

# 4-2 Reteaching (continued)

## Standard Form of a Quadratic Function

- Standard form of a quadratic function is  $y = ax^2 + bx + c$ . Vertex form of a quadratic function is  $y = a(x - h)^2 + k$ .
- For a parabola in vertex form, the coordinates of the vertex are  $(h, k)$ .

### Problem

What is the vertex form of  $y = 3x^2 - 24x + 50$ ?

$$y = ax^2 + bx + c$$

$$y = 3x^2 - 24x + 50$$

$$b = -24, a = 3$$

$$\begin{aligned} x\text{-coordinate} &= -\frac{b}{2a} \\ &= -\frac{-24}{2(3)} \\ &= 4 \end{aligned}$$

$$\begin{aligned} y\text{-coordinate} &= 3(4)^2 - 24(4) + 50 \\ &= 2 \end{aligned}$$

$$y = 3(x - 4)^2 + 2$$

Verify that the equation is in standard form.

Find  $b$  and  $a$ .

For an equation in standard form, the  $x$ -coordinate of the vertex can be found by using  $x = -\frac{b}{2a}$ .

Substitute.

Simplify.

Substitute 4 into the standard form to find the  $y$ -coordinate.

Simplify.

Substitute 4 for  $h$  and 2 for  $k$  into the vertex form.

Once the conversion to vertex form is complete, check by multiplying.

$$y = 3(x^2 - 8x + 16) + 2$$

$$y = 3x^2 - 24x + 50$$

The result is the standard form of the equation.

### Exercises

Write each function in vertex form. Check your answers.

5.  $y = x^2 - 2x - 3$

$$y = (x - 1)^2 - 4$$

6.  $y = -x^2 + 4x + 6$

$$y = -(x - 2)^2 + 10$$

7.  $y = x^2 + 3x - 10$

$$y = \left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$$

8.  $y = x^2 - 9x$

$$y = \left(x - \frac{9}{2}\right)^2 - \frac{81}{4}$$

9.  $y = x^2 + x$

$$y = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$$

10.  $y = x^2 + 5x + 4$

$$y = \left(x + \frac{5}{2}\right)^2 - \frac{9}{4}$$

11.  $y = 4x^2 + 8x - 3$

$$y = 4(x + 1)^2 - 7$$

12.  $y = \frac{3}{4}x^2 + 9x$

$$y = \frac{3}{4}(x + 6)^2 - 27$$

13.  $y = -2x^2 + 2x + 1$

$$y = -2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$$

Write each function in standard form.

14.  $y = (x - 3)^2 + 1$

$$y = x^2 - 6x + 10$$

15.  $y = 2(x - 1)^2 - 3$

$$y = 2x^2 - 4x - 1$$

16.  $y = -3(x + 4)^2 + 1$

$$y = -3x^2 - 24x - 47$$

# 4-4 Reteaching

## Factoring Quadratic Expressions

### Problem

What is  $6x^2 - 5x - 4$  in factored form?

$a = 6$ ,  $b = -5$ , and  $c = -4$  Find  $a$ ,  $b$ , and  $c$ ; they are the coefficients of each term.

$ac = -24$  and  $b = -5$  We are looking for factors with product  $ac$  and sum  $b$ .

Factors of $-24$	1, $-24$	$-1$ , 24	2, $-12$	$-2$ , 12	3, $-8$	$-3$ , 8	4, $-6$	$-4$ , 6
Sum of factors	$-23$	23	$-10$	10	$-5$	5	$-2$	2

The factors 3 and  $-8$  are the combination whose sum is  $-5$ .

$$\underbrace{6x^2 + 3x} - \underbrace{8x - 4}$$

Rewrite the middle term using the factors you found.

$$3x(2x + 1) - 4(2x + 1)$$

Find common factors by grouping the terms in pairs.

$$(3x - 4)(2x + 1)$$

Rewrite using the Distributive Property.

**Check**  $(3x - 4)(2x + 1)$  You can check your answer by multiplying the factors together.

$$6x^2 + 3x - 8x - 4$$

$$6x^2 - 5x - 4$$

Remember that not all quadratic expressions are factorable.

### Exercises

Factor each expression.

- $x^2 + 6x + 8$   $(x + 4)(x + 2)$
- $x^2 - 4x + 3$   $(x - 3)(x - 1)$
- $2x^2 - 6x + 4$   $2(x - 2)(x - 1)$
- $2x^2 - 11x + 5$   $(2x - 1)(x - 5)$
- $2x^2 - 7x - 4$   $(2x + 1)(x - 4)$
- $4x^2 + 16x + 15$   $(2x + 5)(2x + 3)$
- $x^2 - 5x - 14$   $(x + 2)(x - 7)$
- $7x^2 - 19x - 6$   $(7x + 2)(x - 3)$
- $x^2 - x - 72$   $(x - 9)(x + 8)$
- $2x^2 + 9x + 7$   $(2x + 7)(x + 1)$
- $x^2 + 12x + 32$   $(x + 4)(x + 8)$
- $4x^2 - 28x + 49$   $(2x - 7)(2x - 7)$
- $x^2 - 3x - 10$   $(x - 5)(x + 2)$
- $2x^2 + 9x + 4$   $(2x + 1)(x + 4)$
- $9x^2 - 6x + 1$   $(3x - 1)(3x - 1)$
- $x^2 - 10x + 9$   $(x - 1)(x - 9)$
- $x^2 + 4x - 12$   $(x + 6)(x - 2)$
- $x^2 + 7x + 10$   $(x + 5)(x + 2)$
- $x^2 - 8x + 12$   $(x - 6)(x - 2)$
- $2x^2 - 5x - 3$   $(2x + 1)(x - 3)$
- $x^2 - 6x + 5$   $(x - 1)(x - 5)$
- $3x^2 + 2x - 8$   $(3x - 4)(x + 2)$

# 4-4 Reteaching (continued)

## Factoring Quadratic Expressions

- $a^2 + 2ab + b^2 = (a + b)^2$  Factoring perfect square trinomials  
 $a^2 - 2ab + b^2 = (a - b)^2$
- $a^2 - b^2 = (a + b)(a - b)$  Factoring a difference of two squares

### Problem

What is  $25x^2 - 20x + 4$  in factored form?

There are three terms. Therefore, the expression may be a perfect square trinomial.

$$a^2 = 25x^2 \text{ and } b^2 = 4 \quad \text{Find } a^2 \text{ and } b^2.$$

$$a = 5x \text{ and } b = 2 \quad \text{Take square roots to find } a \text{ and } b.$$

Check that the choice of  $a$  and  $b$  gives the correct middle term.

$$2ab = 2 \cdot 5x \cdot 2 = 20x$$

Write the factored form.

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$25x^2 - 20x + 4 = (5x - 2)^2$$

- Check**  $(5x - 2)^2$  You can check your answer by multiplying the factors together.  
 $(5x - 2)(5x - 2)$  Rewrite the square in expanded form.  
 $25x^2 - 10x - 10x + 4$  Distribute.  
 $25x^2 - 20x + 4$  Simplify.

### Exercises

Factor each expression.

23.  $x^2 - 12x + 36$

$$(x - 6)^2$$

26.  $x^2 - 64$

$$(x + 8)(x - 8)$$

29.  $27x^2 - 12$

$$3(3x + 2)(3x - 2)$$

32.  $9x^2 - 16$

$$(3x + 4)(3x - 4)$$

35.  $125x^2 - 100x + 20$

$$5(5x - 2)^2$$

24.  $x^2 + 30x + 225$

$$(x + 15)^2$$

27.  $9x^2 - 42x + 49$

$$(3x - 7)^2$$

30.  $49x^2 + 42x + 9$

$$(7x + 3)^2$$

33.  $8x^2 - 18$

$$2(2x + 3)(2x - 3)$$

36.  $-x^2 + 196$

$$-(x + 14)(x - 14)$$

25.  $9x^2 - 12x + 4$

$$(3x - 2)^2$$

28.  $25x^2 - 1$

$$(5x + 1)(5x - 1)$$

31.  $16x^2 - 32x + 16$

$$16(x - 1)^2$$

34.  $81x^2 + 126x + 49$

$$(9x + 7)^2$$

37.  $-16x^2 - 24x - 9$

$$-(4x + 3)^2$$