**Exponential Models**

Clues in the word problems tell you which formula to use. If there's no mention of compounding, use a growth or decay model. If your interest is compounded, check for the word continuous. That's your clue to use the "Pert" Formula.

| Simple Interest Growth | Simple Interest Decay | Compound Interest | Continuously Compounded Interest |
|---|---|---|---|
| $A(t) = a(1+r)^t$ | $A(t) = a(1-r)^t$ | $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$ | $A(t) = Pe^{rt}$ |

| | | | | | |
|---|---|---|---|---|---|
| $A(t)$ | Amount after time t. | $a$ | Initial amount | $r$ | Rate expressed as a decimal |
| $t$ | Time | $n$ | Number of interest payments in one year | $P$ | Initial investment |

**Growth**

Example baseball card bought for $150 **increases in value** at a rate of 3% each year. How much is the card worth in 10 years?

$$A = 150(1+.03)^{10}$$

1.) The yellow bellied sapsucker has a population growth rate of approximately 4.7% If the population was 8,530 in 2000 and this growth rate continues, about how many yellow bellied sapsuckers will there be in 2006?

**Decay**

You bought a new Ford truck for $40,000 yesterday. The truck **depreciates** a rate of 11% each year. How much is your truck worth 8 years from now?

$$A = 40000(1-.11)^8$$

2.) Amy Farah Fowler bought a new car for $25,000. Suppose the car depreciates at a rate of 13% per year. How much will the car be worth in 4 years?

**Compound Interest**

Your favorite Aunt gives you a quick pick. It's your lucky day! You win $1500. You give $500 to your Aunt and put the rest in a savings account that pays 3% interest **compounded *monthly***. How much money will you have in 10 years?

$$A = 1000(1+\frac{.03}{12})^{(12)(10)}$$

3.) If you put $2400 in an account that pays 6.2% interest compounded quarterly. How much will you have in eight years?

**Continuous Compounding**

Your Aunt decides to deposit the$500 you gave her into a savings account at her bank. This account pays 3.5% interest and compounds **continuously**. How much money will she have in this account in 8 years?

$$A = 500e^{(.035)(8)}$$

4.) If you put the same $2400 in an account that pays 5.7% interest compounded continuously. How much will you have in eight years?

## Inverse Functions

To find the inverse of a function,

1. Switch x and y values
2. Solve for y

Inverse notation: $f^{-1}(x)$

Find the inverse of each function:

5.) $f(x) = 2x^2 - 8$

6.) $f(x) = \frac{x}{4} + 3$

For logs and exponents, put the equation in the "other form". Then switch x and y, solve for y.

| $y=\log_4(16x)$ | Find the inverse |
|---|---|
| $x=\log_4(16y)$ | Switch x and y |
| $4^x=16y$ | Put in exponent form |
| $4^x/16=y$ | Solve for y |
| $4^{x-2}=y$ | Simplify if possible |

| $y=4^x$ | Find the inverse |
|---|---|
| $x=4^y$ | Switch x and y |
| $y=\log_4 x$ | Put in log form |

7.) $f(x) = 3^{x-2}$

8.) $f(x) = \log(2x - 1)$

## Definition of Logarithms

THE Relationship

$$If\ y = b^x, then\ log_b y = x$$

Write $6^2 = 36$ in log form

$$log_6 36 = 2$$

Write $log_2 64 = 6$ in exponential form

$$2^6 = 64$$

Write the following in log form:

9.) $6^2 = 36$ 10.) $5^3 = 125$ 11.) $2^4 = 32$

Write the following in exponential form:

12.) $\log_2 8 = 3$ 13.) $\log_3 81 = 4$

14.) $\log_4 16 = 2$

**Change of base formula**

$$log_b x = \frac{\log x}{\log b}$$

Evaluate $log_6 32$

$$log_6 32 = \frac{\log 32}{\log 6} = 1.9343$$

**Common log**

$log_{10}$ *is written as* $log$

15.) Evaluate $log_2 8$

**Natural Log**

$log_e$ *is written as* $ln$

16.) Evaluate $log_{0.25} 0.0625$

**Properties of Logarithms**

PROPERTIES

$log_b b = 1$
$lob_b 1 = 0$

$log_b mn = log_b m + lob_b n$

$log_b \frac{m}{n} = log_b m - log_b n$

$log_b m^n = n\ log_b m$

*To condense log statements, they must have the same base.*

**EX 1**: Condense the following into one log statement.

$3\ log_4 x + 2\ log_4 y$

Step 1: Move the constants in front of the log statements into the exponent position.

$log_4 x^3 + log_4 y^2$

Step 2: Combine the arguments. Change subtraction to multiplication and addition to multiplication.

$log_4 x^3 y^2$

**EX2:** Expand the expression $log \frac{x}{yz^2}$

Step 1: Deal with the division operation first. Split the argument into two logs.

$\log x - log\ yz^2$

Step 2: Split any statements with multiplication into addition operations. Be sure to distribute the negative from the division.

$\log x - (\log y + \log z^2)$
$\log x - \log y - \log z^2$

Step 3: Move any exponents in front of the log statement.

$\log x - \log y - 2 \log z$

**Condense the following Log Statements**

17.) $\log_5 4 + \log_5 3$

18.) $\frac{1}{3}\log 3x + \frac{2}{3}\log 3x$

19.) $\log_3 2x - 5\log_3 y$

20.) $\log_5 y - 4(\log_5 r + 2\log_5 t)$

**Expand the following Log Statements**

21.) $\log 6x^3 y$

22.) $\log_2 \frac{x}{yz}$

23.) $\log \sqrt{\frac{2rst}{5w}}$

| **Solve Exponential and Logarithmic Equations** | | |
|---|---|---|
| To **solve** an **exponential equation**, take the log of both sides, and **solve** for the variable.<br><br>To **solve** a **logarithmic equation**, rewrite the **equation** in exponential form and **solve** for the variable.<br><br>Other helpful properties:<br>$log_b b^x = x$<br>$b^{log_b x} = x$ | Solve the equation $3^{x-2} + 5 = 74$.<br><br>$3^{x-2} = 69.$ Subtract 5 from both sides.<br>$\log(3^{x-2}) = \log 69$ Take the log of both sides<br>$(x-2)\log 3 = \log 69$ Simplify the left side<br>$x - 2 = \frac{\log 69}{\log 3}$ Evaluate logs<br>$x - 2 = 3.85$<br>$x = 5.85$ Solve for x | Solve the equation $log_2 4x = 5$<br><br>$4x = 2^5$ Put in exponential form.<br>$4x = 32$ Simplify right side<br>$x = 16$ Divide both sides by log 4. |
| **Solve the following equations** | | |
| 24.) $8^{n+1} = 3$ | 25.) $10^{3y} = 5$ | 26.) $4^x - 5 = 12$ |
| 27.) $\log(2x + 5) = 3$ | 28.) $\log 4x = 2$ | 29). $2\log(2x + 5) = 4$ |

**Laws of Exponents**

| Law | Example |
|---|---|
| $x^1 = x$ | $6^1 = 6$ |
| $x^0 = 1$ | $7^0 = 1$ |
| $x^{-1} = 1/x$ | $4^{-1} = 1/4$ |
| $x^m x^n = x^{m+n}$ | $x^2x^3 = x^{2+3} = x^5$ |
| $x^m/x^n = x^{m-n}$ | $x^6/x^2 = x^{6-2} = x^4$ |
| $(x^m)^n = x^{mn}$ | $(x^2)^3 = x^{2\times3} = x^6$ |
| $(xy)^n = x^n y^n$ | $(xy)^3 = x^3y^3$ |
| $(x/y)^n = x^n/y^n$ | $(x/y)^2 = x^2 / y^2$ |
| $x^{-n} = 1/x^n$ | $x^{-3} = 1/x^3$ |
| And the law about Fractional Exponents: | |
| $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$ | $x^{\frac{2}{3}} = \sqrt[3]{x^2} = (\sqrt[3]{x})^2$ |