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7-3 Reteaching

Logarithmic Functions as Inverses

A logarithmic function is the inverse of an exponential function.

To evaluate logarithmic expressions, use the fact that $x = \log_b y$ is the same as $y = b^x$. Keep in mind that $x = \log y$ is another way of writing $x = \log_{10} y$.

Problem

What is the logarithmic form of $6^3 = 216$?

Step 1 Determine which equation to use.

The equation is in the form $b^x = y$.

Step 2 Find x, y, and b.

$b = 6, x = 3$, and $y = 216$

Step 3 Because $y = b^x$ is the same as $x = \log_b y$, rewrite the equation in logarithmic form by substituting for x, y, and b.

$3 = \log_6 216$

Exercises

Write each equation in logarithmic form.

1. $4^{-3} = \frac{1}{64}$ **2.** $5^{-2} = \frac{1}{25}$ **3.** $8^{-1} = \frac{1}{8}$ **4.** $11^0 = 1$

5. $6^1 = 6$ **6.** $6^{-3} = \frac{1}{216}$ **7.** $17^0 = 1$ **8.** $17^1 = 17$

Problem

What is the exponential form of $4 = \log_5 625$?

Step 1 Determine which equation to use.

The equation is in the form $x = \log_b y$.

Step 2 Find x, y, and b.

$x = 4$, $b = 5$, and $y = 625$

Step 3 Because $x = \log_b y$ is the same as $y = b^x$, rewrite the equation in exponential form by substituting for x, y, and b.

$625 = 5^4$

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7-3 Reteaching (continued)

Logarithmic Functions as Inverses

Exercises

Write each equation in exponential form.

9. $3 = \log_2 8$ **10.** $2 = \log_5 25$

11. $\log 0.1 = -1$ **12.** $\log 7 \square 0.845$

13. $\log 1000 = 3$ **14.** $-2 = \log 0.01$

15. $\log_3 81 = 4$ **16.** $\log_{49} 7 = \frac{1}{2}$

17. $\log_8 \frac{1}{4} = -\frac{2}{3}$ **18.** $\log_2 128 = 7$

19. $\log_5 \frac{1}{625} = -4$ **20.** $\log_6 36 = 2$

Problem

What is the value of $\log_4 32$?

$x = \log_4 32$	Write the equation in logarithmic form $x = \log_b y$.
$32 = 4^x$	Rewrite in exponential form $y = b^x$.
$2^5 = (2^2)^x$	Rewrite each side of the equation with like bases in order to solve the equation.
$2^5 = 2^{2x}$	Simplify.
$5 = 2x$	Set the exponents equal to each other.
$x = \frac{5}{2}$	Solve for x.
$\log_4 32 = \frac{5}{2}$	

Exercises

Evaluate the logarithm.

21. $\log_2 64$ **22.** $\log_2 64$ **23.** $\log_3 3^4$

24. $\log 10$ **25.** $\log 0.1$ **26.** $\log 1$

27. $\log_8 2$ **28.** $\log_{32} 2$ **29.** $\log_9 3$

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7-4 Reteaching

Properties of Logarithms

You can write a logarithmic expression containing more than one logarithm as a single logarithm as long as the bases are equal. You can write a logarithm that contains a number raised to a power as a logarithm with the power as a coefficient. To understand the following properties, remember that logarithms are powers.

Name	Formula	Why?
Product Property	$\log_b mn = \log_b m + \log_b n$	When you multiply two powers, you add the exponents. Example: $2^6 \cdot 2^2 = 2^{(6+2)} = 2^8$
Quotient Property	$\log_b \frac{m}{n} = \log_b m + \log_b n$	When you divide two powers, you subtract the exponents. Example: $\frac{2^6}{2^2} = 2^{(6-2)} = 2^4$
Power Property	$\log_b m^n = n\log_b m$	When you raise a power to a power, you multiply the exponents. Example: $(2^6)^2 = 2^{(6 \cdot 2)} = 2^{12}$

Problem

What is $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27$ written as a single logarithm?

$2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27 = \log_2 6^2 - \log_2 9 + \log_2 27^{\frac{1}{3}}$	Use the Power Property twice.
$= \log_2 36 - \log_2 9 + \log_2 3$	$6^2 = 36, 27^{\frac{1}{3}} = \sqrt[3]{27} = 3$
$= (\log_2 36 - \log_2 9) + \log_2 3$	Group two of the logarithms. Use order of operations.
$= \log_2 \frac{32}{9} + \log_2 3$	Quotient Property
$= \log_2 \left(\frac{36}{9} \bullet 3\right)$	Product Property
$= \log_2 12$	Simplify.

As a single logarithm, $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27 = \log_2 12$.

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7-4 Reteaching (continued)

Properties of Logarithms

To evaluate logarithms with any base, you can rewrite the logarithm as a quotient of two logarithms with the same base.

$$\log_b m = \frac{\log_c m}{\log_c b}$$

Move the base to the bottom.

Move the number to the numerator.

Problem

What is $\log_4 8$ written as a quotient of two logarithms with base 2? Simplify your answer, if possible.

$\log_4 8 = \frac{\log_2 8}{\log_2 4}$	The base is 4 and the number is 8. Move the base to the bottom and the number to the numerator.
$= \frac{3}{2}$	Evaluate the logarithms in the numerator and the denominator.

Exercises

Write each logarithmic expression as a single logarithm.

1. $\log_3 13 + \log_3 3$ **2.** $2 \log x + \log 5$ **3.** $\log_4 2 - \log_4 6$

4. $3\log_3 3 - \log_3 3$ **5.** $\log_5 8 + \log_5 x$ **6.** $\log 2 - 2\log x$

7. $\log_2 x + \log_2 y$ **8** $3\log_7 x - 5\log_7 y$ **9.** $4\log x + 3\log x$

10. $\log_5 x + 3\log_5 y$ **11.** $3\log_2 x - \log_2 y$ **12.** $\log_2 16 - \log_2 8$

Write each logarithm as a quotient of two common logarithms. Simplify your answer, if possible. (*Hint:* Common logarithms are logarithms with base 10.)

13. $\log_4 12$ **14.** $\log_2 1000$ **15.** $\log_5 16$

16. $\log_{11} 205$ **17.** $\log_9 32$ **18.** $\log_{100} 5$

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7-5 Reteaching

Exponential and Logarithmic Equations

Use logarithms to solve exponential equations.

Problem

What is the solution of $7 - 5^{2x-1} = 4$?

$7 - 5^{2x-1} = 4$	
$-5^{2x-1} = -3$	First isolate the term that has the variable in the exponent. Begin by subtracting 7 from each side.
$5^{2x-1} = 3$	Multiply each side by –1.
$\log_5 5^{2x-1} = \log_5 3$	Because the variable is in the exponent, use logarithms. Take $\log_5$ of each side because 5 is the base of the exponent.
$(2x-1)\log_5 5 = \log_5 3$	Use the Power Property of Logarithms.
$2x - 1 = \log_5 3$	Simplify. (Recall that $\log_b b = 1$.)
$2x - 1 = \frac{\log 3}{\log 5}$	Apply the Change of Base Formula.
$2x = \frac{\log 3}{\log 5} + 1$	Add 1 to each side.
$x = \frac{1}{2}\left(\frac{\log 3}{\log 5} + 1\right)$	Divide each side by 2.
$x \approx 0.84$	Use a calculator to find a decimal approximation.

Exercises

Solve each equation. Round the answer to the nearest hundredth.

1. $2^x = 5$ **2.** $10^{2x} = 8$ **3.** $5^{x+1} = 25$

4. $2^{x+3} = 9$ **5.** $3^{2x-3} = 7$ **6.** $4^x - 5 = 3$

7. $5 + 2^{x+6} = 9$ **8.** $4^{3x} + 2 = 3$ **9.** $1 - 3^{2x} = -5$

10. $2^{3x} - 2 = 13$ **11.** $5^{2x+7} - 1 = 8$ **12.** $7 - 2^{x+7} = 5$

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7-5 Reteaching (continued)

Exponential and Logarithmic Equations

Use exponents to solve logarithmic equations.

Problem

What is the solution of $8 - \log(4x - 3) = 4$?

$8 - 2\log(4x - 3) = 4$	
$-\log(4x - 3) = -4$	First isolate the term that has the variable in the logarithm. Begin by subtracting 8 from each side.
$\log(4x - 3) = 4$	Multiply each side by –1.
$4x - 3 = 10^4$	Write in exponential form.
$4x - 3 = 10{,}000$	Simplify.
$4x = 10{,}003$	Add 3 to each side.
$x = \frac{10{,}003}{4}$	Solve for x.
$x = 2500.75$	Divide.

Exercises

Solve each equation. Round the answer to the nearest thousandth.

13. $\log x = 2$ **14.** $\log 3x = 3$

15. $\log 2x + 2 = 6$ **16.** $5 + \log(2x + 1) = 6$

17. $\log 5x + 62 = 62$ **18.** $6 - \log\frac{1}{2}x = 3$

19. $\log(4x - 3) + 6 = 4$ **20.** $\frac{2}{3}\log 5x = 2$

21. $2\log 250x - 6 = 4$ **22.** $5 - 2\log x = \frac{1}{2}$

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7-6 Reteaching

Natural Logarithms

The **natural logarithmic function** is a logarithm with base e, an irrational number.

You can write the natural logarithmic function as $y = \log_e x$, but you usually write it as $y = \ln x$.

$y = e^x$ and $y = \ln x$ are inverses, so if $y = e^x$, then $x = \ln y$.

To solve a natural logarithm equation:

- If the term containing the variable is an exponential expression, rewrite the equation in logarithmic form.
- If term containing the variable is a logarithmic expression, rewrite the equation in exponential form.

Problem

What is the solution of $4e^{2x} - 2 = 3$?

Step 1 Isolate the term containing the variable on one side of the equation.

$4e^{2x} - 2 = 3$

$4e^{2x} = 5$ — Add 2 to each side of the equation.

$e^{2x} = \frac{5}{4}$ — Divide each side of the equation by 4.

Step 2 Take the natural logarithm of each side of the equation.

$\ln(e^{2x}) = \ln\left(\frac{5}{4}\right)$

$2x = \ln\left(\frac{5}{4}\right)$ — Definition of natural logarithm

Step 3 Solve for the variable.

$x = \frac{\ln\left(\frac{5}{4}\right)}{2}$ — Divide each side of the equation by 2.

$x \approx 0.112$ — Use a calculator.

Step 4 Check the solution.

$4e^{2(0.112)} - 2 \stackrel{?}{=} 3$

$4e^{0.224} - 2 \stackrel{?}{=} 3$

$3.004 \approx 3$

The solution is $x \approx 0.112$.

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7-6 Reteaching (continued)

Natural Logarithms

Problem

What is the solution of $\ln(t - 2)^2 + 1 = 6$? Round your answer to the nearest thousandth.

Step 1 Isolate the term containing the variable on one side of the equation.

$\ln(t-2)^2 + 1 = 6$

$\ln(t-2)^2 = 5$ — Subtract 1 from each side of the equation.

Step 2 Raise each side of the equation to the base e.

$e^{\ln}(t-2)^2 = e^5$

$(t-2)^2 = e^5$ — Definition of natural logarithm

Step 3 Solve for the variable.

$t - 2 = \pm e^{\frac{5}{2}}$ — Take the square root of each side of the equation.

$t = 2 \pm e^{\frac{5}{2}}$ — Add 2 to each side of the equation.

$t \approx 14.182$ or -10.182 — Use a calculator.

Step 4 Check the solution.

$\ln(14.182 - 2)^2 \stackrel{?}{=} 5$ | $\ln(-10.182 - 2)^2 \stackrel{?}{=} 5$

$4.9999 \approx 5$ | $4.9999 \approx 5$

The solutions are $t \approx 14.182$ and -10.182.

Exercises

Use natural logarithms to solve each equation. Round your answer to the nearest thousandth. Check your answers.

1. $2e^x = 4$

2. $e^{4x} = 25$

3. $e^x = 72$

4. $e^{3x} = 124$

5. $12e^{3x-2} = 8$

6. $\frac{1}{2}e^{6x} = 5$

Solve each equation. Round your answer to the nearest thousandth. Check your answers.

7. $\ln(x - 3) = 2$

8. $\ln 2t = 4$

9. $1 + \ln x^2 = 2$

10. $\ln(2x - 5) = 3$

11. $\frac{1}{3}\ln 2t = 1$

12. $\ln(t - 4)^2 + 2 = 5$