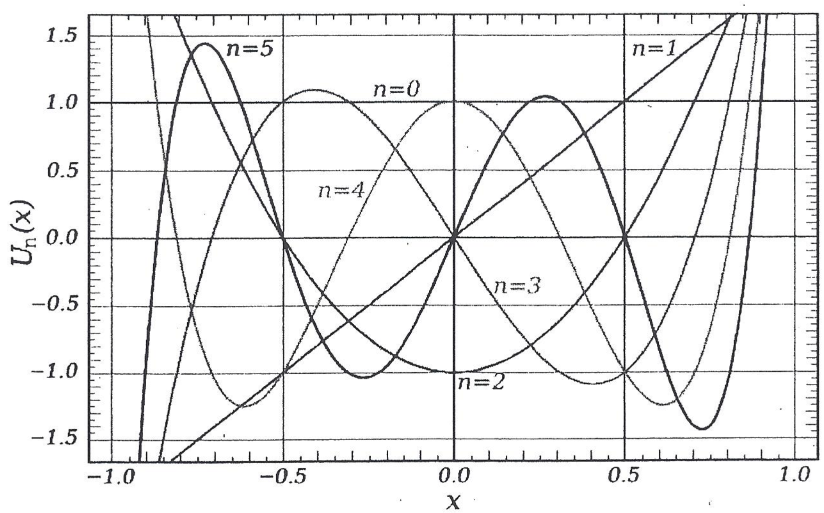
POLYNOMIALS



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5-1 Reteaching

Polynomial Functions

Problem

What is the classification of the following polynomial by its degree? by its number of terms? What is its end behavior? $5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

Step 1 Write the polynomial in standard form. First, combine any like terms. Then, place the terms of the polynomial in descending order from greatest exponent value to least exponent value.

$5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

$8x^4 - 3x + 3x^6 + 9x^3 - 12$ Combine like terms.

$3x^6 + 8x^4 + 9x^3 - 3x - 12$ Place terms in descending order.

Step 2 The degree of the polynomial is equal to the value of the greatest exponent. This will be the exponent of the first term when the polynomial is written in standard form.

$(3x^6) + 8x^4 + 9x^3 - 3x - 12$ The first term is $3x^6$.

$3x^{(6)}$ The exponent of the first term is 6.

This is a sixth-degree polynomial.

Step 3 To determine the end behavior of the polynomial (the directions of the graph to the far left and to the far right), look at the degree of the polynomial (n) and the coefficient of the leading term (a).

If a is positive and n is even, the end behavior is up and up.

If a is positive and n is odd, the end behavior is down and up.

If a is negative and n is even, the end behavior is down and down.

If a is negative and n is odd, the end behavior is up and down.

The leading term in this polynomial is $3x^6$.
a (+3) is positive and n (6) is even, so the end behavior is up and up.

Exercises

What is the classification of each polynomial by its degree? by its number of terms? What is its end behavior?

1. $8 - 6x^3 + 3x + x^3 - 2$ **2.** $15x^7 - 7$ **3.** $2x - 6x^2 - 9$

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5-2 Reteaching

Polynomials, Linear Factors, and Zeros

The Factor Theorem tells you that if you know the zeros of a polynomial function, you can write the polynomial.

Factor Theorem

The expression $x - a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

Problem

What is a cubic polynomial function in standard form with zeros 0, 4, and −2?

Each zero (a) is part of a linear factor of the polynomial, so you can write each factor as $(x - a)$.

$(x - a_1)(x - a_2)(x - a_3)$	Set up the cubic polynomial factors.
$a_1 = 0, a_2 = 4, a_3 = -2$	Assign the zeros.
$(x - 0)(x - 4)[x - (-2)]$	Substitute the zeros into the factors.
$f(x) = x(x - 4)(x + 2)$	Write the polynomial function in factored form.
$f(x) = x(x^2 - 2x - 8)$	Multiply $(x - 4)(x + 2)$.
$f(x) = x^3 - 2x^2 - 8x$	Multiply by x using the Distributive Property.

The polynomial function written in standard form is $f(x) = x^3 - 2x^2 - 8x$.

Exercises

Write a polynomial function in standard form with the given zeros.

1. 5,−1, 3
2. 1, 7,−5
3. −1, 1,−6
4. 2,−2,−3
5. 2, 1, 3
6. 2, 3,−3,−1
7. 0,−8, 2
8. −10, 0, 2
9. $-2, 2, -\frac{3}{2}$
10. $-1, \frac{2}{3}$

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5-2 Reteaching (continued)

Polynomials, Linear Factors, and Zeros

You can use a polynomial function to find the minimum or maximum value of a function that satisfies a given set of conditions.

Problem

Your school wants to put in a swimming pool. The school wants to maximize the volume while keeping the sum of the dimensions at 40 ft. If the length must be 2 times the width, what should each dimension be?

Step 1 First, define a variable x. Let x = the width of the pool.

Step 2 Determine the length and depth of the pool using the information in the problem.

The length must be 2 times the width, so length = $2x$.

The length plus width plus depth must equal 40 ft, so depth = $40 - x - 2x = 40 - 3x$.

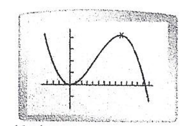
Step 3 Create a polynomial in standard form using the volume formula

V = length · width · depth

$= 2x(x)(40 - 3x)$

$= -6x^3 + 80x^2$

Step 4 Graph the polynomial function. Use the MAXIMUM feature. The maximum volume is 2107 ft^3 at a width of 8.9 ft.



Step 5 Evaluate the remaining dimensions: width = x □8.9 ft

length = $2x$ □17.8 ft

depth = $40 - 3x$ □13.3 ft

Exercises

11. Find the dimensions of the swimming pool if the sum must be 50 ft and the length must be 3 times the depth.
12. Find the dimensions of the swimming pool if the sum must be 40 ft and the depth must be one tenth of the length.
13. Find the dimensions of the swimming pool if the sum must be 60 ft and the length and width are equal.

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5-4 Reteaching

Dividing Polynomials

Problem

What is the quotient and remainder? Use polynomial long division to divide $2x^2 + 6x - 7$ by $x + 1$.

Step 1 To find the first term of the quotient, divide the highest-degree term of $2x^2 + 6x + 7$ by the highest-degree term of the divisor, $x + 1$. Circle these terms before dividing.

$$\begin{array}{r} 2x \phantom{{}+6x+7} \\ x + 1\overline{)2x^2 + 6x + 7} \end{array}$$

Step 2 Multiply $x + 1$ by the new term, $2x$, in the quotient. $2x(x + 1) = 2x^2 + 2x$. Align like terms.

$$\begin{array}{r} 2x \phantom{{}+6x+7} \\ x + 1\overline{)2x^2 + 6x + 7} \\ \underline{2x^2 + 2x} \phantom{{}+7} \end{array}$$

Step 3 Subtract to get $4x$. Bring down the next term, 7.

$$\begin{array}{r} 2x \phantom{{}+6x+7} \\ x + 1\overline{)2x^2 + 6x + 7} \\ \underline{2x^2 + 2x} \phantom{{}+7} \\ 4x + 7 \end{array}$$

Step 4 Divide the highest-degree term of $4x + 7$ by the highest-degree term of $x + 1$. Circle these terms before dividing.

Step 5 Repeat Steps 2 and 3. The *remainder* is 3 because its degree is less than the degree of $x + 1$.

$$\begin{array}{r} 2x + 4 \\ x + 1\overline{)2x^2 + 6x + 7} \\ \underline{2x^2 + 2x} \phantom{{}+7} \\ 4x + 7 \\ \underline{4x + 4} \\ 3 \end{array}$$

$2x^2 + 6x + 7$ divided by $x + 1$ is $2x + 4$, with a remainder of 3. The quotient is $2x + 4$ with remainder 3.

Check the answer by multiplying $(x + 1)$ by $(2x + 4)$ and adding 3.
$(x + 1)(2x + 4) + 3 = 2x^2 + 6x + 7$

Exercises

Divide using polynomial long division.

1. $(3x^2 - 8x + 7) \div (x - 1)$
2. $(x^3 + 5x^2 - 3x - 4) \div (x + 6)$
3. $(x^2 + 3x - 8) \div (x - 5)$
4. $(x^2 + 6x + 14) \div (x + 3)$
5. $(x^3 - 7x^2 + 11x + 3) \div (x - 3)$
6. $(2x^3 - 3x^2 - x - 2) \div (x - 2)$
7. $(2x^2 - 4x + 7) \div (x - 3)$
8. $(x^3 + 2x^2 - 20x + 4) \div (x + 7)$
9. $(x^2 - 5x + 2) \div (x - 1)$
10. $(2x^3 + 3x^2 + x + 6) \div (x + 3)$

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5-4 Reteaching (continued)

Dividing Polynomials

Problem

Use synthetic division to divide $x^3 + 13x^2 + 46x + 48$ by $x + 3$. What is the quotient and remainder?

Step 1 Set up your polynomial division.
$(x^3 + 13x^2 + 46x + 48) \div (x + 3)$

Step 2 Reverse the sign of the constant, 3, in the divisor. Write the coefficients of the dividend: 1 13 46 48.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ \hline & & & & \end{array}$$

Step 3 Bring the first coefficient, 1, down to the bottom line.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ \hline & 1 & & & \end{array}$$

Step 4 Multiply the coefficient, 1, by the divisor, –3. Put this product, –3, underneath the second coefficient 13, and add those two numbers: $13 + (-3) = 10$.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ & & -3 & & \\ \hline & 1 & 10 & & \end{array}$$

Step 5 Continue multiplying and adding through the last coefficient. The final sum is the remainder.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ & & -3 & -30 & -48 \\ \hline & 1 & 10 & 16 & 0 \end{array}$$

The quotient is $x^2 + 10x + 16$. Since the remainder is 0, $x + 3$ is a factor of $x^3 + 13x^2 + 46x + 48$.

Exercises

What is the quotient and remainder of the following polynomials?

11. $(x^3 - 2x + 8) \div (x + 2)$
12. $(12x^3 - 71x^2 + 57x - 10) \div (x - 5)$
13. $(3x^4 + x^3 - 6x^2 - 9x + 12) \div (x + 1)$
14. $(2x^3 - 15x + 23) \div (x - 2)$
15. $(x^3 + x + 10) \div (x + 2)$
16. $(x^4 - 12x^3 - 18x^2 + 10) \div (x + 4)$

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MATHEMATICS SUPPORT CENTRE

Title: Remainder Theorem and Factor Theorem

Target: On completion of this worksheet you should be able to use the remainder and factor theorems to find factors of polynomials.

Generally when a polynomial is divided by a linear expression there is a remainder.

e.g. $(3x^3 + 4x^2 - 5x + 3) \div (x + 2)$

$$\begin{array}{r} 3x^2 - 2x - 1 \\ x+2 \overline{)\, 3x^3 + 4x^2 - 5x + 3} \\ \underline{3x^3 + 6x^2} \qquad\qquad \\ -2x^2 - 5x \qquad \\ \underline{-2x^2 - 4x} \qquad \\ -x + 3 \\ \underline{-x - 2} \\ 5 \end{array}$$

$(3x^3 + 4x^2 - 5x + 3) = (x + 2)(3x^2 - 2x - 1) + 5$

divisor — quotient — remainder

Any polynomial can be written in the following form:

polynomial ≡ divisor × quotient + remainder.

In particular if the divisor is $(x - a)$ and the polynomial is $f(x)$ then

$f(x) \equiv (x - a) \times$ quotient + remainder.

If $x = a$ then

$f(a) = (a - a) \times$ quotient + remainder.

$f(a) =$ remainder

This gives an easy way of finding the remainder when a polynomial is divided by $(x - a)$

Examples

1. Using previous example

$f(x) = 3x^3 + 4x^2 - 5x + 3$

$\quad = (x + 2)(3x^2 - 2x - 1) + 5$

Now using $x = -2$

$f(-2) = (-2 + 2)(3 \times (-2)^2 - 2 \times (-2) - 1) + 5$

$\quad = 0 \times (3 \times (-2)^2 - 2 \times (-2) - 1) + 5$

$\quad = 5$

i.e. the remainder.

2. Find the remainder when $(2x^3 - 5x^2 + x - 3)$ is divided by $(x - 1)$

Let $f(x) = 2x^3 - 5x^2 + x - 3$

Substitute $x = 1$ since we require $(x - 1) = 0$

$f(1) = 2 \times 1^3 - 5 \times 1^2 + 1 - 3 = -5$

The remainder is -5

3. Find the remainder when $(x^4 + 4x^2 + 3x - 7)$ is divided by $(x + 3)$

Let $f(x) = x^4 + 4x^2 + 3x - 7$

Substitute $x = -3$ since we require $(x + 3) = 0$

$f(-3) = (-3)^4 + 4 \times (-3)^2 + 3 \times (-3) - 7 = 101$

The remainder is 101

Exercise

Find the remainders for the following:

1. $(x^3 - 5x^2 + 6x - 4) \div (x - 2)$
2. $(4x^3 + 3x^2 + x + 2) \div (x - 1)$
3. $(2x^4 - x^3 + 3x^2 - 1) \div (x + 1)$
4. $(2x^3 - 6x - 5) \div (x + 3)$
5. $(x^3 - 4x^2 - x) \div (x - 4)$

Example

Find the remainder when $(3x^3 + 4x^2 - 5x - 2)$ is divided by $(x - 1)$

Let $f(x) = 3x^3 + 4x^2 - 5x - 2$ and $x = 1$

$f(1) = 3 \times 1^3 + 4 \times 1^2 - 5 \times 1 - 2 = 0$

The remainder is 0.

$3x^3 + 4x^2 - 5x - 2 = (x - 1) \times \text{quotient} + 0$

$\quad = (x - 1) \times \text{quotient}$

so $(x - 1)$ is a factor of $(3x^3 + 4x^2 - 5x - 2)$

We can use the remainder theorem to check for factors of a polynomial.

As before

$f(x) = (x - a) \times \text{quotient} + \text{remainder}$

and $f(a) = \text{remainder}$

If $(x - a)$ is a factor then the remainder is 0

ie $f(a) = 0$

This is called the factor theorem.

Examples

1. Is $(x - 3)$ a factor of $(2x^3 - 3x^2 - 8x - 3)$?

Let $f(x) = (2x^3 - 3x^2 - 8x - 3)$ and $x = 3$ as we are checking whether $(x - 3)$ is a factor.

$f(3) = 2 \times 3^3 - 3 \times 3^2 - 8 \times 3 - 3 = 0$

so $(x - 3)$ is a factor of $(2x^3 - 3x^2 - 8x - 3)$

2. Is $(x - 1)$ a factor of $(2x^3 - 3x^2 - 8x - 3)$?

Using $f(x)$ as above and $x = 1$

$f(1) = 2 \times 1^3 - 3 \times 1^2 - 8 \times 1 - 3 = -12 \neq 0$

so $(x - 1)$ is **not** a factor of $(2x^3 - 3x^2 - 8x - 3)$

Exercise

1. Is $(x - 1)$ a factor of $f(x) = (x^3 + 2x^2 - 2x - 1)$?
2. Is $(x + 2)$ a factor of $f(x) = (4x^2 + 13x + 10)$?
3. Is $(x - 2)$ a factor of $f(x) = (4x^2 + 13x + 10)$?
4. Is $(x + 3)$ a factor of $f(x) = (3x^3 + 10x^2 + x - 6)$?
5. Is $(x - 1)$ a factor of $f(x) = (3x^3 + 10x^2 + x - 6)$?

We can use the factor theorem to factorise polynomials, although some trial and error is involved.

Example

Factorise $(2x^3 + 5x^2 - x - 6)$.

Let $f(x) = 2x^3 + 5x^2 - x - 6$. Since the constant is -6 we will consider factors of this ie. $\pm 1, \pm 2, \pm 3, \pm 6$. We will try $(x - 1)$

$f(1) = 2 \times 1^3 + 5 \times 1^2 - 1 - 6 = 0$

so $(x - 1)$ is a factor.

Now we can find the quadratic factor by division or by repeating the above.

$$\begin{array}{r} 2x^2 + 7x + 6 \\ (x-1) \overline{)\, 2x^3 + 5x^2 - x - 6} \\ \underline{2x^3 - 2x^2} \qquad\qquad \\ 7x^2 - x \qquad \\ \underline{7x^2 - 7x} \qquad \\ 6x - 6 \\ \underline{6x - 6} \\ 0 \end{array}$$

$f(x) = 2x^3 + 5x^2 - x - 6$

$\quad = (x - 1)(2x^2 + 7x + 6)$

$\quad = (x - 1)(x + 2)(2x + 3)$

The quadratic factor is factorised in the normal way.

Exercise

Factorise the following:

1. $f(x) = x^3 + 2x^2 - 5x - 6$
2. $f(x) = 2x^3 + x^2 - 2x - 1$
3. $f(x) = x^3 - 3x^2 - 3x - 4$
4. $f(x) = 3x^3 + 6x^2 + x + 2$
5. $f(x) = 4x^3 - 15x^2 + 17x - 6$

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5-4 Practice
Dividing Polynomials

Form K

Divide using long division. Check your answers.

1. $(2x^2 + 7x - 5) \div (x + 1)$

   To start, divide $\frac{2x^2}{x} = 2x$

   $$\begin{array}{r} 2x \\ x+1\overline{)2x^2+7x-5} \\ \underline{2x^2+2x} \end{array}$$

   Then, multiply $2x(x + 1) = 2x^2 + 2x$.

2. $(x^3 + x^2 - 14x - 27) \div (x + 3)$

3. $(2x^3 + 13x^2 + 16x + 5) \div (x + 5)$

4. $(x^2 + 9x + 22) \div (x + 2)$

5. $(6x^2 + 4x - 16) \div (2x - 2)$

6. $(8x^3 + 18x^2 + 7x - 3) \div (4x - 1)$

7. $(12x^2 + 18x - 17) \div (6x - 3)$

Determine whether each binomial is a factor of $x^3 - 3x^2 - 4x$.

8. $x - 4$

9. $x + 2$

10. $x - 3$

11. $x + 1$

Determine whether each binomial is a factor of $x^3 - 9x^2 + 15x + 25$.

12. $x - 2$

13. $x + 1$

14. $x - 5$

15. $x - 3$

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5-4 Practice (continued)
Dividing Polynomials

Form K

Divide using synthetic division.

16. $(x^3 - 7x^2 - 36) \div (x - 2)$

   To start, write the coefficients of the polynomial. Use 2 for the divisor.

   $$\begin{array}{r|rrrr} 2 & 1 & -7 & 0 & -36 \\ & & 2 & -10 & -20 \\ \hline & 1 & -5 & -10 & -56 \end{array}$$

17. $(x^3 + x^2 - 14x - 27) \div (x + 3)$

18. $(x^3 - 6x^2 + 3x - 2) \div (x - 2)$

19. $(x^3 - 15) \div (x - 1)$

20. $(x^2 + 8) \div (x - 4)$

21. $(3x^3 - 70x + 2) \div (x - 5)$

22. $(2x^3 + x^2 - 8x + 4) \div (x + 2)$

Use synthetic division and the given factor to completely factor each polynomial function.

23. $y = 2x^3 + 9x^2 + 13x + 6$; $(x + 1)$

24. $y = x^3 + 4x^2 - 7x - 10$; $(x - 2)$

Use synthetic division and the Remainder Theorem to find $P(a)$.

25. $P(x) = 5x^3 - 12x^2 + 2x + 1$, $a = 3$

26. $P(x) = 2x^3 - 4x^2 + 3x - 6$, $a = -2$

27. $P(x) = x^3 + 6x^2 - 2$, $a = 3$

28. $P(x) = 7x^3 + x^2 - 2x + 10$, $a = 1$

29. $P(x) = x^3 - 412$, $a = 8$

30. $P(x) = 2x^3 + x^2 - 3x - 3$, $a = -3$

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5-6 Reteaching

Problem

What are all the complex roots of $x^4 + x^3 - 2x^2 + 4x - 24 = 0$?

Because this is a fourth-degree polynomial, you know it will have four roots.

Step 1 Use your calculator to find the real roots. Put the equation into y_1 and graph.

Step 2 Find the real zeros using 2^{nd} TRACE ZERO. You'll find x = -3 and x = 2

Step 3 Use synthetic division with a divisor of 2 to begin factoring the polynomial.

2	1	1	−2	4	−24
		2	6	8	24
	1	3	4	12	0

$x^3 + 3x^2 + 4x + 12 = 0$

Step 4 Repeat Steps 3 until you have a polynomial of degree 2 or less.

−3	1	3	4	12
		−3	0	−12
	1	0	4	0

$x^2 + 4 = 0$

Step 5 If the answer to your division is a quadratic use the quadratic formula to find the remaining roots.

$$\frac{-0 \pm \sqrt{0^2 - 4(1)(4)}}{2(1)} = \frac{\pm\sqrt{-16}}{2} = \frac{\pm 4i}{2} = \pm 2i$$

The four roots of $x^4 + x^3 - 4x^2 + 2x - 24 = 0$ are 2, –3, $2i$, and $-2i$.

Exercises

Find all the complex roots of each polynomial.

1. $x^4 - 8x^3 + 11x^2 + 40x - 80$

2. $4x^4 - x^3 - 12x^2 + 4x - 16$

3. $x^6 + 2x^5 + 7x^4 + 20x^3 - 21x^2 + 18x - 27$

4. $x^3 - 4x^2 + 4x - 16$

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5-6 Reteaching (continued)

Problem

What are the zeros of $f(x) = x^3 + 4x^2 - x - 10$?

Use your calculator to find the real roots. You will find that –2 is one of the roots.

−2	1	4	−1	−10
		−2	−4	10
	1	2	−5	0

Use synthetic division to test each possible rational root until you get a remainder of zero.

$x^3 + 4x^2 - x - 10 = (x+2)(x^2 + 2x - 5)$

Use the coefficients from synthetic division to obtain the quadratic factor.

Because $X^2 + 2X - 5$ cannot be factored, use the Quadratic Formula to solve $X^2 + 2X - 5 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{24}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{6}}{2}$$

$$x = -1 \pm \sqrt{6}$$

The polynomial function $f(x) = x^3 + 4x^2 - x - 10$ has one rational zero, – 2, and two irrational zeros, $-1 + \sqrt{6}$ and $-1 - \sqrt{6}$.

Exercises

What are the zeros of each function?

5. $f(x) = x^3 - 2x^2 + 4x - 3$

6. $f(x) = x^3 - 3x^2 - 15x + 125$

7. $f(x) = 3x^3 - 2x^2 - 15x + 10$

8. $f(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

9. $f(x) = x^4 - 3x^2 + 2$

10. $f(x) = x^3 - 2x^2 - 17x - 6$

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5-7 Reteaching
The Binomial Theorem

You can find the coefficients of a binomial expansion in Pascal's Triangle.

To create Pascal's Triangle, start by writing a triangle of 1's. This triangle forms the first two rows. Each row has one more element than the one above it. Each row begins with a 1, and then each element is the sum of the two closest elements in the row above. The last element in each row is a 1.

Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Problem

What is the expansion of $(x+y)^5$? Use Pascal's Triangle.

Step 1 The power of the binomial corresponds to the second number in each row of Pascal's Triangle. Because the power of this binomial is 5, use the row of Pascal's Triangle with 5 as the second number. The numbers of this row are the coefficients of the expansion.

Pascal's Triangle

1

1 1

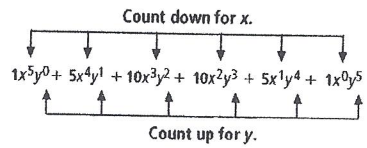
1 2 1

1 3 3 1

1 4 6 4 1

| 1 | 5 | 10 | 10 | 5 | 1 |

Step 2 The exponents of the x-terms of the expansion begin with the power of the binomial and decrease to 0. The exponents of the y-terms of the expansion begin with 0 and increase to the power of the binomial.



Step 3 Simplify all terms to write the expansion in standard form.

$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Exercises

Write the expansion of each binomial.

1. $(a+b)^3$
2. $(x-y)^4$
3. $(r+1)^5$
4. $(a-b)^6$

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5-7 Reteaching (continued)
The Binomial Theorem

- The *Binomial Theorem* states that for any binomial $(a+b)$ and any positive integer n, $(a+b)^n = {}_nC_0a^n + {}_nC_1a^{n-1}b + {}_nC_2a^{n-2}b^2 + \ldots + {}_nC_{n-1}ab^{n-1} + {}_nC_nb^n$.
- The theorem provides an effective method for expanding any power of a binomial.

Evaluate the combination ${}_nC_k$ as $\dfrac{n!}{k!(n-k)!}$.

Problem

What is the expansion of $(3x+2)^3$? Use the Binomial Theorem.

Step 1 Determine a, b, and n.

$a = 3x,\ b = 2,\ n = 3$

Step 2 Use the formula to write the equation.

$(3x+2)^3 = {}_3C_0(3x)^3 + {}_3C_1(3x)^2(2) + {}_3C_2(3x)(2)^2 + {}_3C_3(2)^3$

Step 3 Simplify.

$= 1(27x^3) + 3(9x^2)(2) + 3(3x)(4) - 1(8)$

$= 27x^3 + 54x^2 + 36x + 8$

Exercises

Fill in the correct coefficients, variables, and exponents for the expanded form of each binomial.

5. $(x+y)^4 = x\square + \square x^3y + 6x\square y^2 + \square xy\square + \square^4$
6. $(z-y)^3 = z\square - \square z^2y + \square zy\square - \square^3$
7. $(x+z)^5 = x\square + \square x^4z + 10x\square z^2 + \square x^2z\square + \square xz^4 + \square^5$

Write the expansion of each binomial. Use the Binomial Theorem.

8. $(x+y)^5$
9. $(x-y)^5$
10. $(2x+y)^3$
11. $(x+3y)^4$
12. $(x-2y)^5$
13. $(2x-y)^5$
14. $(x-3y)^4$
15. $(4x-y)^3$
16. $(x-1)^5$
17. $(1-x)^3$
18. $(x^2+1)^3$
19. $(y^2+a)^4$

5-2 Puzzle: Made in the Shade

Polynomials, Linear Factors, and Zeros

Find the zeros of each polynomial below. For each corresponding row, shade in each number that is a zero. The illustration made from shading the squares suggests the answer to the riddle below.

A. $P(x) = x(x^2 - 1)$ ____________

B. $P(x) = x(x + 2)(x + 1)(x^2 + 2x - 3)$ ____________

C. $P(x) = x(x + 4)(x + 3)(x + 1)(x - 1)$ ____________

D. $P(x) = x(x^2 - 25)(x^2 + 4x + 3)$ ____________

E. $P(x) = (x^2 + x - 20)(x + 2)(x^2 + 4x + 3)$ ____________

F. $P(x) = (x^2 - 9)(x^2 - 25)$ ____________

G. $P(x) = (x^2 + 9x + 20)(x^2 - 5x + 6)(x - 5)$ ____________

H. $P(x) = (x^2 - 5x + 6)(x^2 - 9x + 20)$ ____________

I. $P(x) = x^2 - 6x + 9$ ____________

J. $P(x) = (x^2 - 4x + 4)(x^2 - 4x + 4)$ ____________

K. $P(x) = x(x^2 - 2x + 1)(x - 2)$ ____________

A	–5	–4	–3	–2	–1	0	1	2	3	4	5
B	5	–5	–4	–3	–2	–1	0	1	2	3	4
C	4	5	–5	–4	–3	–2	–1	0	1	2	3
D	3	4	5	–5	–4	–3	–2	–1	0	1	2
E	2	3	4	5	–5	–4	–3	–2	–1	0	1
F	1	2	3	4	5	–5	–4	–3	–2	–1	0
G	0	1	2	3	4	5	–5	–4	–3	–2	–1
H	–1	0	1	2	3	4	5	–5	–4	–3	–2
I	–2	–1	0	1	2	3	4	5	–5	–4	–3
J	–3	–2	–1	0	1	2	3	4	5	–5	–4
K	–4	–3	–2	–1	0	1	2	3	4	5	–5

Riddle: This grows above the ground, but the solutions to the polynomials above lie beneath. And as it grows, it provides shade to those underneath. What is it?

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E	2	3	4	5	–5	–4	–3	–2	–1	0	1
F	1	2	3	4	5	–5	–4	–3	–2	–1	0
G	0	1	2	3	4	5	–5	–4	–3	–2	–1
H	–1	0	1	2	3	4	5	–5	–4	–3	–2
I	–2	–1	0	1	2	3	4	5	–5	–4	–3
J	–3	–2	–1	0	1	2	3	4	5	–5	–4
K	–4	–3	–2	–1	0	1	2	3	4	5	–5

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