \frac{2}{3}$.

Factors are $(x-4), (x+3), and$ $(3x-2)$

$f(x) = (x-4)(x+3)(3x-2)$

26. Write an equation of a polynomial with the roots $x = 0, x = -\frac{3}{2}, x = 1, x = \frac{2}{3}$.

27. Write an equation of a polynomial with the roots $x = 5, x = 2, x = -\frac{3}{7}$.

REGRESSION MODELING

Use data and the graphing calculator to create a quadratic equation that will model real world situations.

x-value	1	2	3
y-value	1	4	8

First enter your data
[STAT] [EDIT]
type x-values in L1
type y values in L2

Next create regression equation
[STAT][CALC][5:QuadReg] [VARS] [Y-VARS] [1:Function][Y1][ENTER]

To find an x-value,
Type the given y-value into $y_2 =$
Press [2nd] [TRACE] [5] – intersect
Press [ENTER] [ENTER] [ENTER]

To find a y-value
Press [2nd][TRACE][VALUE]
Type given x-value
Press [ENTER]

28. Use the table below to create a quadratic model. The answer the questions below.

x	-1	0	1	2	4
y	6	1	-2	2	21

What is the predicted value of x when $y = 3$?

What will y equal when $x = -2$?

29. A ball is thrown in the air. The table below represents the height of the ball after a number of seconds. Create a quadratic model from the data and answer the questions below.

time	0	1	3	4
height	0	13	100	200

What is the predicted height after 12 seconds?

At what time(s) would the object be 490 feet high?

In addition to quadratic regression models, Cubic (6) and Quartic (7) models can be created. Based on the data different models may fit more closely. Use the R^2 value to determine the best model. Select the model with the R^2 value closest to 0. Make sure diagnostics is on to see the R^2 value. ([2nd][0] on your calculator then scroll down to diagnosticsOn).