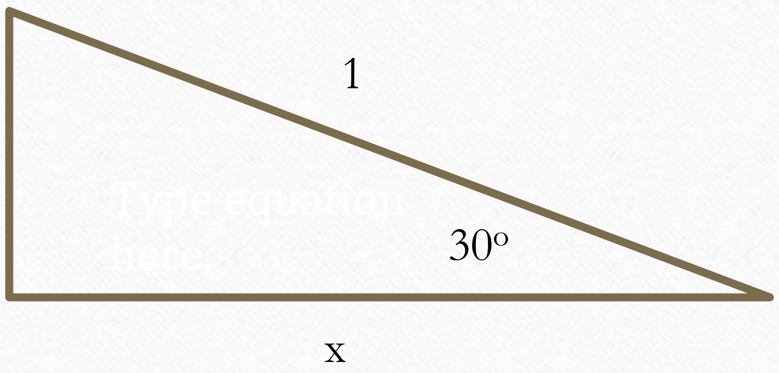
Monday, December 8, 2014

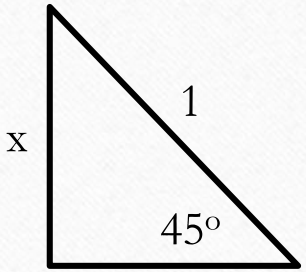
WARM UP

EXACT ANSWERS ONLY
No Decimals

1. Find the missing side length.



2. Find the missing side length.



3. Given triangle ABC with angle B a right angle.
If $\tan A = \frac{3}{5}$, find the remaining 5 trig functions for this angle.

Objectives

- Prove the Pythagorean Identities
- Use trigonometric functions to simplify trigonometric expressions
- Use trigonometric functions to verify trigonometric identities

Homework

- WBP 371, 2-38 even

Homework Review

Unit Circle Worksheet A

#	Ans
1	1
2	$\frac{\sqrt{2}}{2}$
3	$-\frac{\sqrt{2}}{2}$
4	$-\frac{\sqrt{2}}{2}$
5	1
6	0
7	$-\frac{\sqrt{2}}{2}$
8	0

Unit Circle Worksheet B

#	Ans
1	$\frac{1}{2}$
2	$-\frac{\sqrt{3}}{2}$
3	$\frac{1}{2}$
4	$-\frac{\sqrt{2}}{2}$
5	Undefined
6	-1
7	$-\frac{\sqrt{3}}{2}$
8	$-\frac{1}{2}$

Homework Review

Unit Circle Worksheet C

Angle	120
Quadrant	II
Sin	$\frac{\sqrt{3}}{2}$
Cos	$-\frac{1}{2}$
Tan	$-\sqrt{3}$

Angle	270
Quadrant	
Sin	-1
Cos	0
Tan	Undefined

Angle	225
Quadrant	III
Sin	$-\frac{\sqrt{2}}{2}$
Cos	$-\frac{\sqrt{2}}{2}$
Tan	1

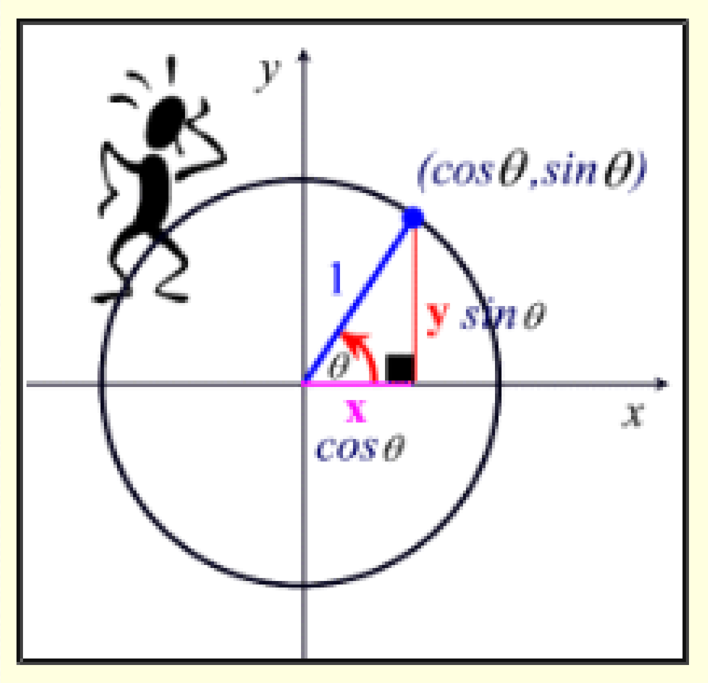
4	$\frac{\sqrt{2}}{2}$
5	$-\frac{1}{2}$
6	$-\frac{\sqrt{3}}{2}$
7	$-\frac{1}{2}$
8	1
9	Undefined
10	$-\frac{\sqrt{2}}{2}$
11	$\frac{\sqrt{2}}{2}$
12	1
13	1
14	$-\frac{\sqrt{2}}{2}$
15	$-\frac{\sqrt{3}}{2}$

What is an “identity”

In **mathematics** an **identity** is an equality relation A = B, such that A and B contain some variables and A and B produce the same value as each other regardless of what values (usually numbers) are substituted for the variables.

In trigonometry we frequently need to prove two things are equal to each other.

The Pythagorean Identity



Remember what the coordinates of any point on the unit circle represent?

We can translate that to the Pythagorean Theorem which gives us the **Pythagorean Identity**…

$a^2 + b^2 = c^2$ Pythagorean Theorem

$x^2 + y^2 = 1^2$ Substitute corresponding parts

$\sin^2\theta + \cos^2\theta = 1$ Substitute corresponding trig functions

$\boxed{\cos^2\theta + \sin^2\theta = 1}$ Rearrange

The Pythagorean Identity, other forms

There are actually 2 more Pythagorean Identities.

What happens when you divide each term by $\cos^2\theta$?

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{1 + tan^2\theta = sec^2\theta}$$

Another Pythagorean identity

The Pythagorean Identity, other forms

What happens when you divide each term by $\sin^2\theta$?

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{\cot^2\theta + 1 = \csc^2\theta}$$

Yet another Pythagorean identity

The Pythagorean Identity, other forms

Pythagorean Identity	Variations	
$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\theta = 1 - \cos^2\theta$	$\cos^2\theta = 1 - \sin^2\theta$
$\tan^2\theta + 1 = \sec^2\theta$	$\tan^2\theta = \sec^2\theta - 1$	
$1 + \cot^2\theta = \csc^2\theta$	$\cot^2\theta = \csc^2\theta - 1$	

You may see them rearranged but they are all the same identity.

Make sure you have all these identities in your notes.

TRIGONOMETRIC IDENTITIES

Reciprocal Identities:

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Quotient Identities:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities:

$$\sin^2 x + \cos^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

When we <u>verify identities</u>, we are proving one side of the equation is equal to the other.

Only work on one side at a time. You can not move terms from one side to the other.

Start by putting everything in terms of sine and cosine, then simplify

$$\sin\theta \sec\theta \cot\theta = 1$$

$$\cancel{\sin\theta}\,\frac{1}{\cancel{\cos\theta}}\,\frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}$$

$$1$$

Verify the identity $\sin\theta\tan\theta + \cos\theta = \sec\theta$

$$\sin\theta\frac{\sin\theta}{\cos\theta} + \cos\theta$$

Put everything in terms of sine and cosine

$$\frac{sin^2\,\theta}{\cos\theta} + \cos\theta$$

Simplify the fraction

$$\frac{sin^2\,\theta}{\cos\theta} + \cos\theta\frac{\cos\theta}{\cos\theta}$$

Combine fractions. You need a common denominator.

$$\frac{sin^2\,\theta}{\cos\theta} + \frac{cos^2\theta}{\cos\theta}$$

$$\frac{sin^2\,\theta + cos^2\theta}{\cos\theta}$$

Use the Pythagorean Identity to replace the numerator with 1.

$$\frac{1}{\cos\theta} = \sec\theta$$

Use the Reciprocal Identity

Verify the identity $\dfrac{\sin\theta + \cos\theta}{\sin\theta} = 1 + cot\,\theta$

Work on the more complicated side.

$$\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

Distribute the denominator

$$1 + \frac{\cos\theta}{\sin\theta}$$

Simplify the first fraction

$$1 + cot\theta \quad ✓$$

Reciprocal Identity for Cotangent

Simplify the expression $\csc\theta\tan\theta$

When we <u>simplify expressions</u> the objective is to get down to a single expression with no fractions.

$$\csc\theta\tan\theta = \frac{1}{\cancel{\sin}\theta}\frac{\cancel{\sin}\theta}{\cos\theta}$$

Put everything in terms of sine and cosine

$$= \frac{1}{\cos\theta}$$

Simplify

$$\boxed{= \sec\theta}$$

Final Answer

Use a Reciprocal Identity

Remember to try these approaches when you are verifying identities or simplifying expressions…

Put the expression in terms of sine and cosine

Split fractions with a single term denominator by distributing the denominator to each term in the numerator.

Combine fractions with different denominators by finding a common denominator.

Practice is <u>THE ONLY WAY</u> you get better at these!

Work on your homework problems.

If you finish them in class I will add one point to your trig unit test.

Make sure I initial your work book page.