

# WARM UP

1. Draw a diagram showing two triangles are similar using the ASA similarity Theorem

2. Draw a diagram showing two triangles are congruent using the AAS Theorem.

3. If an inscribed angle cuts an arc of  $120^\circ$  on a circle, what is the measure of the inscribed angle?

1

2

3

4

5

6

7

8

9

10

# Objectives

- Use properties external angles to determine the measure of intercepted arcs.
- Use properties of Chords and Secants to determine segment length
- Solve real world problems involving circles.

# Homework

Circle Packet, Sections IV, V and VI (4, 5, and 6 😊)

ALL Retakes for the Log and Exponents Unit must be completed by Friday November 21<sup>st</sup>.

No exceptions.

You **MUST** bring your test corrections with you to be eligible for a retake.

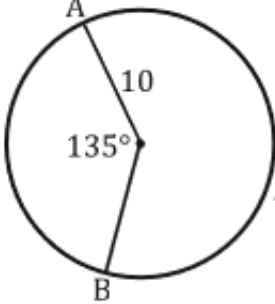
**CIRCLES Quiz Wednesday**

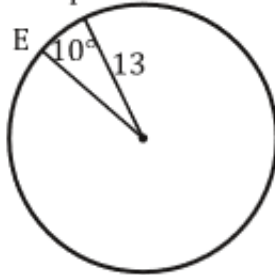
**UNIT TEST THIS FRIDAY**

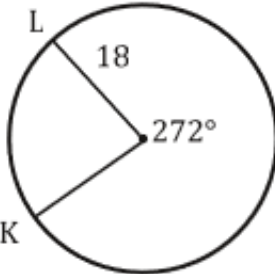
**New location for Monday afternoon tutoring! MC 1114**

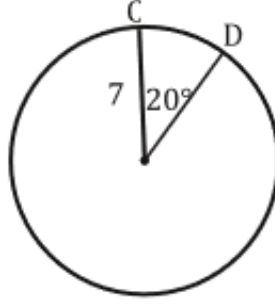
# Check your homework

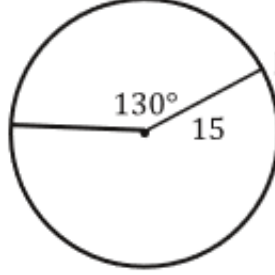
Calculate the length of each arc...

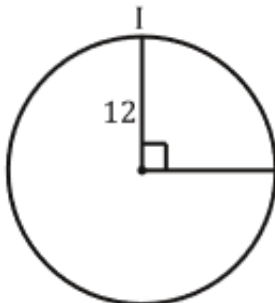
1.   $\frac{135^\circ}{360^\circ} = \frac{3}{8}$   
 $C = 2\pi r$   
 $C = 2\pi 10$   
 $C = 20\pi$   
 $20\pi * \frac{3}{8} = \frac{60\pi}{8} =$   
 $\frac{15\pi}{2} \approx 23.56$

3.   $\frac{10^\circ}{360^\circ} = \frac{1}{36}$   
 $C = 2\pi r$   
 $C = 2\pi 13$   
 $C = 26\pi$   
 $26\pi * \frac{1}{36} = \frac{26\pi}{36} =$   
 $\frac{13\pi}{18} \approx 2.27$

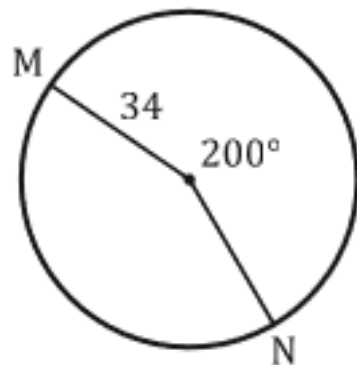
5.   $\frac{272^\circ}{360^\circ} = \frac{34}{45}$   
 $C = 2\pi r$   
 $C = 2\pi 18$   
 $C = 36\pi$   
 $36\pi * \frac{34}{45} = \frac{1224\pi}{45} =$   
 $\frac{136\pi}{5} \approx 85.45$

2.   $\frac{20^\circ}{360^\circ} = \frac{1}{18}$   
 $C = 2\pi r$   
 $C = 2\pi 7$   
 $C = 14\pi$   
 $14\pi * \frac{1}{18} = \frac{14\pi}{18} =$   
 $\frac{7\pi}{9} \approx 2.44$

4.   $\frac{130^\circ}{360^\circ} = \frac{13}{36}$   
 $C = 2\pi r$   
 $C = 2\pi 15$   
 $C = 30\pi$   
 $30\pi * \frac{13}{36} = \frac{390\pi}{36} =$   
 $\frac{65\pi}{6} \approx 34.03$

6.   $\frac{90^\circ}{360^\circ} = \frac{1}{4}$   
 $C = 2\pi r$   
 $C = 2\pi 12$   
 $C = 24\pi$   
 $24\pi * \frac{1}{4} = \frac{24\pi}{4} =$   
 $6\pi \approx 18.85$

7.



$$\frac{200^\circ}{360^\circ} = \frac{5}{9}$$

$$C = 2\pi r$$

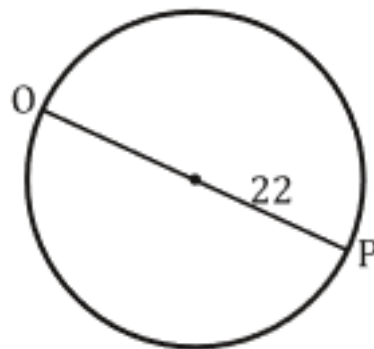
$$C = 2\pi 34$$

$$C = 68\pi$$

$$68\pi * \frac{5}{9} = \frac{340\pi}{9} =$$

$$\frac{340\pi}{9} \approx 118.68$$

8.



$$\frac{180^\circ}{360^\circ} = \frac{1}{2}$$

$$C = 2\pi r$$

$$C = 2\pi 22$$

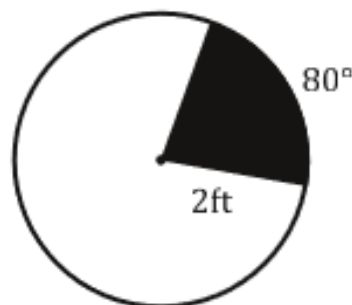
$$C = 44\pi$$

$$44\pi * \frac{1}{2} = \frac{44\pi}{2} =$$

$$22\pi \approx 69.12$$

Find the area of each shaded sector..

9.



$$\frac{80^\circ}{360^\circ} = \frac{2}{9}$$

$$C = \pi r^2$$

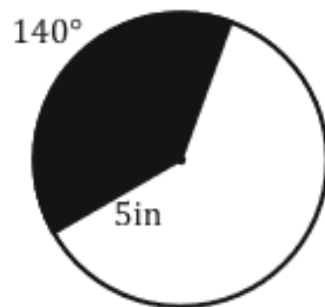
$$C = \pi 2^2$$

$$C = 4\pi$$

$$4\pi * \frac{2}{9} = \frac{8\pi}{9} =$$

$$\frac{8\pi}{9} \approx 2.79 \text{ft}^2$$

10.



$$\frac{140^\circ}{360^\circ} = \frac{7}{18}$$

$$C = \pi r^2$$

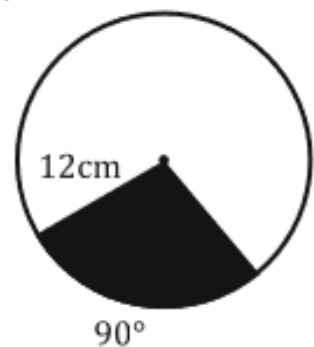
$$C = \pi 5^2$$

$$C = 25\pi$$

$$25\pi * \frac{7}{18} = \frac{175\pi}{18} =$$

$$\frac{175\pi}{18} \approx 30.54 \text{in}^2$$

11.



$$\frac{90^\circ}{360^\circ} = \frac{1}{4}$$

$$C = \pi r^2$$

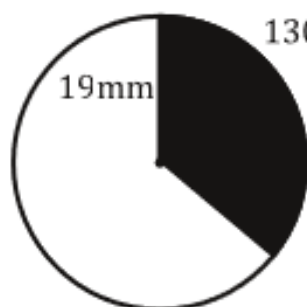
$$C = \pi 12^2$$

$$C = 144\pi$$

$$144\pi * \frac{1}{4} = \frac{144\pi}{4} =$$

$$\frac{144\pi}{4} \approx 113.1 \text{cm}^2$$

12.



$$\frac{130^\circ}{360^\circ} = \frac{13}{36}$$

$$C = \pi r^2$$

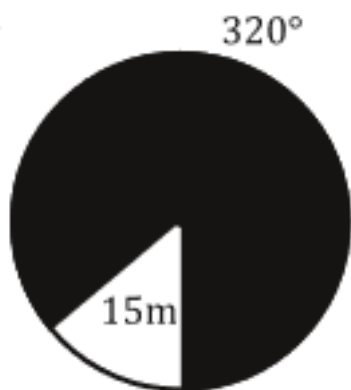
$$C = \pi 19^2$$

$$C = 361\pi$$

$$361\pi * \frac{1}{36} = \frac{361\pi}{36} =$$

$$\frac{361\pi}{36} \approx 31.50 \text{mm}^2$$

13.



$$\frac{320^\circ}{360^\circ} = \frac{8}{9}$$

$$C = \pi r^2$$

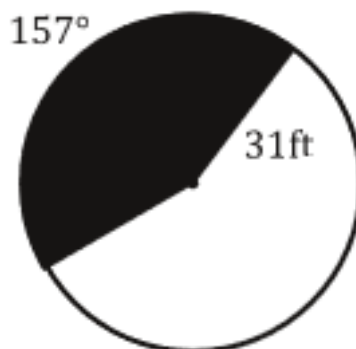
$$C = \pi 15^2$$

$$C = 225\pi$$

$$225\pi * \frac{8}{9} = \frac{1800\pi}{9}$$

$$200\pi \approx 628.32 \text{ m}^2$$

14.



$$\frac{157^\circ}{360^\circ} = \frac{157}{360}$$

$$C = \pi r^2$$

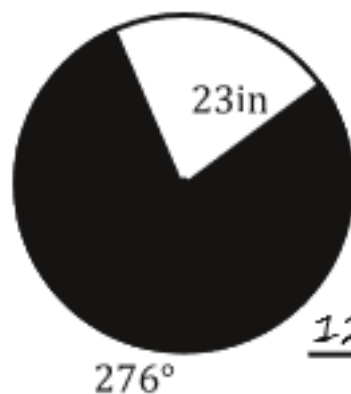
$$C = \pi 31^2$$

$$C = 961\pi$$

$$961\pi * \frac{157}{360} = \frac{7688\pi}{360}$$

$$\frac{961\pi}{45} \approx 67.09 \text{ ft}^2$$

15.



$$\frac{276^\circ}{360^\circ} = \frac{23}{30}$$

$$C = \pi r^2$$

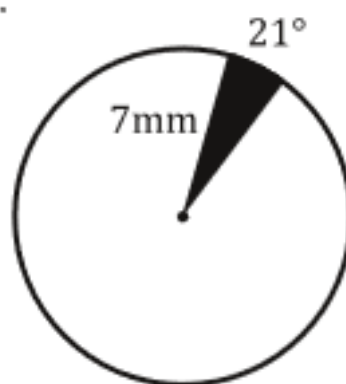
$$C = \pi 23^2$$

$$C = 529\pi$$

$$529\pi * \frac{23}{30} = \frac{12167\pi}{30}$$

$$\frac{12167\pi}{30} \approx 1274.13 \text{ in}^2$$

16.



$$\frac{21^\circ}{360^\circ} = \frac{7}{120}$$

$$C = \pi r^2$$

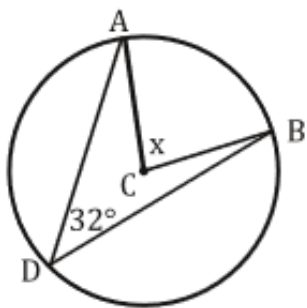
$$C = \pi 7^2$$

$$C = 49\pi$$

$$49\pi * \frac{7}{120} = \frac{343\pi}{120}$$

$$\frac{343\pi}{120} \approx 8.98 \text{ mm}^2$$

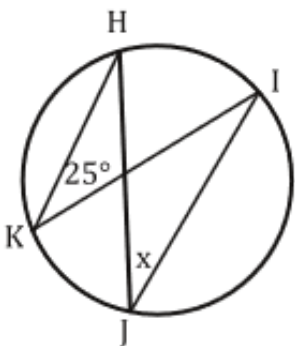
9.



$$\begin{aligned} 2(m\angle D) &= m\widehat{AB} \\ 2(32^\circ) &= m\widehat{AB} \\ 64^\circ &= m\widehat{AB} \end{aligned}$$

$$\begin{aligned} m\angle C &= m\widehat{AB} \\ x &= 64^\circ \end{aligned}$$

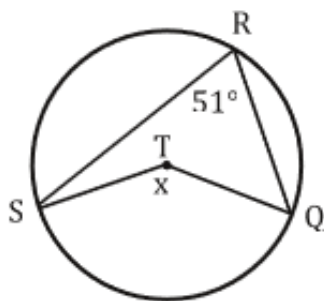
11.



$$\begin{aligned} 2(m\angle K) &= m\widehat{HI} \\ 2(25^\circ) &= m\widehat{HI} \\ 50^\circ &= m\widehat{HI} \end{aligned}$$

$$\begin{aligned} m\angle J &= 1/2 m\widehat{KI} \\ x &= 1/2(50^\circ) \\ x &= 25^\circ \end{aligned}$$

13.

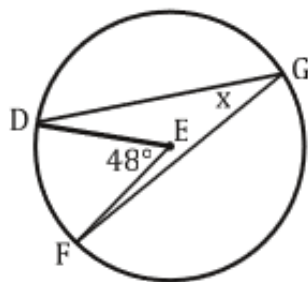


$$\begin{aligned} 2(m\angle R) &= m\widehat{QS} \\ 2(51^\circ) &= m\widehat{QS} \\ 102^\circ &= m\widehat{QS} \end{aligned}$$

$$\begin{aligned} m\angle T &= m\widehat{QS} \\ x &= 102^\circ \end{aligned}$$

15

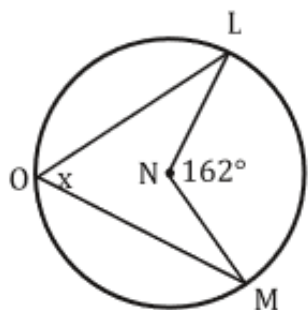
10.



$$\begin{aligned} m\angle E &= m\widehat{DF} \\ 48^\circ &= m\widehat{DF} \end{aligned}$$

$$\begin{aligned} m\angle G &= 1/2 m\widehat{DF} \\ x &= 1/2(48^\circ) \\ x &= 24^\circ \end{aligned}$$

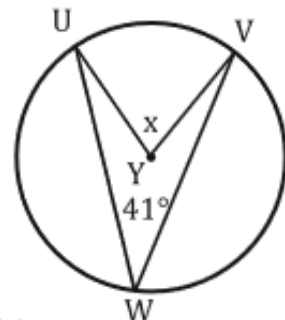
12.



$$\begin{aligned} m\angle N &= m\widehat{LM} \\ 162^\circ &= m\widehat{LM} \end{aligned}$$

$$\begin{aligned} m\angle O &= 1/2 m\widehat{LM} \\ x &= 1/2(162^\circ) \\ x &= 81^\circ \end{aligned}$$

14.

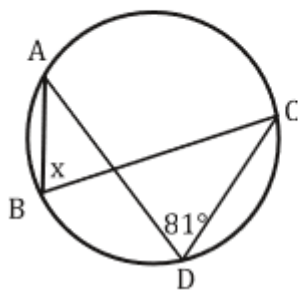


$$\begin{aligned} 2(m\angle W) &= m\widehat{UV} \\ 2(41^\circ) &= m\widehat{UV} \\ 82^\circ &= m\widehat{UV} \end{aligned}$$

$$\begin{aligned} m\angle Y &= m\widehat{UV} \\ x &= 82^\circ \end{aligned}$$

16

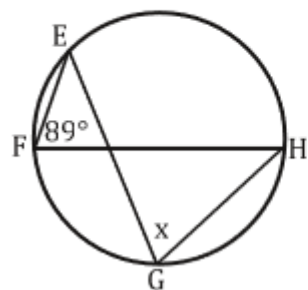
15.



$$\begin{aligned} 2(m\angle D) &= m\widehat{AC} \\ 2(81^\circ) &= m\widehat{AC} \\ 162^\circ &= m\widehat{AC} \end{aligned}$$

$$\begin{aligned} m\angle B &= 1/2 m\widehat{AC} \\ x &= 1/2(162^\circ) \\ x &= 81^\circ \end{aligned}$$

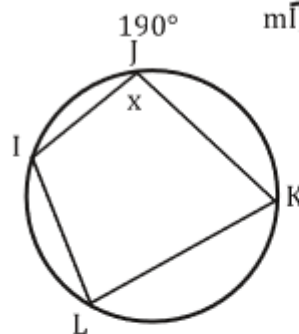
16.



$$\begin{aligned} 2(m\angle F) &= m\widehat{EH} \\ 2(89^\circ) &= m\widehat{EH} \\ 178^\circ &= m\widehat{EH} \end{aligned}$$

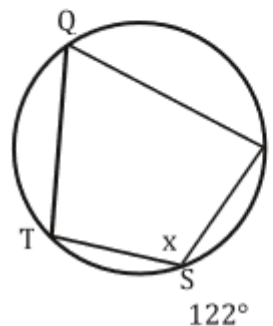
$$\begin{aligned} m\angle G &= 1/2 m\widehat{EH} \\ x &= 1/2(178^\circ) \\ x &= 89^\circ \end{aligned}$$

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17.  $m\widehat{JK} = 190^\circ$ 

$$\begin{aligned} \text{First find } m\widehat{KL}. \\ m\widehat{JK} + m\widehat{KL} &= 360^\circ \\ m\widehat{JK} + m\widehat{KL} &= 360^\circ \\ 190^\circ + m\widehat{KL} &= 360^\circ \\ -190^\circ & \quad -190^\circ \\ m\widehat{KL} &= 170^\circ \end{aligned}$$

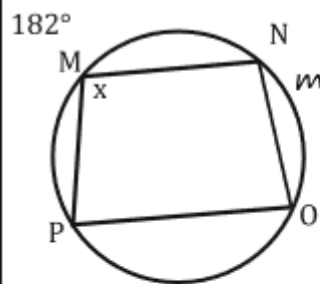
$$\begin{aligned} \text{Then find } m\angle J. \\ m\angle J &= 1/2 m\widehat{KL} \\ x &= 1/2(170^\circ) \\ x &= 85^\circ \end{aligned}$$

19.  $m\widehat{RST} = 122^\circ$ 

$$\begin{aligned} m\widehat{RST} + m\widehat{QR} &= 360^\circ \\ 122^\circ + m\widehat{QR} &= 360^\circ \\ -122^\circ & \quad -122^\circ \\ m\widehat{QR} &= 238^\circ \end{aligned}$$

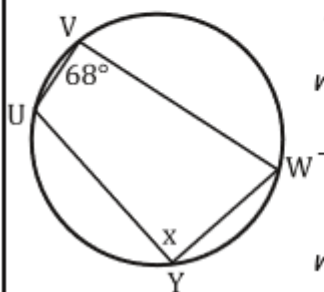
$$\begin{aligned} m\angle S &= 1/2 m\widehat{QR} \\ x &= 1/2(238^\circ) \\ x &= 119^\circ \end{aligned}$$

W

18.  $m\widehat{PMN} = 182^\circ$ 

$$\begin{aligned} m\widehat{PMN} + m\widehat{NOP} &= 360^\circ \\ 182^\circ + m\widehat{NOP} &= 360^\circ \\ -182^\circ & \quad -182^\circ \\ m\widehat{NOP} &= 178^\circ \end{aligned}$$

$$\begin{aligned} m\angle M &= 1/2 m\widehat{NOP} \\ x &= 1/2(178^\circ) \\ x &= 89^\circ \end{aligned}$$

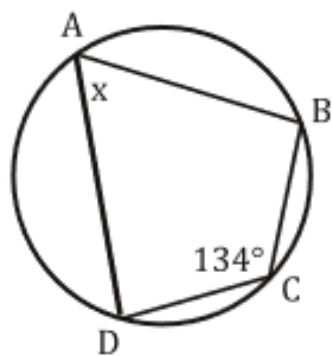
20. First find  $m\widehat{UYW}$ .

$$\begin{aligned} 2(m\angle V) &= m\widehat{UYW} \\ 2(68^\circ) &= m\widehat{UYW} \\ 136^\circ &= m\widehat{UYW} \\ m\widehat{UYW} + m\widehat{UVW} &= 360^\circ \\ 136^\circ + m\widehat{UVW} &= 360^\circ \\ -136^\circ & \quad -136^\circ \\ m\widehat{UVW} &= 224^\circ \end{aligned}$$

$$\begin{aligned} m\angle Y &= 1/2 m\widehat{UVW} \\ x &= 1/2(224^\circ) \\ x &= 112^\circ \end{aligned}$$



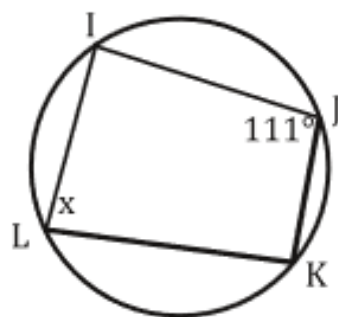
21.



$$\begin{aligned}
 2(m\angle C) &= m\widehat{DAB} \\
 2(134^\circ) &= m\widehat{DAB} \\
 268^\circ &= m\widehat{DAB} \\
 m\widehat{DAB} + m\widehat{DCB} &= 360^\circ \\
 268^\circ + m\widehat{DCB} &= 360^\circ \\
 -268^\circ & \quad -268^\circ \\
 m\widehat{DCB} &= 92^\circ
 \end{aligned}$$

$$\begin{aligned}
 m\angle A &= 1/2 m\widehat{DCB} \\
 x &= 1/2(92^\circ) \\
 x &= 46^\circ
 \end{aligned}$$

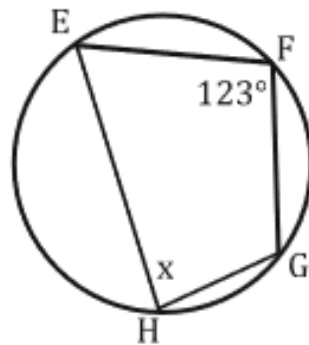
23.



$$\begin{aligned}
 2(m\angle J) &= m\widehat{ILK} \\
 2(111^\circ) &= m\widehat{ILK} \\
 222^\circ &= m\widehat{ILK} \\
 m\widehat{ILK} + m\widehat{IJK} &= 360^\circ \\
 222^\circ + m\widehat{IJK} &= 360^\circ \\
 -222^\circ & \quad -222^\circ \\
 m\widehat{IJK} &= 138^\circ
 \end{aligned}$$

$$\begin{aligned}
 m\angle L &= 1/2 m\widehat{IJK} \\
 x &= 1/2(138^\circ) \\
 x &= 69^\circ
 \end{aligned}$$

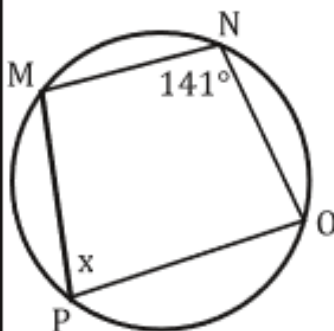
22.



$$\begin{aligned}
 2(m\angle F) &= m\widehat{EHG} \\
 2(123^\circ) &= m\widehat{EHG} \\
 246^\circ &= m\widehat{EHG} \\
 m\widehat{EHG} + m\widehat{EFG} &= 360^\circ \\
 246^\circ + m\widehat{EFG} &= 360^\circ \\
 -246^\circ & \quad -246^\circ \\
 m\widehat{EFG} &= 114^\circ
 \end{aligned}$$

$$\begin{aligned}
 m\angle H &= 1/2 m\widehat{EFG} \\
 x &= 1/2(114^\circ) \\
 x &= 57^\circ
 \end{aligned}$$

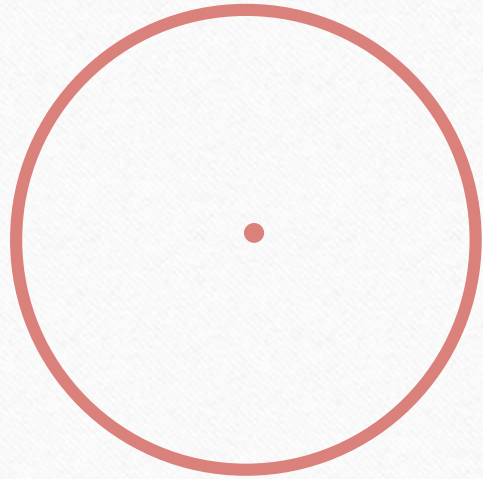
24.



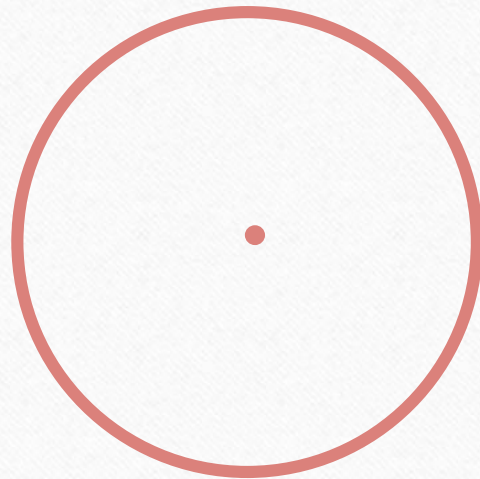
$$\begin{aligned}
 2(m\angle N) &= m\widehat{MPO} \\
 2(141^\circ) &= m\widehat{MPO} \\
 282^\circ &= m\widehat{MPO} \\
 m\widehat{MPO} + m\widehat{MNO} &= 360^\circ \\
 282^\circ + m\widehat{MNO} &= 360^\circ \\
 -282^\circ & \quad -282^\circ \\
 m\widehat{MNO} &= 78^\circ
 \end{aligned}$$

$$\begin{aligned}
 m\angle P &= 1/2 m\widehat{MNO} \\
 x &= 1/2(78^\circ) \\
 x &= 39^\circ
 \end{aligned}$$

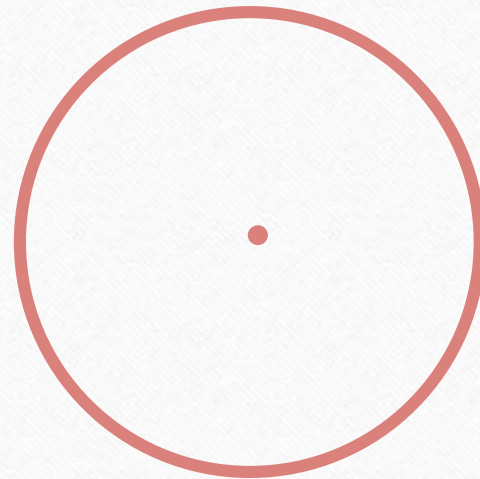
To Summarize our adventures from yesterday...



Central  
Angle



Inscribed  
Angle



Intersecting  
Chord  
Angles

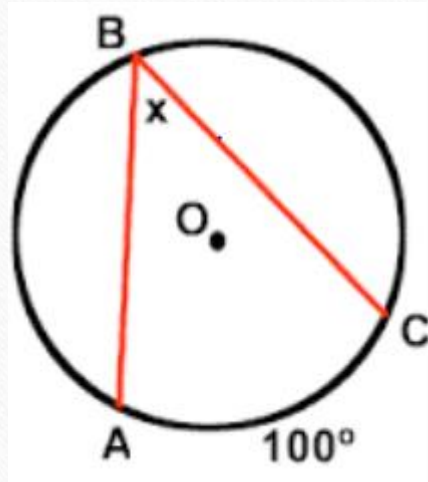


Tangent  
Angle

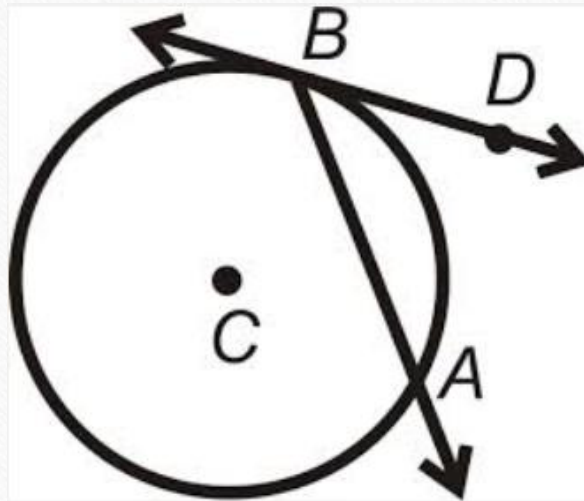
Yesterday, all our troubles seemed so far away.

What do the angles pictured below have in common?

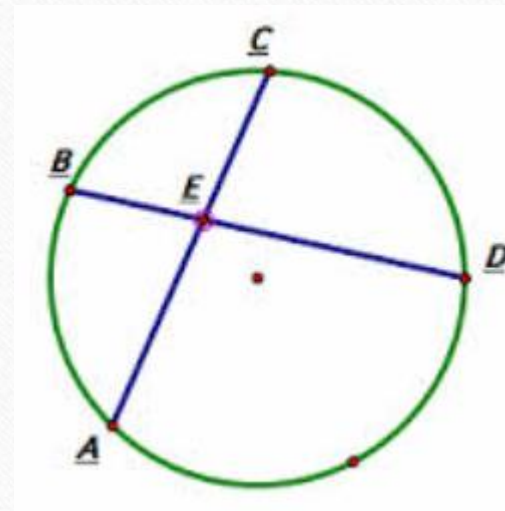
The angles are formed inside the circle.



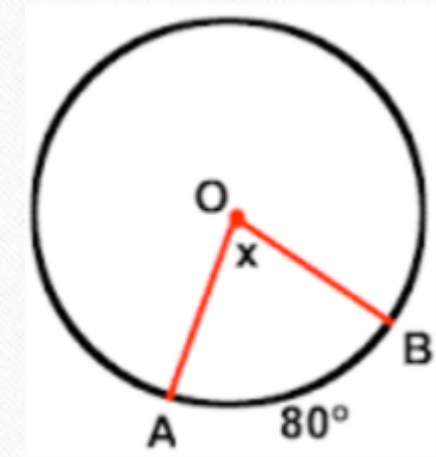
*Inscribed angle*



*Tangent angle*

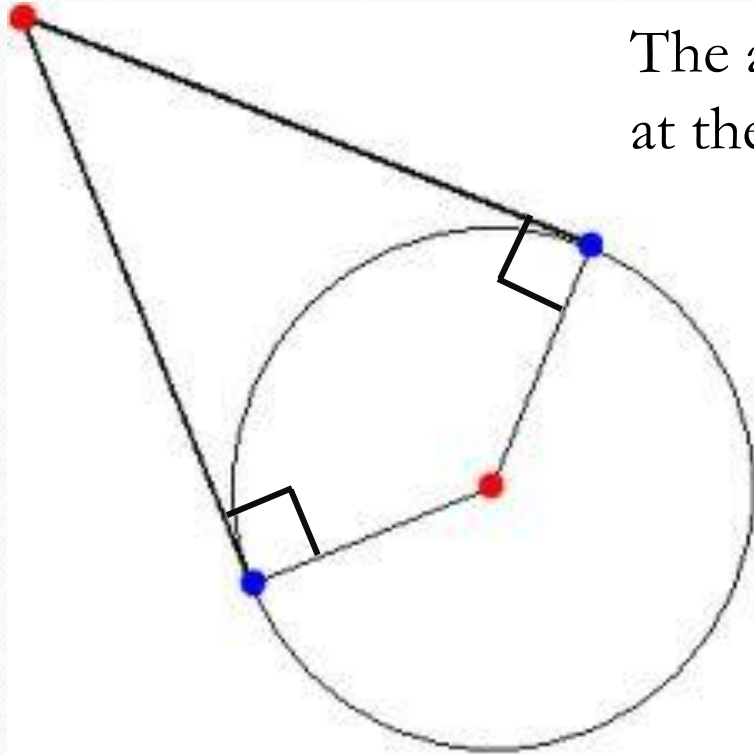


*Intersecting Chords*



*Central Angle*

One more thing about tangents and circles.

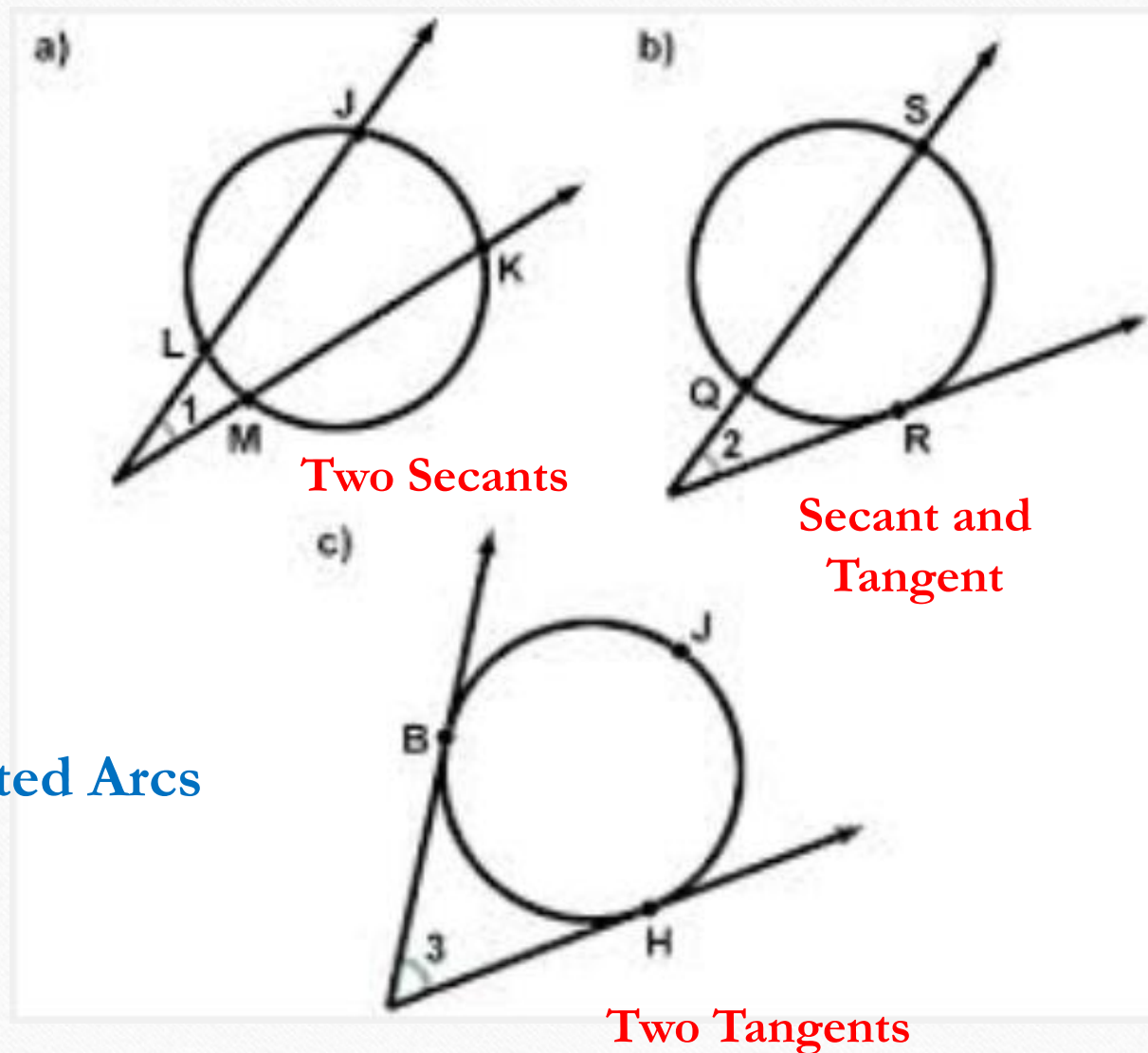


The angle formed with the tangent line and the radius at the point of tangency is a right angle. *ALWAYS.*

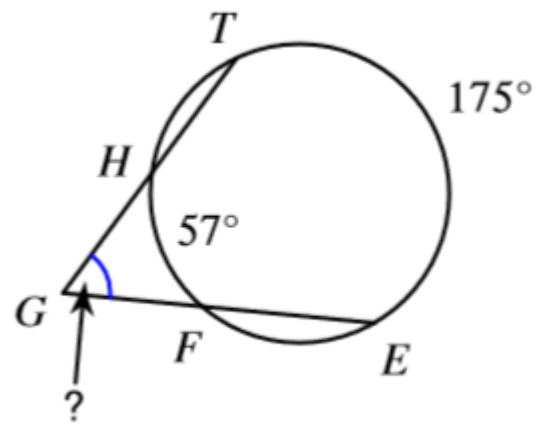
What's different about the angles pictured here?

Angles are formed outside of the circle.

Angle Formed Outside  
 $= \frac{1}{2}$  Difference of Intercepted Arcs



## Angle Formed Outside = $\frac{1}{2}$ Difference of Intercepted Arcs

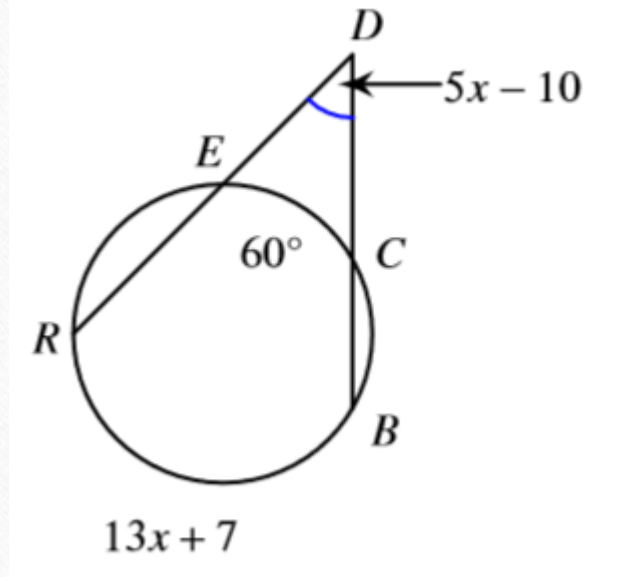


$$\angle G = \frac{1}{2}(\widehat{TE} - \widehat{HF})$$

$$\angle G = \frac{1}{2}(175 - 57)$$

$$\angle G = \frac{1}{2}(175 - 57)$$

$$\angle G = 59$$



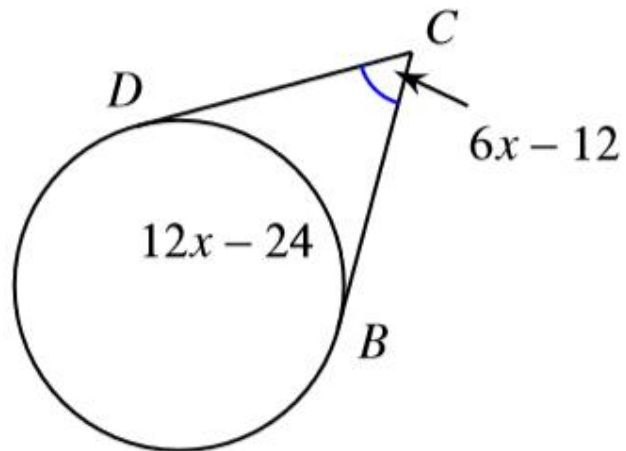
$$5x - 10 = \frac{1}{2}(13x + 7 - 60)$$

$$10x - 20 = 13x - 53$$

$$33 = 3x$$

$$11 = x$$

# Scooby Doo? No you do!



$$6x - 12 = \frac{1}{2}(-12 + 21x - (12x - 24))$$

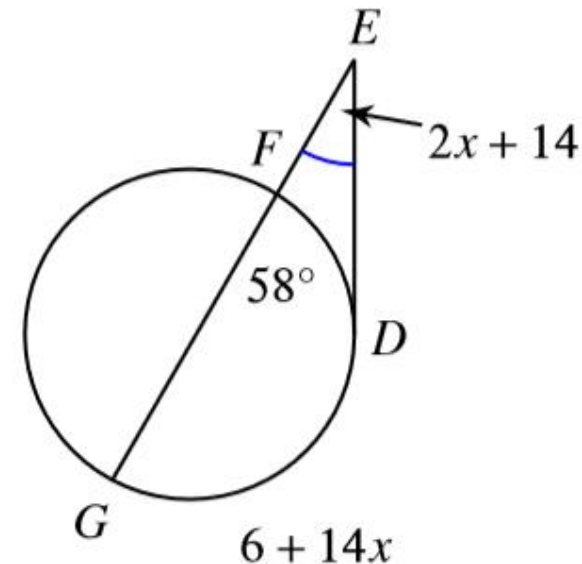
$$6x - 12 = \frac{1}{2}(-12 + 21x - 12x + 24)$$

$$6x - 12 = \frac{1}{2}(9x + 12)$$

$$12x - 24 = 9x + 12$$

$$3x = 36$$

$$x = 12$$



$$6x + 14 = \frac{1}{2}(6 + 14x - 58)$$

$$6x + 14 = \frac{1}{2}(14x - 52)$$

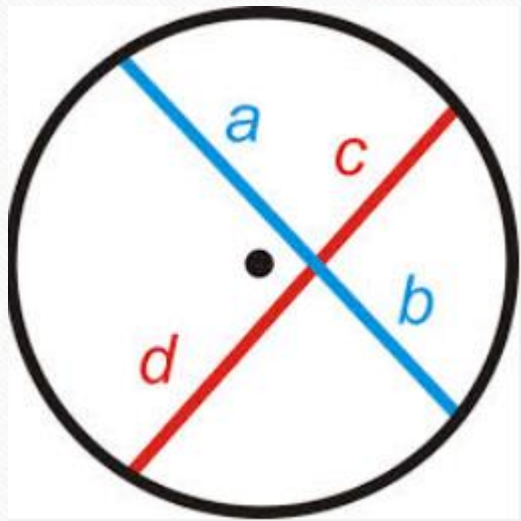
$$12x + 28 = 14x - 52$$

$$80 = 2x$$

$$40 = x$$



Enough about angles, lets talk chord and segment lengths.



## Intersecting Chords

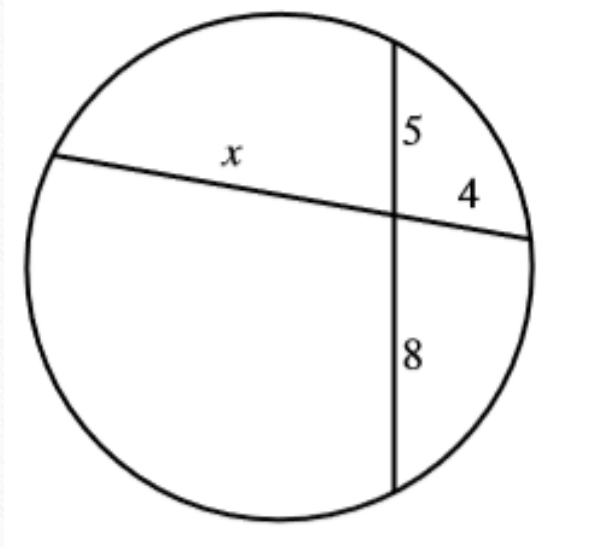
There is a relationship between the segments created when chords intersect within a circle.

(Segment Piece)(Segment Piece)=(Segment Piece)(Segment Piece)

$$ab=dc$$



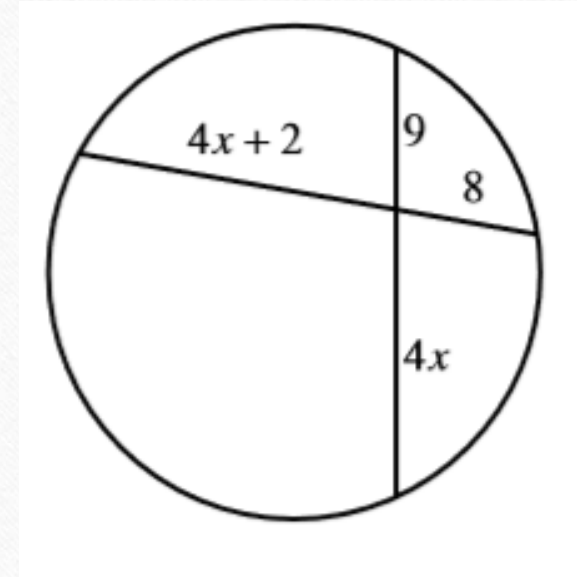
**(Segment Piece)(Segment Piece) = (Segment Piece)(Segment Piece)**



$$(5)(8) = 4x$$

$$40 = 4x$$

$$10 = x$$



$$(9)(4x) = (8)(4x + 2)$$

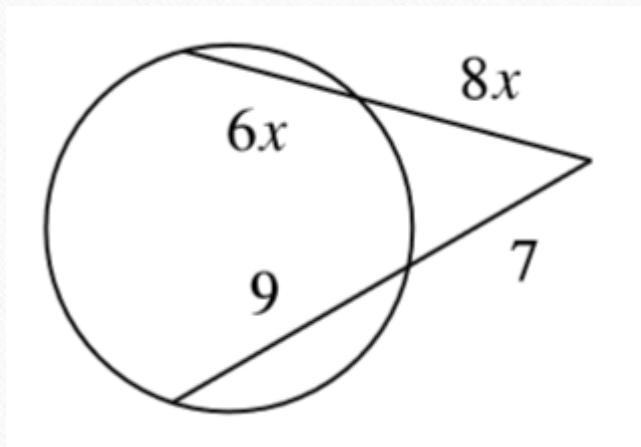
$$36x = 32x + 16$$

$$4x = 16$$

$$x = 4$$

# Secant-Secant Rule

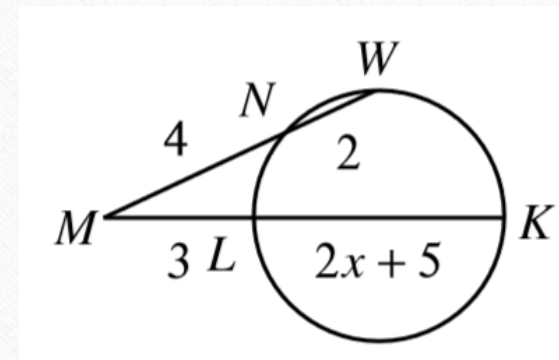
$$(\text{Whole Secant})(\text{External Part}) = (\text{Whole Secant})(\text{External Part})$$



$$(16)(7) = (6x + 8x)(8x)$$

$$112 = 112x$$

$$x = 1$$



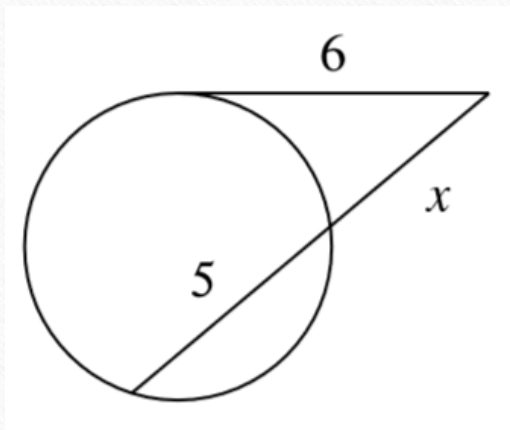
$$(6)(4) = (2x + 5 + 3)(3)$$

$$24 = 6x + 24$$

$$x = 0$$

# Secant-Tangent Rule

$$(\text{Whole Secant})(\text{External Part}) = (\text{Tangent})^2$$



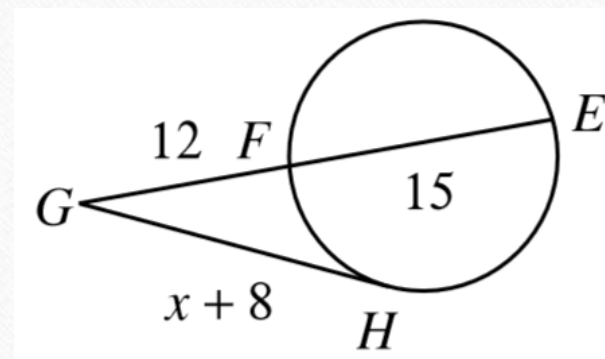
$$(5 + x)x = 6^2$$

$$5x + x^2 = 36$$

$$x^2 + 5x - 36 = 0$$

$$(x - 4)(x + 8) = 0$$

$$x = 4, x = -8$$



$$(27)(12) = (x + 8)^2$$

$$324 = (x + 8)^2$$

$$18 = x + 8$$

$$10 = x$$



**Practice you nuts and bolts. 😊**

Intentional typo!

Finish your circles packet and the new handout.