OBJECTIVE

Find a quadratic equation that has the given roots/solutions. Up to this point we have found solutions to quadratic equations by methods such as factoring, graphing, or the quadratic formula. Building functions from roots is basically working in reverse. We'll start with the solutions and determine the quadratic function that has those solutions.

REAL SOLUTIONS

Rational Solutions

For rational solutions we can use factoring in reverse. Set each solution equal to x and then make the equation equal to zero by adding or subtracting. These expressions become the factors of the quadratic.

EXAMPLE 1 – Find the quadratic equations whose roots are 4 and -2.

$x = 4 \qquad x = -2$ Set each solution equal to x.

$x - 4 = 0 \qquad x + 2 = 0$ Subtract the constant from both sides to make the equation equal to zero.

$(x - 4)(x + 2) = 0$ Create a factor equation. (Look familiar?)

$x^2 + 2x - 4x - 8 = 0$ Foil the factors.

$x^2 - 2x - 8 = 0$ Combine Like Terms. This is our Quadratic Equation.

PRACTICE 1 – Find the quadratic equations whose roots are -3 and 6.

Set each solution equal to x.

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Subtract the constant from both sides to make the equation equal to zero.

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Create a factor equation.

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Foil the factors

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Combine Like Terms. This is our Quadratic Equation.

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More Practice

1. Find the quadratic equations with the roots 20 and 2.

2. Find the quadratic equations with the roots -4 and 0.

Rational Solutions Continued

If one or both of the solutions are fractions, we will clear the fractions by multiplying the denominators.

EXAMPLE 2 – Find the quadratic equations whose solutions are $\frac{2}{3}$ and $\frac{3}{4}$.

$x = \frac{2}{3}$	$x = \frac{3}{4}$	Set each solution equal to x.
$3x = 2$	$4x = 3$	Clear the fractions by multiplying by the denominators.
$3x - 2 = 0$	$4x - 3 = 0$	Subtract the constant from both sides to make the equation equal to zero.
$(3x - 2)(4x - 3) = 0$		Create a factor equation.
$12x^2 - 9x - 8x + 6 = 0$		Foil the factors.
$12x^2 - 17x + 6 = 0$		Combine Like Terms. This is our Quadratic Equation.

PRACTICE 2– Find the quadratic equations whose solutions are $\frac{5}{3}$ and $-\frac{1}{2}$.

______________________ Set each solution equal to x.

______________________ Clear the fractions by multiplying by the denominators.

______________________ Subtract the constant from both sides to make the equation equal to zero.

______________________ Create a factor equation.

______________________ Foil the factors

______________________ Combine Like Terms. This is our Quadratic Equation.

More Practice

1. Find the quadratic equations with the roots 0 and $-\frac{2}{5}$.

2. Find the quadratic equations with the roots 2 and $\frac{2}{9}$.

Irrational Solutions (Radicals)

When there are radicals, the solutions will always come in pairs (one with a plus, one with a minus) that can be combined into "one" solution using ±. Set the solution equal to x and square both sides. This will clear the radical from the problem. If one or both of the solutions are fractions, clear the fractions by multiplying the denominators.

EXAMPLE 3 – Find the quadratic equations whose solutions are $\sqrt{3}$ and $-\sqrt{3}$.

$x = \pm\sqrt{3}$	Write as one expression equal to x.
$x^2 = 3$	Square both sides.
$x^2 - 3 = 0$	Subtract the constant from both sides to make the equation equal to zero. Our solution.

EXAMPLE 4 – Find the quadratic equations whose solutions are $2 - 5\sqrt{2}$ and $2 + 5\sqrt{2}$.

$x = 2 \pm 5\sqrt{2}$	Write as one expression equal to x.
$x - 2 = \pm 5\sqrt{2}$	Isolate the radical term.
$(x-2)^2 = 25(2)$ $x^2 - 4x + 4 = 50$	Square both sides and simplify.
$x^2 - 4x - 46 = 0$	Subtract the constant from both sides to make the equation equal to zero. Our solution.

PRACTICE 3 – Find the quadratic equations whose solutions are $\frac{2+\sqrt{3}}{4}$ and $\frac{2-\sqrt{3}}{4}$.

______________________	Write as one expression equal to x.
______________________	Clear the fractions by multiplying by the denominators.
______________________	Isolate the radical term.
______________________	Square both sides and simplify
______________________	Subtract the constant from both sides to make equation equal to zero. Our solution.

More Practice – Find the quadratic equations with the following roots.

1. $-3 \pm \sqrt{2}$

2. $\pm\sqrt{7}$

Complex Solutions

Complex solutions also come in pairs. The process for complex solutions is identical to the process used for radicals. Just remember $i = \sqrt{-1}$ and $i^2 = -1$.

EXAMPLE 5 – Find the quadratic equations whose solutions are $4 - 5i$ and $4 + 5i$.

$x = 4 \pm 5i$	Write as one expression equal to x.
$x - 4 = \pm 5i$	Isolate the i term.
$(x-4)^2 = 25i^2$ $x^2 - 8x + 16 = -25$	Square both sides and simplify.
$x^2 - 8x + 41 = 0$	Subtract the constant from both sides to make the equation equal to zero. Our solution.

PRACTICE 4 – Find the quadratic equations whose solutions are $\frac{3+5i}{2}$ and $\frac{3-5i}{2}$.

______________________	Write as one expression equal to x.
______________________	Clear the fractions by multiplying by the denominators.
______________________	Isolate the i term.
______________________	Square both sides and simplify
______________________	Subtract the constant from both sides to make equation equal to zero. Our solution.

More Practice – Find the quadratic equations with the following roots.

1. $\pm 5i\sqrt{2}$

2. $6 \pm 4i$

3. $\frac{6 \pm i\sqrt{2}}{8}$