

Warm-up

1. Write the quadratic equation in standard form that one of its roots equal to $3 + \sqrt{5}$.
2. Write the quadratic equation in standard form that one of its roots equal to $2 - 4i$.



Objectives

Use the Binomial Theorem and Pascal's Triangle to expand binomials raised to any power.

Use the Binomial Theorem and Pascal's Triangle to find specific terms in a expansion polynomial expansion.

Homework

Hand out 5-7 page 65; problems 2, 4, 8, 10, 12, 18, 20 and 22

Hand out 5-3 page 23: 1-8 all

Homework? We'll check it later.

Let's do a little review. Remember this?

$$(3x + 2)^4$$

We had two tools to help us...

Pascal's Triangle and the Binomial Theorem

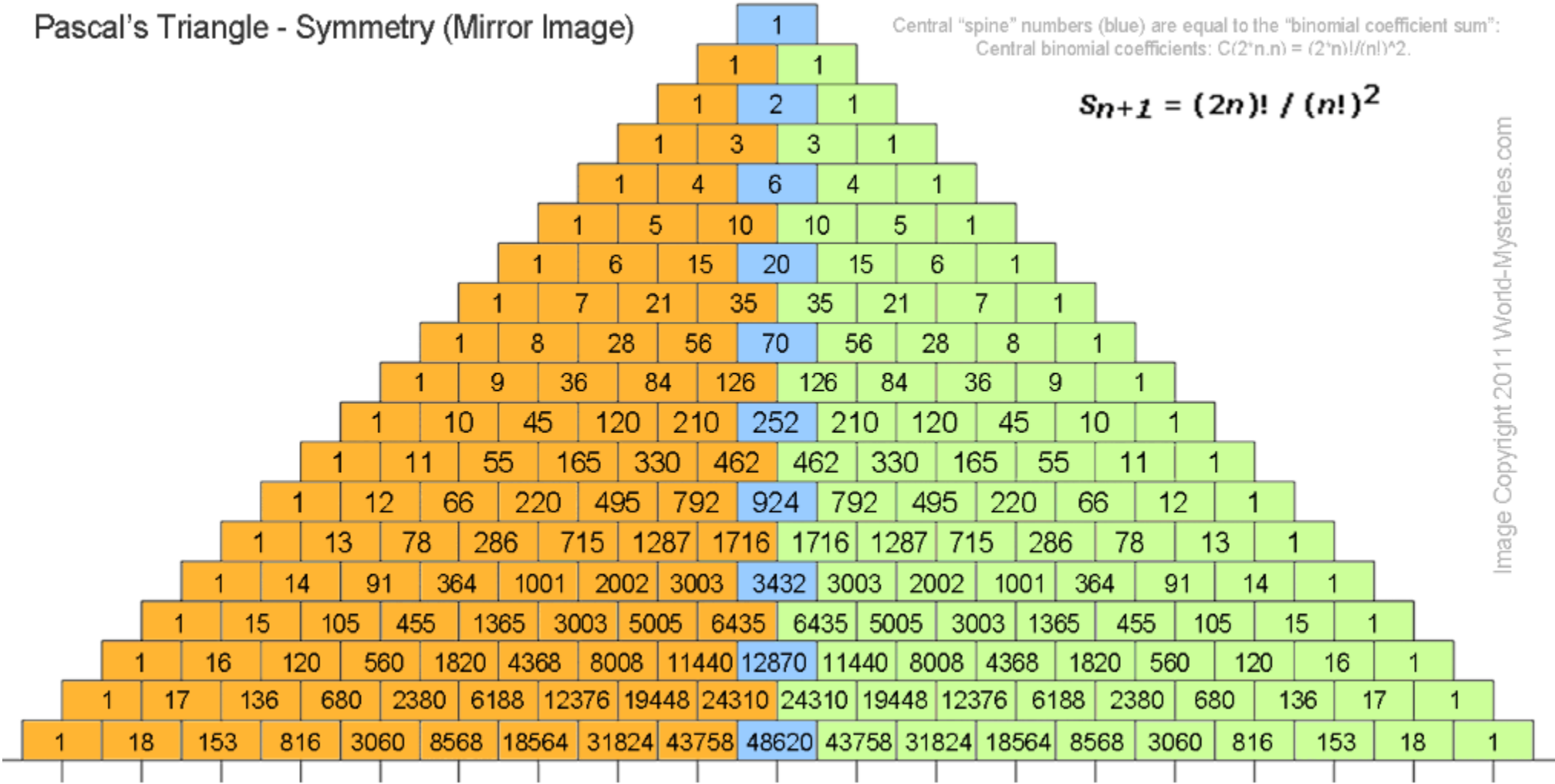


Pascal's Triangle has some interesting properties

Pascal's Triangle - Symmetry (Mirror Image)

Central "spine" numbers (blue) are equal to the "binomial coefficient sum":
 Central binomial coefficients: $C(2^n, n) = \frac{(2^n)!}{(n!)^2}$.

$$S_{n+1} = \frac{(2n)!}{(n!)^2}$$



Pascal's Triangle has some interesting properties

Pascal's Triangle and Fibonacci Numbers

Fibonacci's Sequence can also be located in Pascal's Triangle. The sum of the numbers in the consecutive rows shown in the diagram are the first numbers of the Fibonacci Sequence:

- 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,
- 144, 233, 377, 610, 987,
- 1597, 2584, 4181, 6765,
- 10946, 17711, 28657, 46368, 75025, ...

The Fibonacci numbers occur in the sums of "shallow" diagonals in Pascal's triangle

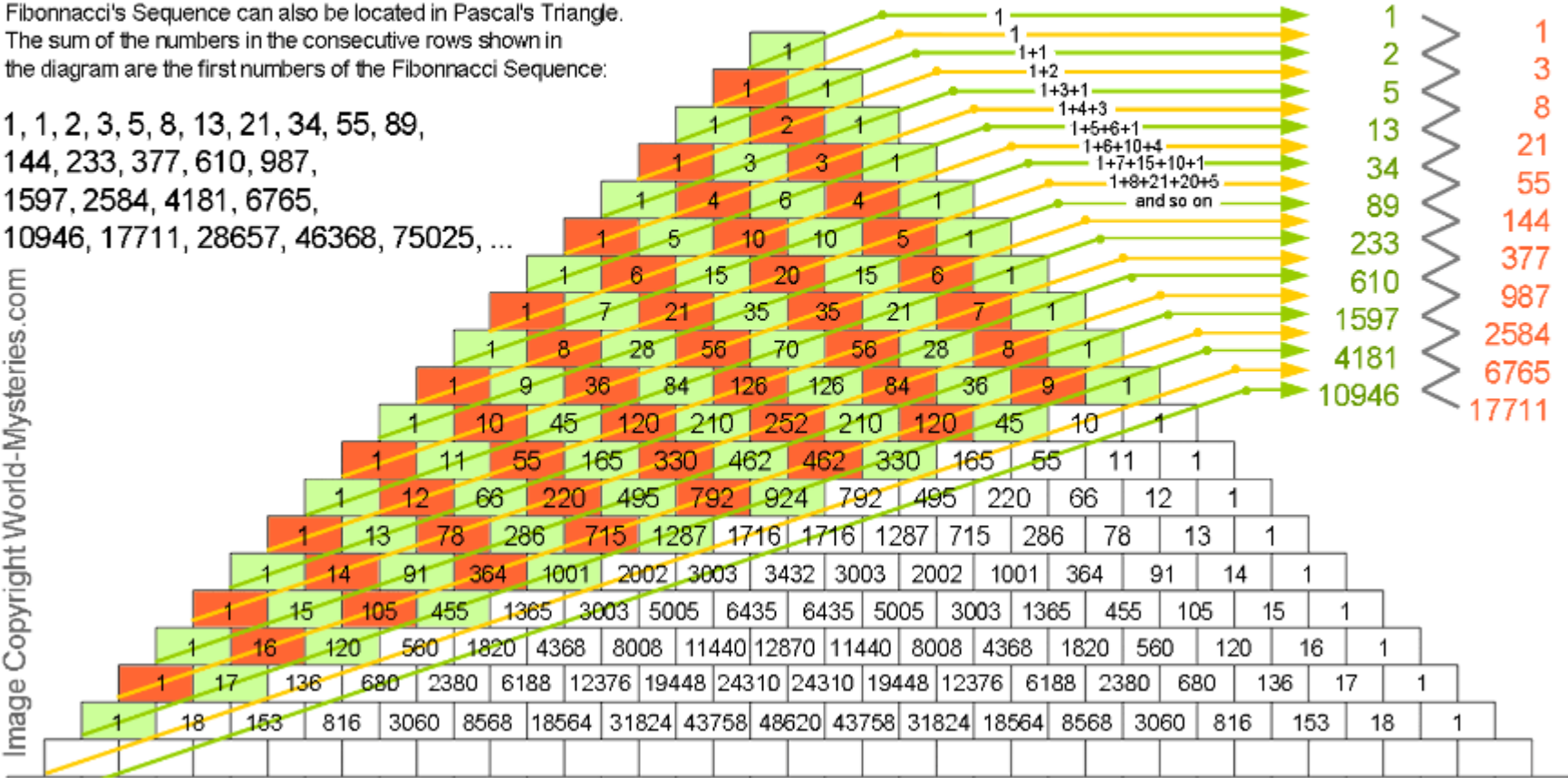


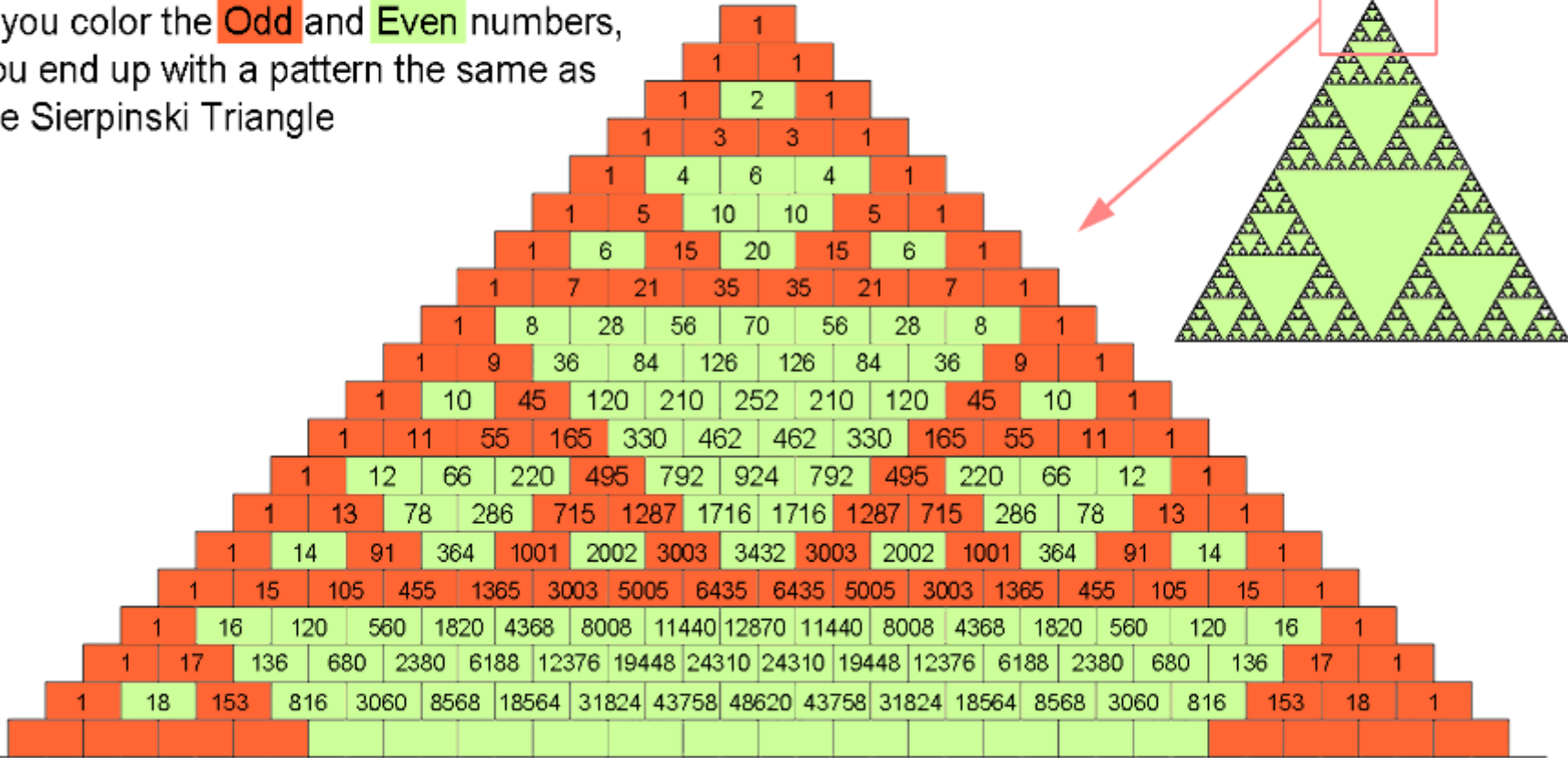
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Pascal's Triangle has some interesting properties

Pascal's Triangle and Sierpinski Triangle

If you color the **Odd** and **Even** numbers, you end up with a pattern the same as the Sierpinski Triangle

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take note

Theorem Binomial Theorem

For every positive integer n ,

$$(a + b)^n = P_0 a^n + P_1 a^{n-1} b + P_2 a^{n-2} b^2 + \cdots + P_{n-1} a b^{n-1} + P_n b^n$$

where P_0, P_1, \dots, P_n are the numbers in the n th row of Pascal's Triangle.

The Binomial Theorem just pointed out the patterns we can use when we expand a polynomial.

Some other things to keep in mind...

The number of terms in an expansion is always the degree of the polynomial plus 1.

The first term of an expansion is always the first term of the binomial raised to the indicated power.

The last term of an expansion is always the last term of the binomial raised to the indicated power.

Expand the polynomial $(3x + 2)^4$

Terms: 5

$$a = 3x \quad b = 2$$

Pascal's Coefficients: 1 4 6 4 1

$$\boxed{a^4} + \boxed{4a^3b} + \boxed{6a^2b^2} + \boxed{4ab^3} + \boxed{b^4}$$

What are our **first** and **last** terms?

Write **Pascal's** coefficients in the interior terms.

Working left to right, fill in the **a** terms with **descending** powers.

Working left to right, fill in the **b** terms with **ascending** powers.

Now substitute our values for **a** and **b**.

$$\boxed{(3x)^4} + \boxed{4(3x)^3(2)} + \boxed{6(3x)^2(2)^2} + \boxed{4(3x)(2)^3} + \boxed{(2)^4}$$

Simplify.

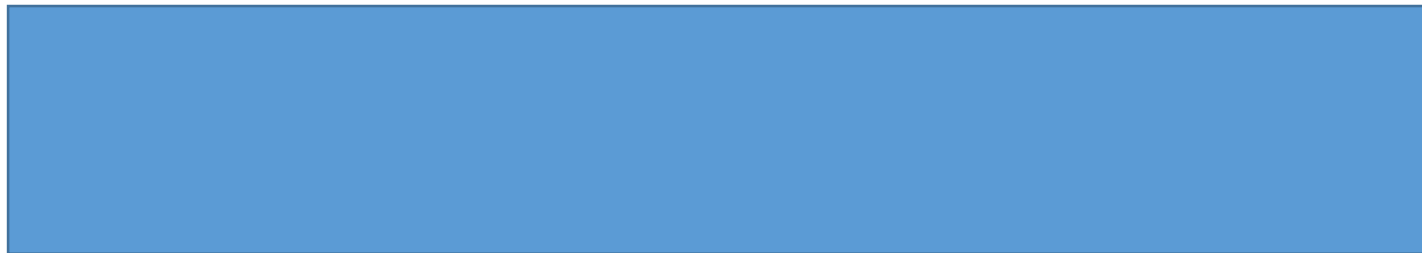
$$\boxed{81x^4} + \boxed{216x^3} + \boxed{216x^2} + \boxed{96x} + \boxed{16}$$

OK, now you expand $(2x - 4)^5$

Terms: 5

$a = 3x$ $b = 2$

Pascal's Coefficients: 1 4 6 4 1



What is the third term of $(2x + 1)^5$.

1. How many terms?

$$\square + \square + \square + \square + \square + \square$$

1a. How many terms do I care about?



What is the eleventh term of $(2x + y^2)^{10}$.

1. How many terms?

1a. How many terms do I care about?

The eleventh term is the last term and the only term we care about.

The last term is just b raised to the 10th power!

So the last term, the eleventh term is $(y^2)^{10} = y^{20}$

OK, You try problems 9 and 13 on your handout.

9. third term of $(x + 3)^{12}$



13. seventh term of $(x - 4y)^6$



Remember our first special case? The **Difference of Squares**?

$$a^2 - b^2 = (a + b)(a - b)$$

Factor the expression $x^2 - 16 = (x - 4)(x + 4)$

There are two more special cases; the **Sum and Difference of Cubes**.

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Factor $x^3 - 8$

$$a = \sqrt[3]{x^3} = x$$

$$b = \sqrt[3]{8} = 2$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\begin{aligned}x^3 - 8 &= (x - 2)(x^2 + x2 + 2^2) \\ &= (x - 2)(x^2 + 2x + 4)\end{aligned}$$

Factor $8x^3 + 27$

$$a = \sqrt[3]{8x^3} = 2x$$

$$b = \sqrt[3]{27} = 3$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\begin{aligned} 8x^3 + 27 &= (2x + 3)((2x)^2 - 2x(3) + 3^2) \\ &= (2x + 3)(4x^2 - 6x + 9) \end{aligned}$$

Factor and then solve.

1. $8x^3 - 27 = 0$



2. $x^3 + 64 = 0$



Now it's time to torture your neighbor.

Depending in which group you are in...

- 1. Create an expansion problem. Keep your exponent under 9.**
- 2. Create a problem in which you have to find a specific term in an expansion.**

Now exchange problems. Then you have to check their answers are correct!