WARM UP

1. Name the vertex for the following function.

$$f(x) = (x + 5)^2 - 4$$

2. Complete the square to find the vertex of the following function

$$f(x) = x^2 + 6x + 10$$

3. List the transformations that were applied to the parent function that resulted in the function $g(x) = 2\sqrt{x - 3} + 2$.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Objectives

- Identify the center and radius of a circle from the equation of a circle.
- Write and graph the equation of a circle given the center and radius
- Put an equation of a circle in standard form to find the center and radius

Homework

- All problems on the worksheet (we'll do these in class)
- WBP 263, Even

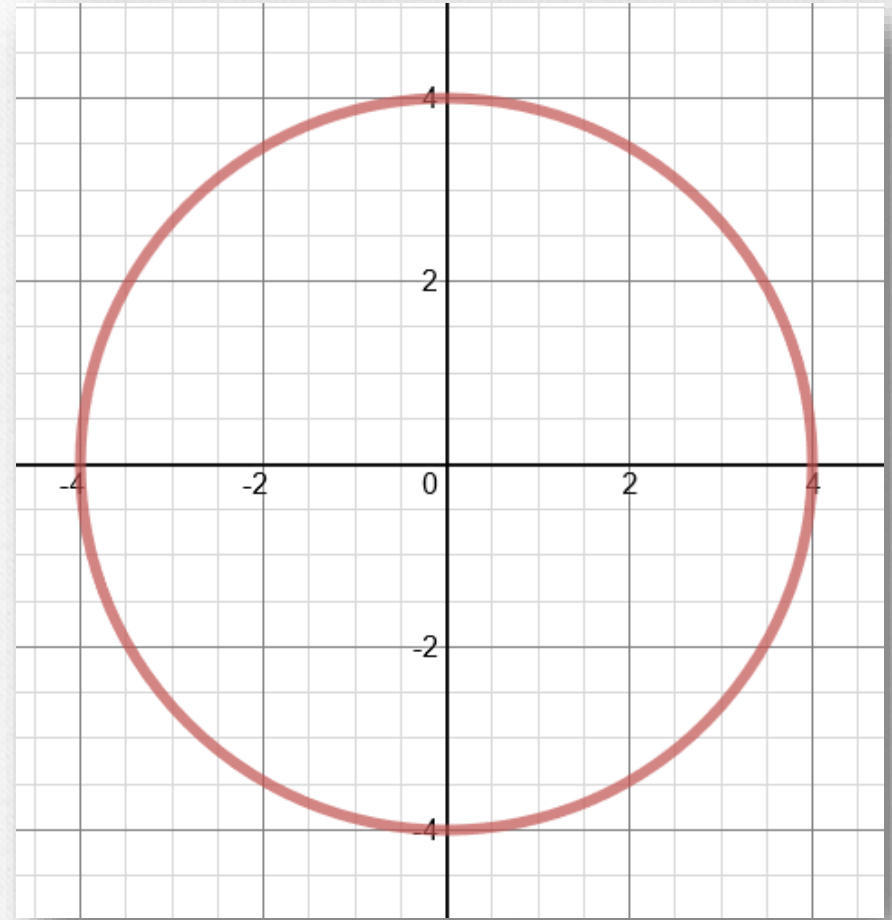A CIRCLE is the set of all points that are the same distance, r, from a fixed point.

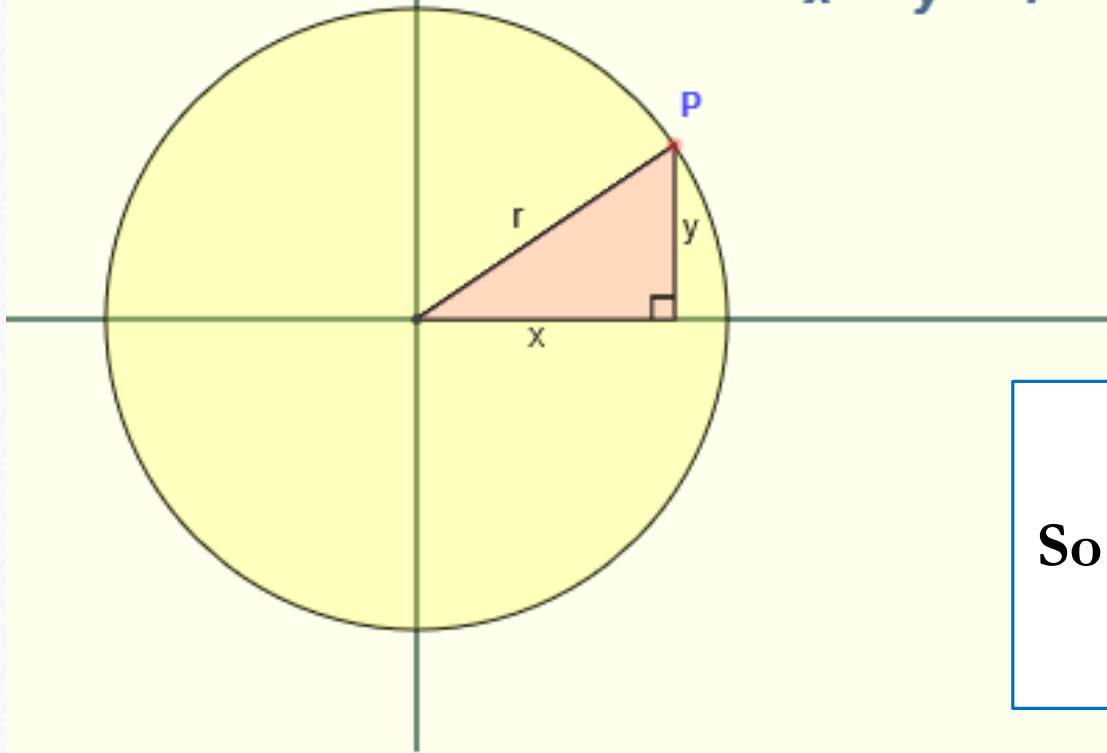What is the radius of this circle?

$r = 4$

Where is the center?

$(0,0)$

What is the equation of this circle?

For any point P at distance r from the origin

$$x^2 + y^2 = r^2$$

P

r

y

x

The world famous **PYTHAGOREAN THEOREM** of course!

So where do we start?

http://www.mathopenref.com/coordbasiccircle.html

Equation of a circle

Standard Form, centered at the origin.
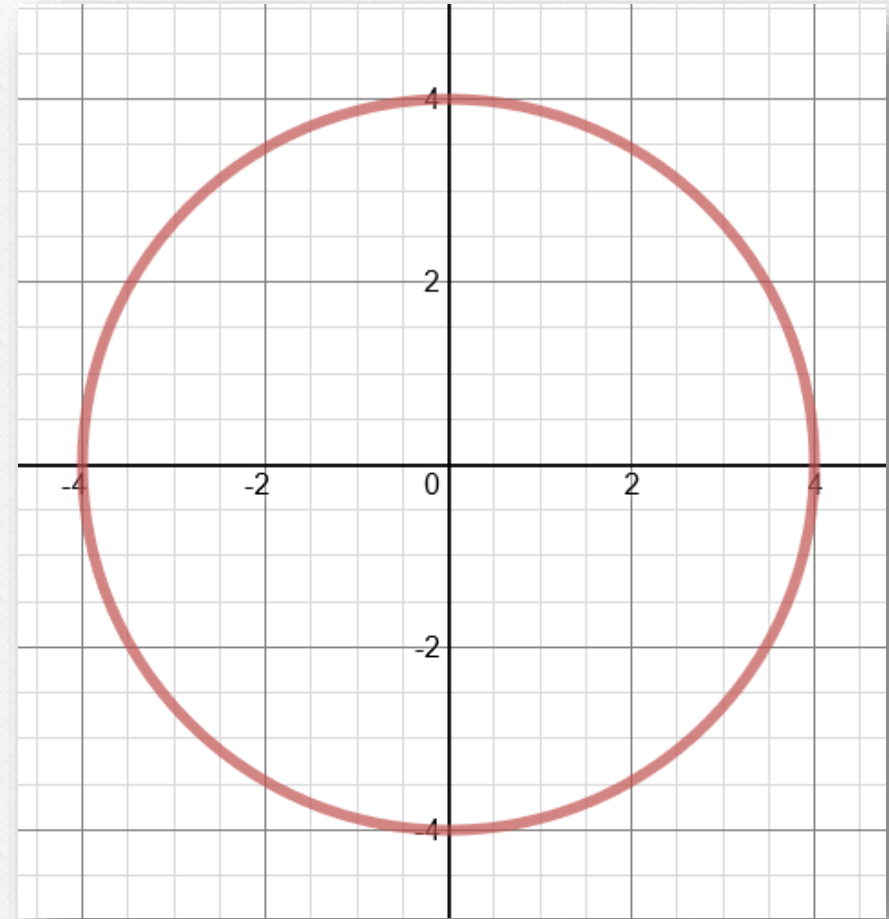
$$x^2 + y^2 = r^2$$

For our circle…

Radius $r = 4$ Center $(0,0)$

Our equation becomes

$$x^2 + y^2 = 16$$

Equation of a circle

Standard Form with center $(\boldsymbol{h}, \boldsymbol{k})$
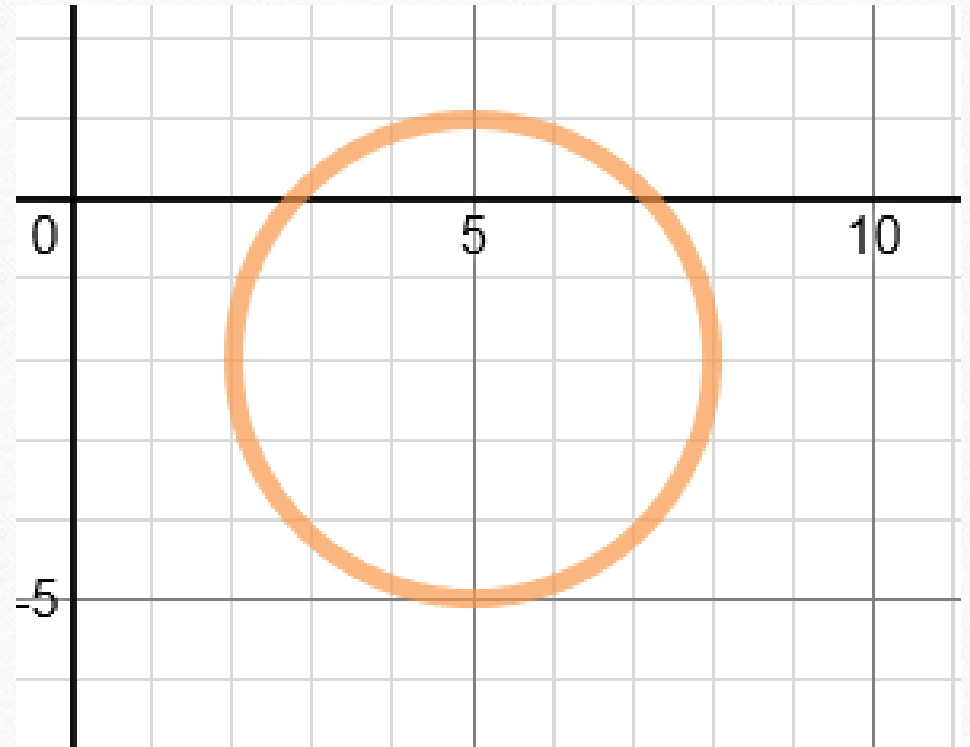
$$(x-h)^2 + (y-k)^2 = r^2$$

For our circle...

Center $(5, -2)$ Radius $r = 3$

Our equation becomes

$$(x-5)^2 + (y+2)^2 = 3^2$$
$$(x-5)^2 + (y+2)^2 = 9$$

What's the difference between these two equation?

$$(x - 2)^2 - y = 4 \qquad\qquad (x - 2)^2 + y^2 = 4$$

Parabola

Circle

Either the x **OR** y term is squared

Both the x **AND** y terms are squared

Vertex is located at the point $(2, -4)$

Center is located at the point $(2, 0)$

$$(x - 2)^2 - 4 = y$$

$$r^2 = 4; r = 2$$

Write the equation of this circle.

Radius $\quad r = 2$

Center $\quad (2, -1)$

$(x - 2)^2 + (y + 1)^2 = 4$

Equation of a circle

General Form

$$ax^2 + by^2 + cx + dy + e = 0$$

Not so easy to figure out where the center is.

We have to "complete the square" for both the x and y variables.

Find the center and radius of the circle $4x^2 + 4y^2 - 16x - 24y + 51 = 0.$

$$4x^2 + 4y^2 - 16x - 24y = -51$$

Move the constant to the other side

$$x^2 + y^2 - 4x - 6y = \frac{-51}{4}$$

Divide out the coefficient of the x^2 term

$$\boxed{x^2 - 4x +4} + \boxed{y^2 - 6y +9} = \frac{-51}{4} +4 +9$$

Group the x and y terms

Complete the square for the x and y terms

$$(x - 2)^2 + (y - 3)^2 = \frac{1}{4}$$

Center (2,3), Radius $\frac{1}{2}$

On your work sheet complete problems 5, 7 and 8.

Two formulas you need to know

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Mid Point Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

For example, find the distance between the points (0,4) and 6,3).

$$d = \sqrt{(6 - 0)^2 + (3 - 4)^2} = \sqrt{37}$$

For example, find the distance between the points (0,4) and 6,3).

$$\left(\frac{0 + 6}{2}, \frac{4 + 3}{2}\right) = \left(3, \frac{7}{2}\right)$$

What if you're only given the center and radius? How will you find the equation of the circle?

What is the equation of a circle with center (4,-2) and a radius of 7.

Equation of a circle – Standard Form $(x - h)^2 + (y - k)^2 = r^2$

Just plug in what you know…

$$(x - 4)^2 + (y + 2)^2 = 7^2$$

$$(x - 4)^2 + (y + 2)^2 = 49$$

Problems 10-14 require you to do some thinking to figure out where the center is and the length of the radius.

10) Center: $(-13, -16)$
    Point on Circle: $(-10, -16)$

What do you know?

What don't you know?

How can you find what you're missing?

Center, which gives you h and k.

Radius of the Circle

The distance formula will give you the radius.

A point on the circle

11) Ends of a diameter: $(18, -13)$ and $(4, -3)$

What do you know?

What don't you know?

How can you find what you're missing?

The points on the diameter of the circle.

Center of circle

Radius of circle

Use the midpoint formula to find the center

Use the distance formula to find the radius.

12) Center: $(10, -14)$
    Tangent to $x = 13$

What do you know?	What don't you know?	How can you find what you're missing?
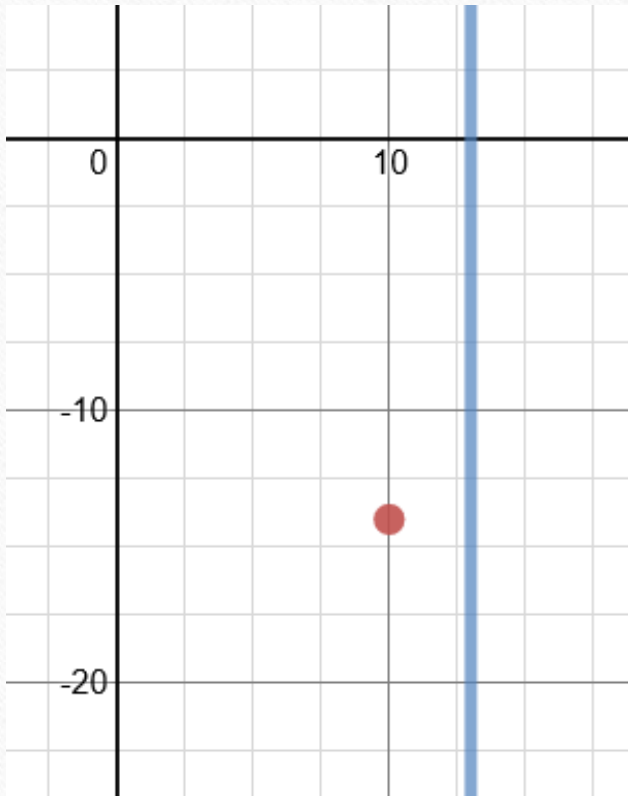| Center, which gives you h and k. | Radius of circle | Use the tangent line to determine the length of the radius

Draw a picture! |

12) Center: $(10, -14)$
    Tangent to $x = 13$

Use the tangent line to determine the length of the radius

What is the horizontal distance from the center point to the tangent line?

$$|10 - 13| = 3$$

Remember that distance is always positive.

13) Center lies in the first quadrant
    Tangent to $x = 8$, $y = 3$, and $x = 14$

What do you know?

What don't you know?

How can you find what you're missing?

"Lies in the first quadrant" tells me that the x and y values are positive.
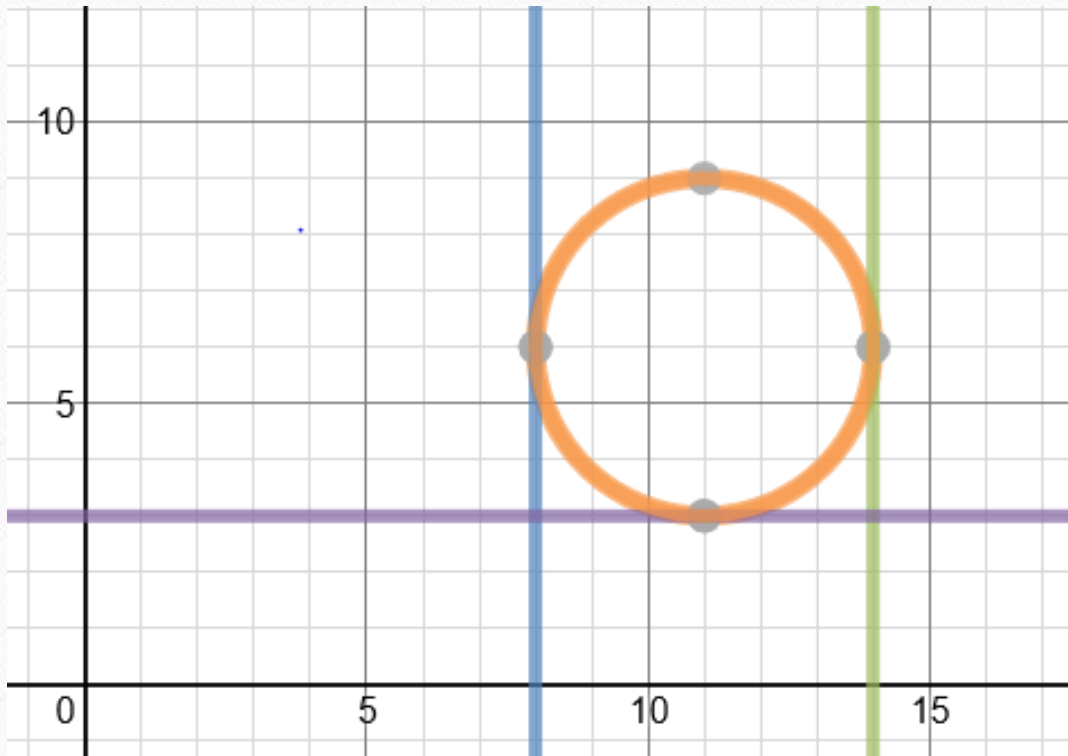
Tangent lines help me put a bound on my circle

Radius of circle

Center of circle

Draw a picture.

13) Center lies in the first quadrant
    Tangent to $x = 8$, $y = 3$, and $x = 14$

Can you find the radius?

Half way between the tangent lines $x = 8$ and $x = 14$.

Is the center above or below the line $y = 3$?

Above. All of the circle must reside in the first quadrant.

14) Center: $(0, 13)$
    Area: 25π

What do you know?	What don't you know?	How can you find what you're missing?
Center of circle	Radius of circle	Use the area formula $A = \pi r^2$ to back into the radius.

Exit Ticket

1. Write an equation in standard form for a circle with a center of (3,12) and radius of 16.

2. Put the equation $x^2 + y^2 + 2x - 4y - 4 = 0$ in standard form.