I. Function Characteristics

Domain: *Interval* of possible x values for a given function. (Left,Right)

Range: *Interval* of possible y values for a given function. (down, up)

End Behavior: What is happening at the far ends of the graph?

For each side	Left side	Right side
	$x \to -\infty$,	$x \to \infty$
Pick one of these	Points Down	Points Up
	$y \to -\infty$	$y \to \infty$

Increasing Intervals: *Interval* of x values for which the corresponding y values are increasing.

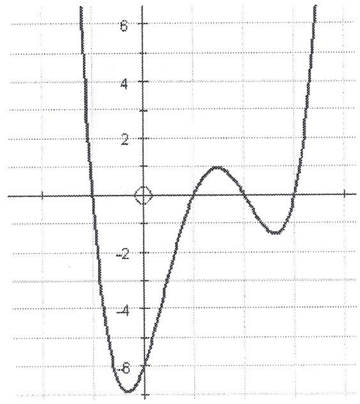
Decreasing Intervals: *Interval* of x values for which the corresponding y values are decreasing.

x-Intercepts: *points* where the graph crosses the x axis. $(x, 0)$

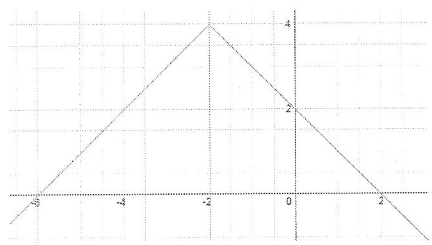
y-Intercepts: *points* where the graph crosses the y axis. $(0, y)$

Maximums: *points* where the graph changes from increasing to decreasing. Peaks in the graph.

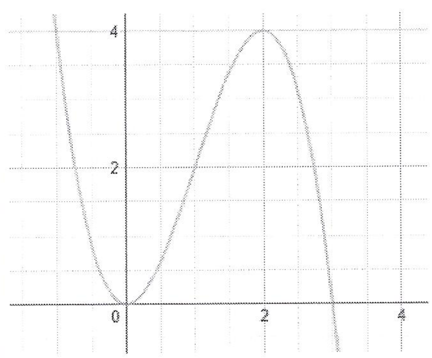
Minimums: *points* where the graph changes from decreasing to increasing. Valleys in the graph.



Domain: $(-\infty, \infty)$
Range: $[-7, \infty)$
End Behavior:
As $x \to -\infty, y \to \infty$
As $x \to \infty, y \to \infty$
Increasing Intervals: $(-0.5, 1.5), (2.5, \infty)$
Decreasing Intervals: $(-\infty, -0.5), (1.5, 2.5)$
x-Intercepts: $(-1,0), (1,0), (2,0), (3,0)$
y-Intercepts: $(0, -6)$
Maximums: $(1.5, 1)$
Minimums: $(-0.5, -7), (2.5, -1.25)$



Domain: $(-\infty, \infty)$
Range: $(-\infty, 4]$
End Behavior:
As $x \to -\infty$, $y \to -\infty$
As $x \to \infty$, $y \to -\infty$
Increasing Intervals: $(-\infty, -2)$
Decreasing Intervals: $(-2, \infty)$
x-Intercepts: $(-6,0), (2,0)$
y-Intercepts: $(0,2)$
Maximums: $(-2,4)$
Minimums: none



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
End Behavior:
As $x \to -\infty$, $y \to \infty$
As $x \to \infty$, $y \to -\infty$
Increasing Intervals: $(0, 2)$
Decreasing Intervals: $(-\infty, 0), (2, \infty)$
x-Intercepts: $(0,0)$ $(3,0)$
y-Intercepts: $(0,0)$
Maximums: $(2,0)$
Minimums: $(0,0)$

II. Function Transformations

General form: $g(x) = a\,f(x-h) + k$

f(x) parent function
g(x) transformed function

a if negative, flip vertically

$0 < |a| < 1$ vertical compression
$|a| > 1$ vertical stretch

h if negative, horizontal shift right
if positive, horizontal shift left

k if negative, vertical shift down
if positive, vertical shift up

Examples

$E1.\ g(x) = x^2 + 2$
Parent Function:
quadratic
Transformations:
shift up 2 units

$E2.\ g(x) = -(x-4)^3 - 1$
Parent Function:
cubic
Transformations:
flip vertically
shift right 4 units
shift down 1 unit

$E3.\ g(x) = 3\sqrt{x+1} - 7$
Parent Function:
Radical (square root)
Transformations:
Stretch by a factor of 3
Shift left 1 unit
Shift down 7 units

$E4.\ g(x) = -\frac{1}{2}(x-3)^2 + 1$
Parent Function:
quadratic
Transformations:
Flip vertically
Compression by a factor of $\frac{1}{2}$
Shift Right 3 units
Shift up 1 unit

1. $g(x) = 2^{x-3} + 5$
Parent Function:
exponential
Transformations:
right 3
up 5

2. $g(x) = -(x+7)^2$
Parent Function:
quadratic
Transformations:
flip over x axis
shift left 7

3. $g(x) = 2\log(x-2) - 1$
Parent Function:
logarithmic
Transformations:
stretch by factor of 2
shift right 2
shift down 1

III. Graphing a function from an equation - Example

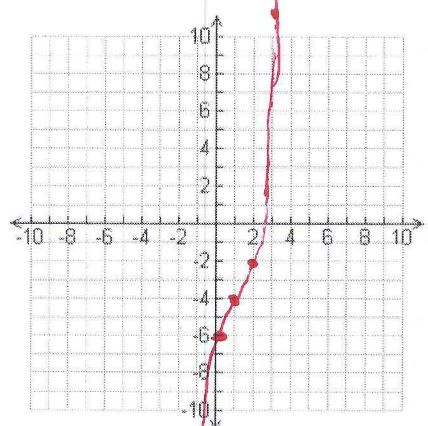
1. Identify the parent function to determine a general shape.
<u>*Cubic*</u>

2. Think about where the vertex or critical points are usually found for the parent function.
<u>*Centered at the origin. Is always increasing from left to right.*</u>

3. Where are the critical points of the new function given the transformations in the equation? <u>*Since there is a horizontal shift right 1 unit and a vertical shift down four units, the center is at the point (1,-4).*</u>

4. Use this information to plan which points to plot on the graph. Make a t table with these points.
<u>*Since the center of the graph is (1,-4), pick two x values on either side of this point and evaluate the f(x) at those x's.*</u>

Graph $f(x) = 2(x-1)^3 - 4$



x	y
-1	-20
0	-6
1	-4
2	-2
3	12

5. Plot the points and connect the dots.

Graphing a function from an equation

1. Identify the parent function to determine a general shape.

cubic

2. Think about where the vertex or critical points are usually found for the parent function.

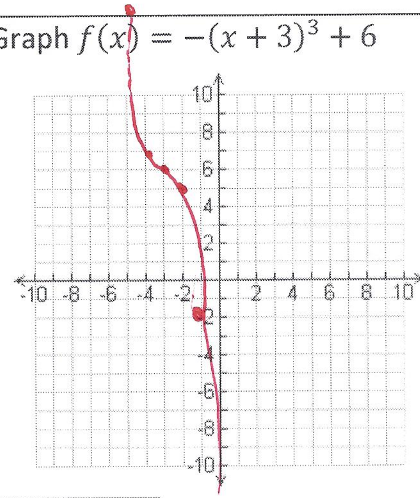
centered at (0,0)
increases from left to right

3. Where are the critical points of the new function given the transformations in the equation?

centered at (-3,6)

4. Use this information to plan which points to plot on the graph. Make a t table with these points.

Graph $f(x) = -(x+3)^3 + 6$



x	y
-1	-2
-2	5
-3	6
-4	7
-5	14

5. Plot the points and connect the dots.

Writing Function Equation from a description of the transformations

How do translations effect equation?

$f(x) = -a(x-h) + k$

"-" flip over x axis

a compression or stretch

h horizontal shift in the opposite direction of the sign

k vertical shift in the same direction of the sign

EXAMPLE

Write the equation for a quadratic function with a vertical shift down 3, left 7 and a vertical stretch by a factor of 4.

Quadratic : x^2

Down 3: -3 from the function (outside)

Left 7: add 7 to x (inside)

V. stretch by 4: multiply by 4

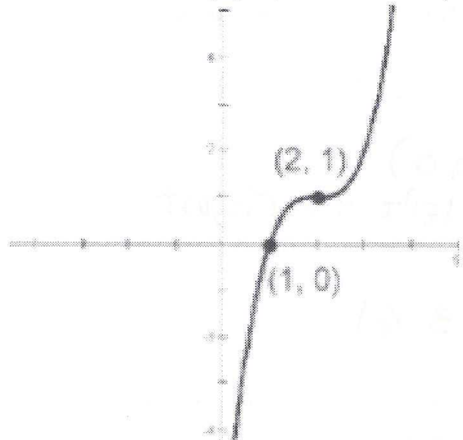
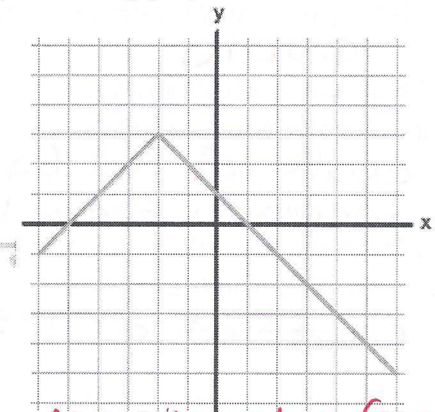
$$y = 4(x+7)^2 - 3$$

Write the equation for an absolute value function that has been compressed by a factor of 2 and shifted down three units

$f(x) = \frac{1}{2}|x| - 3$

Write the equation for a cubic that has been flipped vertically, shifted up 5 units, and shifted right 2 units.

$f(x) = -(x-2)^3 + 5$

Determining Equation from Graph What's the parent function? Where's the vertex or critical point of the parent function? Where's the vertex or critical point of this function? How did we get from the parent function critical point to the critical point of this function? How do I translate those changes into an equation?	Example: Write the equation for the following graph.  Cubic so x^3 vertex is up 2, right 1 $y = (x-1)^3 + 2$	Write the equation for the following graph  Absolute value, flipped & shifted left 2 & up 3 $y = -\|x+2\|+3$
Shifts of Shifts Apply the stated changes to the appropriate parts of the "starting function". $f(x) = (x+1)^2 - 1$ $(-4) \quad +3 \quad +2$ $-4(x+4)+1$	Example: If the function $f(x) = (x+1)^2 - 1$, what would be the equation of g(x) if g(x) is f(x) shifted left 3 units, up 2 units, flipped vertically and stretched by a factor of 4? Left 3: +3 to x Up 2: + 2 Flipped vertically: - in front Stretched by 4: multiplied by 4 $g(x) = -4(x+1+3)^2 - 1 + 2$ $g(x) = -4(x+4)^2 + 1$	$f(x) = 2(x)^3 + 4$ Find g(x) if g(x) is f(x) shifted up 2, right 1 and compressed by a factor of 6. $g(x) = \frac{2}{6}(x-1)^3 + 6$ $= \frac{1}{3}(x-1)^3 + 6$ $f(x) = -\|x-5\|$ Find g(x) if g(x) is f(x) shifted up 4, left 3, stretched by a factor of 2, and flipped vertically. $g(x) = 2\|x-2\|+4$
Shifts of Shifts part 2 State the transformations to f(x) that would yield g(x) Example: $f(x) = -3\sqrt{x-4} + 1$ $g(x) = \frac{3}{5}\sqrt{x+4} + 7$ Was +1, now is +7 so went up 6 Was -4 now is +4 so went left 8 Was 3 now 3/5 so compressed by a factor of 5 Was negative, now positive so flipped vertically	$f(x) = \|x+2\| - 3$ $g(x) = -2\|x+1\| + 2$ flip vertally stretch by factor of 2 shift right 1 shift up 5	$f(x) = -3(x-1)^2 - 3$ $g(x) = -(x+4)^2 - 5$ stretch vertically by factor of 3 left 5 down 2

Real World Functions
Use word problems to create and analyze a function. Decide what information is pertinent, and use it to answer the questions.

Example:
The width of a sandbox is ~~two~~ seven feet greater than the opposite of its length.
Create an equation to represent the area of the sandbox.
A=LW -> $A = x(-x + 7)$
Find a realistic domain
(0,7)many ans. with explanation
Find a realistic range
(0,12.25) many ans. with exp.
What is the maximum area?
12.25 square feet
What length would create that area?
3.5 feet

Mr. Mealey jumps to dunk a basketball. The path followed by his feet forms a parabola following the function $f(t) = -16t^2 + 16t$ where t is the time in seconds after he jumps and f(t) is the height of his feet.

a. What are the realistic domain and range for this graph?
D (0,1] R(0, 4]

b. What is the maximum height of his feet?
4 ft

c. At what time do his feet reach that height?
.5 seconds

d. What are his intervals of increase and decrease?
I (0, .5) D (.5, 1)

e. What are his x and y intercepts? Why?
(0,0) & (1,0)

the intercepts (or zeros) indicate 0 height. This is where Mr. Mealy is on the ground.

A crazy engineer is designing an auditorium to have x sections with x+4 chairs per row and −x+12 chairs per column.

Write an equation for the total number of chairs in the auditorium
$(x)(x+4)(-x+12)$ = #chairs

Find a realistic domain
~~(-4, 12)~~ (0,12)

Find a realistic range
(0, 388)

Find the maximum number of chairs.
388

How many sections would create that number of chairs?
7

graph