Warm Up

Solve the following equation and check for extraneous solutions

$$\sqrt{3x+7} = x - 1$$

Simplify the following expressions

$(3 + 4i)(-2 - 2i)$

$(1 + 2i) - (2 + 7i)$

Focus and Directrix

Objectives Define a parabola in conic terms using the focus and directrix.

Use focus and directrix to define a parabola in algegraic terms.

Homework Finish Worksheet

Quiz TIme

Let’s get the quiz out of the way….

Homework Questions?

Find an equation in standard form of the parabola passing through the points.

1. (2, −20), (−2, −4), (0, −8)

$-x^2+4x-8$

2. (1, −3), (2, 0), (3, 9)

$3x^2-6x+0$

3. (2, −8), (3, −8), (6, 4)

x^2-5x-2

4. (−1, −12), (2, −6), (4, −12)

$-x^2+3x-8$

5. A player hits a tennis ball across the court and records the height of the ball at different times, as shown in the table.

a. Find a quadratic model for the data. $-0.5x^2+x+5.5$

b. Use the model to estimate the height of the ball at 4 seconds. 1.5

c. What is the ball's maximum height? 6

Time(s)	Height (ft)
0	5.5
1	6.0
2	5.5
3	4.0
4	1.5

6. Reasoning Explain why the quadratic model only works up to 4.5 seconds — that height measurements made after 4.5 seconds are not valid. (Remember this is a discrete, real situation.) those measurements are negative. reasonable domain is $[0, 4.621]$

7. The table at the right shows the height of the tides measured at the Santa Monica Municipal Pier in California. Hours are measured from 0.00 at midnight.

a. Find a quadratic model for this data using quadratic regression. $.06x^2-.42x+4.1$

b. Use the model to predict the lowest tide height. 2.49

c. When does the lowest tide occur? 5:17 am

Time	Tide Height (ft)
0.33	3.9
3.30	2.7
11.11	4.6

SOURCE: www.tidesandcurrents.noaa.gov

Homework Questions?

8. The table at the right shows in thousands how many people in the U.S. subscribe to a cellular telephone. $547.3x^2 - 1724.8x - 1098$

a. Find a quadratic model for the data.
Let x = the number of years since 1985.

b. Use the model to estimate the number of subscribers in 1995. x = 10 42,800

c. Describe a reasonable domain and range for this situation.

D [0, 100]

R [0, 30,000,000)

Year	U.S Cellular Telephone Subscribership (in thousands)
1985 0	340
1990 5	5283
2000/5	109,478
2004 /9	182,140

SOURCE: CTIA Semi-Annual Wireless Industry

1. Jason jumped off of a cliff into the ocean in Acapulco while vacationing with some friends. His height as a function of time could be modeled by the $h(t) = -16t^2 + 16t + 480$ where t is the time in seconds and h is the height in feet.

a. How long did it take for Jason to reach his maximum height? 0.5 seconds

b. What was the highest point that Jason reached? 484 ft.

c. Jason hit the water after how many seconds? 6

2. If a toy rocket is launched vertically upward from ground level with an initial velocity of 128 feet per second, then its height *h* after *t* seconds is given by the $h(t) = -16t^2 + 128t$ (if air resistance is neglected).

a. How long will it take for the rocket to return to the ground? 8 seconds

b. After how many seconds will the rocket be 112 feet above the ground? 1 & 7 seconds

c. How long will it take the rocket to hit its maximum height? 4

d. What is the maximum height? 256

Homework Questions?

3. You and a friend are hiking in the mountains. You want to climb to a ledge that is 20 ft. above you. The height of the grappling hook you throw is given by the function $h(t) = -16t^2 - 32t + 5$.

a. What is the maximum height of the grappling hook? 5 ft

b. Can you throw it high enough to reach the ledge? no

4. You are trying to dunk a basketball. You need to jump 2.5 ft. in the air to dunk the ball. The height that your feet are above the ground is given by the function $h(t) = -16t^2 + 12t$.

a. What is the maximum height your feet will be above the ground? 2.25

b. Will you be able to dunk the basketball? no

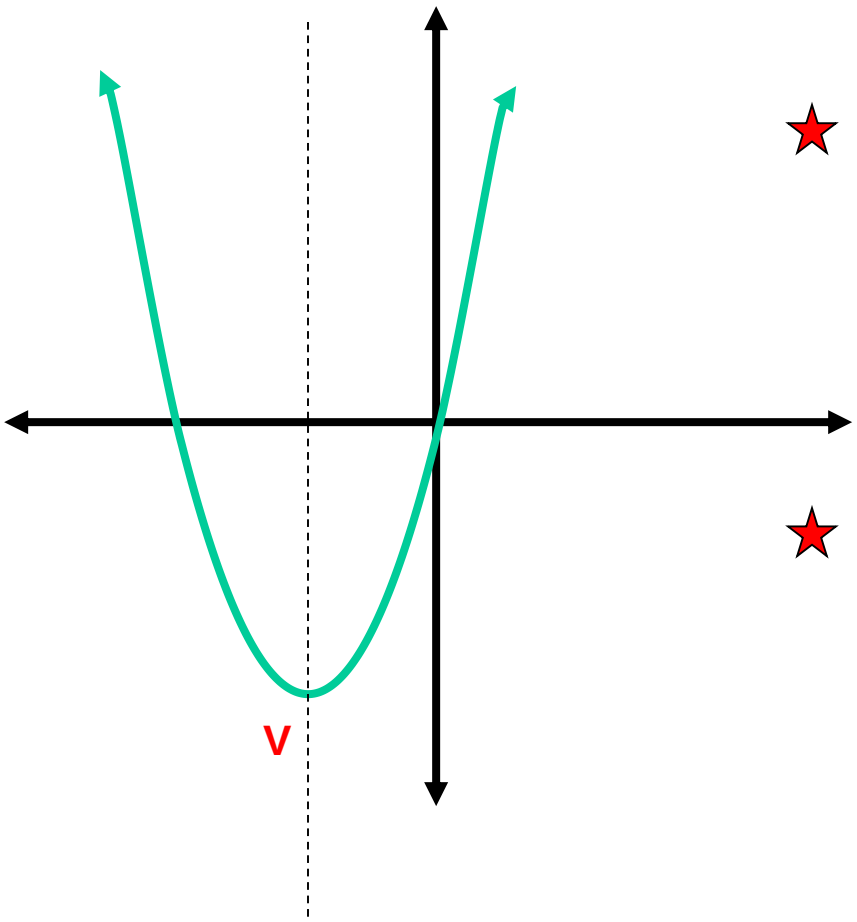
5. A diver is standing on a platform 24 ft. above the pool. He jumps from the platform with an initial upward velocity of 8 ft/s. Use the formula $h(t) = -16t^2 + vt + s$, where h is his height above the water, t is the time, v is his starting upward velocity, and s is his starting height. How long will it take for him to hit the water? $h(t) = -16t^2 + 8t + 24$ 1.5 seconds

6. A trebuchet launches a projectile on a parabolic arc at a velocity of 35 ft/s. Using the function $h(t) = -16t^2 + vt + h_0$ determine when the projectile will first reach a height of 80 ft., and how many seconds later will it again be 80 feet.

$h(t) = -16t^2 + 35t$ won't happen, max height is 15 ft.

7. During World War I, mortars were fired from trenches 3 feet down. The mortars had a velocity of 150 ft/s. Using the function $h(t) = -16t^2 + vt + h_0$ determine how long it will take for the mortar shell to strike its target. $h(t) = -16t^2 + 150t - 3$ 9.35 seconds.

Focus and Directrix



You already know…

Forms of equations

★ $y = a(x - h)^2 + k$

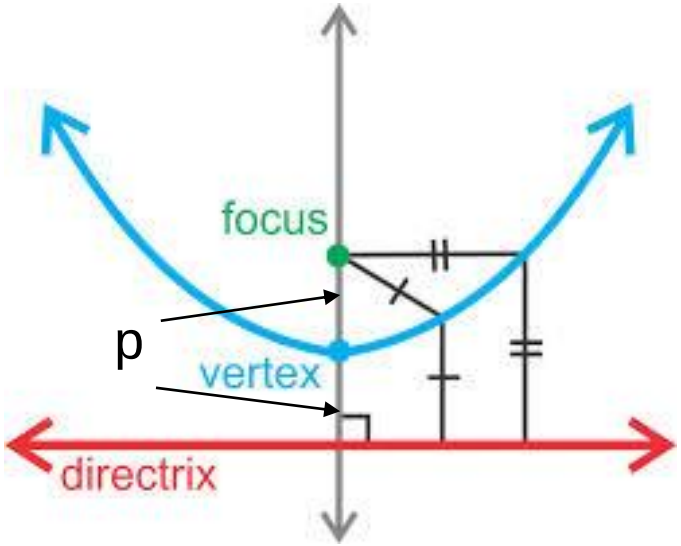
- opens up if a is positive
- opens down if a is negative
- vertex is (h, k)

★ $y = ax^2 + bx + c$

- opens up if a is positive
- opens down if a is negative
- vertex is $\left(\frac{-b}{2a} , f(\frac{-b}{2a}) \right)$

Focus and Directrix

Definition: A **parabola** is the collection of all points (x,y) that have the same distance to the focus and directrix.



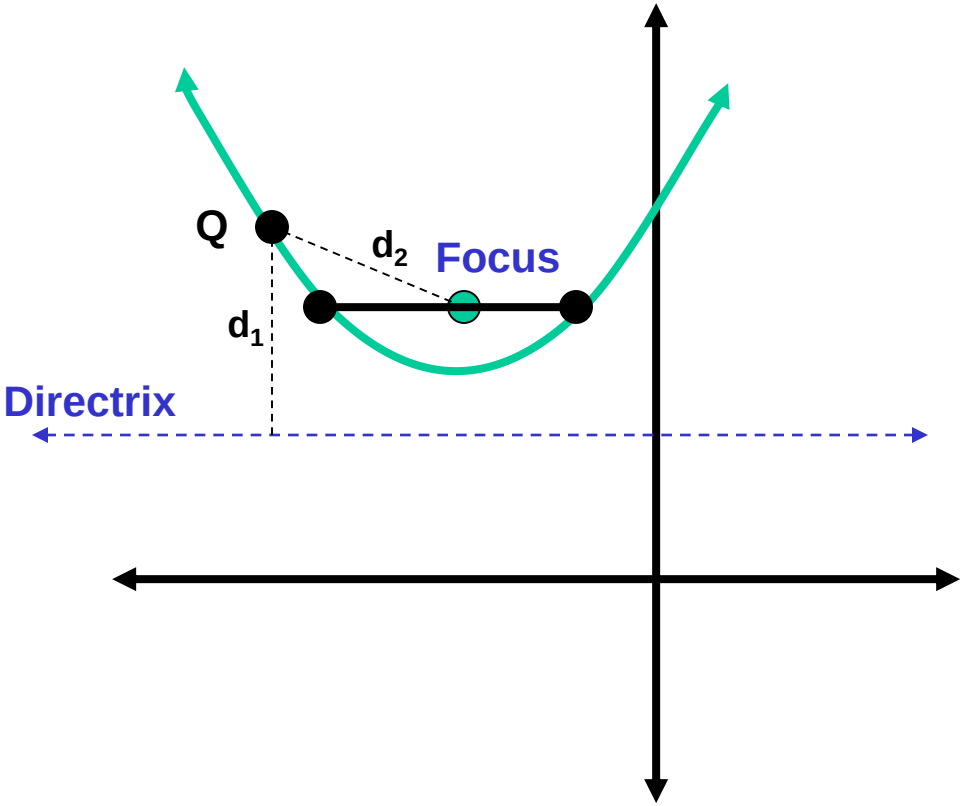
Vertex:	(h, k)
Directrix:	$y = k - p$
Focus:	$(h, k + p)$

When we think about vertex form, $a = \frac{1}{4p}$

A little potty math…

The latus rectum of a parabola is a line segment that passes through the focus, is parallel to the directrix and has its endpoints on the parabola.

The length of the latus rectum is |4p| where p is the distance from the vertex to the focus.



Focus and Directrix

Find the equation of the parabola given: the vertex is (2, -3) and focus is (2, -5)

Because of the location of the vertex and focus this must be a vertical parabola that opens down.

Equation: $\frac{1}{4p}(x - h)^2 + k = y$

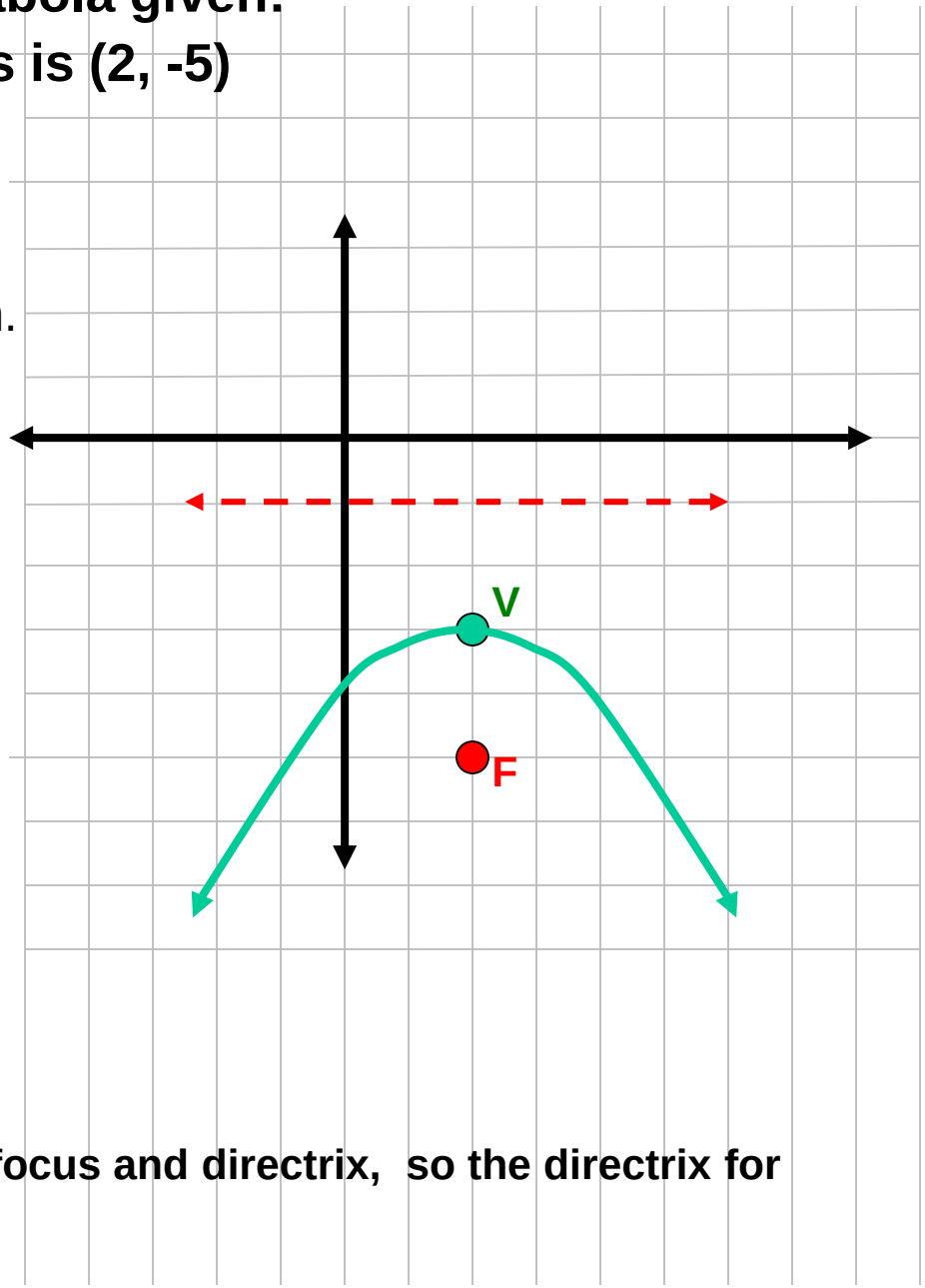
|p| = 2

Equation:

$$-\frac{1}{4(2)}(x-2)^2 - 3 = y$$

$$-\frac{1}{8}(x - 2)^2 - 3 = y$$

Note: **a** is negative because the parabola opens down.



The vertex is midway between the focus and directrix, so the directrix for this parabola is y = -1

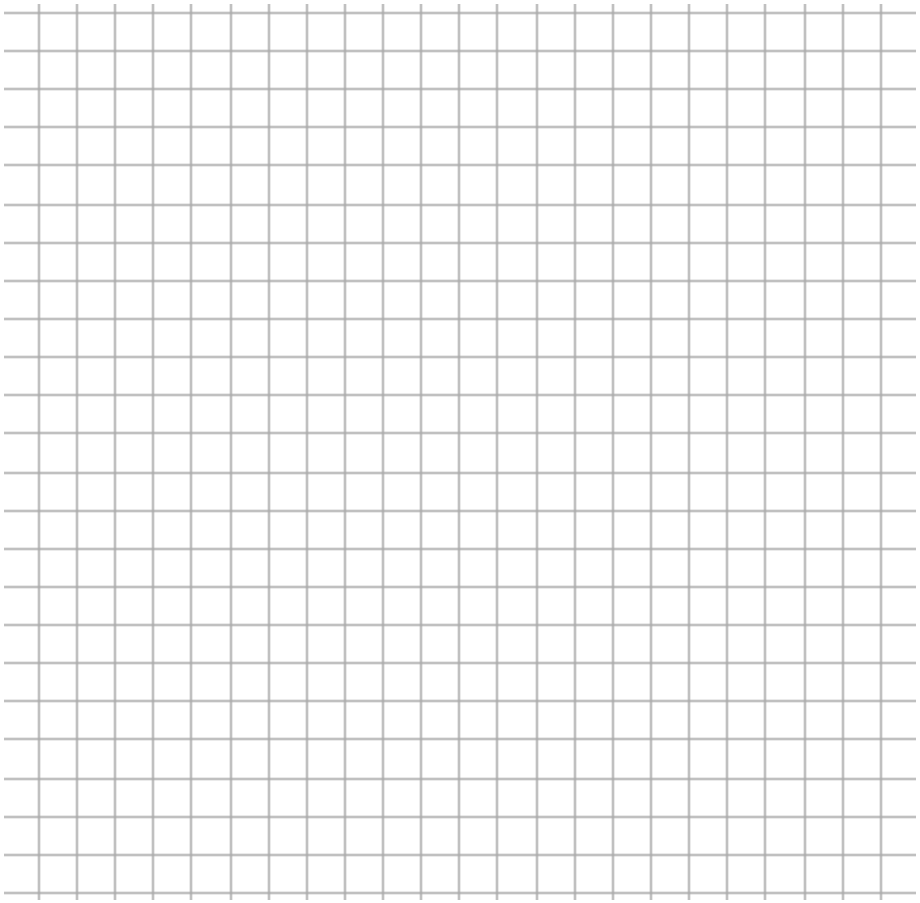
Focus and Directrix

Find the directrix of a parabola with a vertex at (3,4) and a focus at 3,1)

1. Draw a picture.

2. Find p.

3. Use what you know.



Focus and Directrix

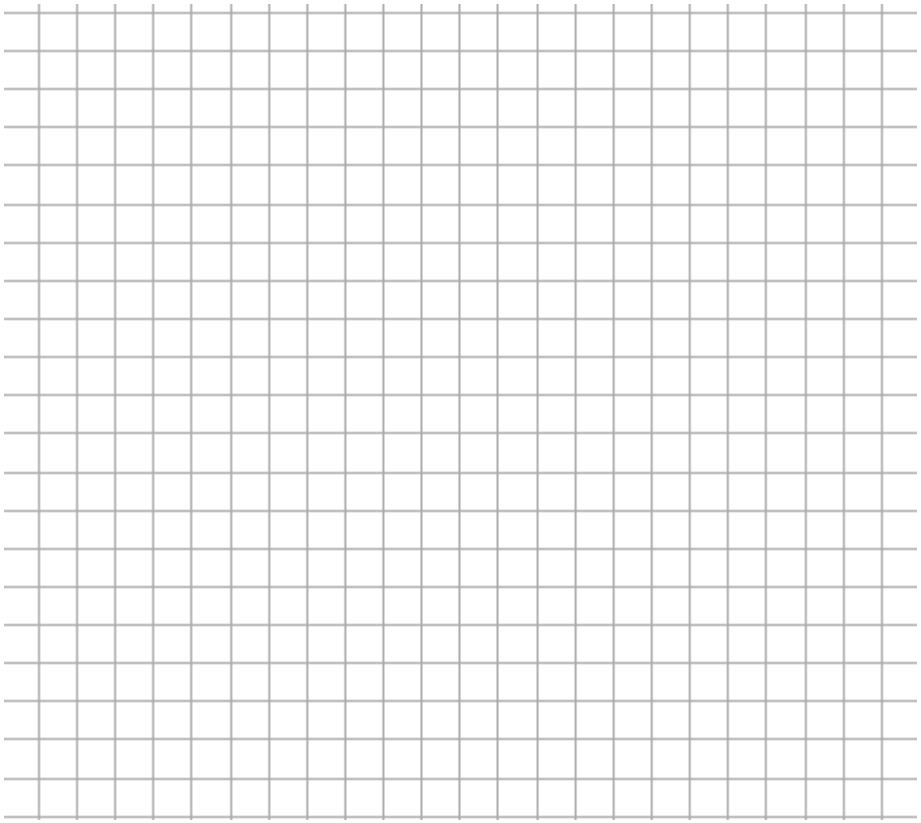
Identify the vertex, focus and directrix of the parabola $y = -2(x - 3)^2 + 5$

h =

k =

a =

p =



Vertex

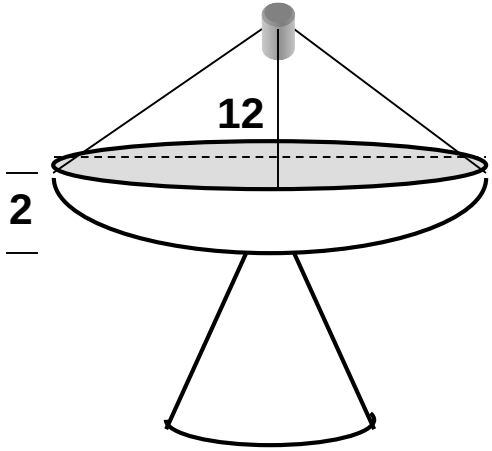
Focus

Directrix

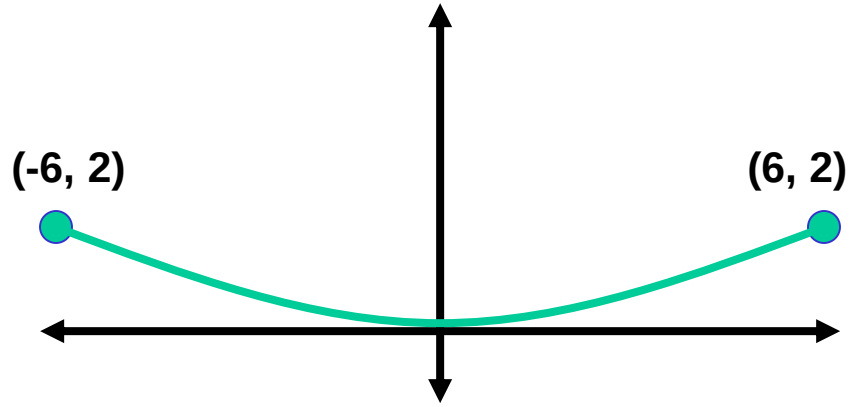
Quiz TIme

Work on the Focus and Directrix work sheet.

Applications



A satellite dish is in the shape of a parabolic surface. The dish is 12 ft in diameter and 2 ft deep. How far from the base should the receiver be placed?



Consider a parabola cross-section of the dish and create a coordinate system where the origin is at the base of the dish.

Since the parabola is vertical and has its vertex at (0, 0) its equation must be of the form:

$$x^2 = 4py$$

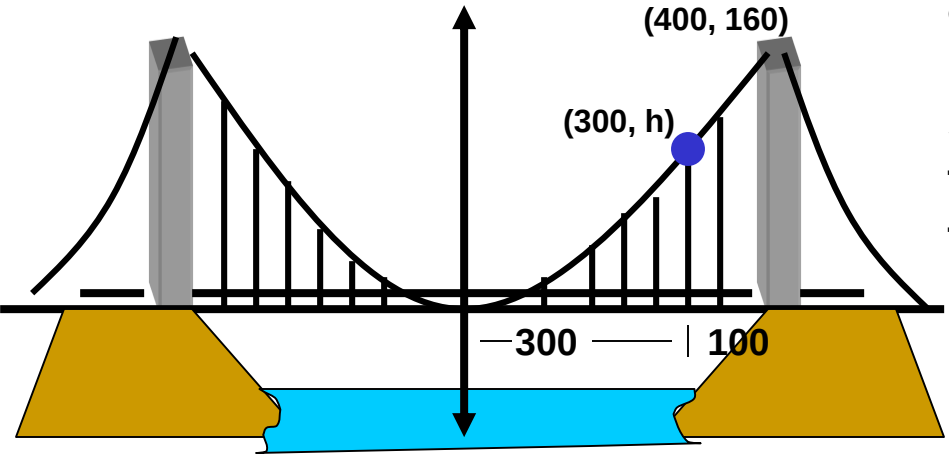
At (6, 2), 36 = 4p(2)

so p = 4.5

thus the focus is at the point (0, 4.5)

The receiver should be placed 4.5 feet above the base of the dish.

Application



The towers of a suspension bridge are 800 ft apart and rise 160 ft above the road. The cable between them has the shape of a parabola, and the cable just touches the road midway between the towers.

What is the height of the cable 100 ft from a tower?

Since the parabola is vertical and has its vertex at (0, 0) its equation must be of the form:

$$x^2 = 4py$$

At (400, 160), $160{,}000 = 4p(160)$

$1000 = 4p$

$p = 250$

thus the equation is $x^2 = 1000y$

At (300, h), $90{,}000 = 1000h$

$h = 90$

The cable would be 90 ft long at a point 100 ft from a tower.