Warm Up

#46 on your worksheet

The expression $p(x) = 25000x - 2x^2$ describes the profit of a company that customizes motorcycles when it customizes x bulldozers in a month.

1. How many bulldozers per month must the company customize to make the maximum possible profit?

2. What is the maximum profit?

3. Describe a reasonable domain and range for the function.

4. For what number of bulldozers per month is the profit at least $750,000?

Warm Up

Objectives Convert a quadratic equation to vertex form by Completing the Square

Homework Packet Pages 59 -60; 1-21 odd

Homework Review

Exercises

What are the solutions for each equation? Use the Quadratic Formula.

1. $-x^2 + 7x - 3 = 0$ $\frac{7 + \sqrt{37}}{2}$ or $\frac{7 - \sqrt{37}}{2}$

2. $x^2 + 6x = 10$ $-3 + \sqrt{19}$ or $-3 - \sqrt{19}$

3. $2x^2 = 4x + 3$ $\frac{2 + \sqrt{10}}{2}$ or $\frac{2 - \sqrt{10}}{2}$

4. $4x^2 + 81 = 36x$ $\frac{9}{2}$

5. $2x^2 + 1 = 5 - 7x$ -4 or $\frac{1}{2}$

6. $6x^2 - 10x + 3 = 0$ $\frac{5 + \sqrt{7}}{6}$ or $\frac{5 - \sqrt{7}}{6}$

Homework Review

Exercises

What is the value of the discriminant and what is the number of real solutions for each equation?

7. $x^2 + x - 42 = 0$
169; two

8. $-x^2 + 13x - 40 = 0$
9; two

9. $x^2 + 2x + 5 = 0$
–16; none

10. $x^2 = 18x - 81$
0; one

11. $-x^2 + 7x + 44 = 0$
225; two

12. $\frac{1}{4}x^2 - 5x + 25 = 0$
0; one

13. $2x^2 + 7 = 5x$
–31; none

14. $4x^2 + 25x = 21$
961; two

15. $x^2 + 5 = 3x$
–11; none

16. $\frac{1}{9}x^2 = 4x - 36$
0; one

17. $\frac{1}{2}x^2 + 2x + 3 = 0$
–2; none

18. $\frac{1}{6}x^2 = 2x + 18$
16; two

Homework Review

Exercises

Simplify each expression.

1. $2i + (-4 - 2i)$
   -4

2. $(3 + i)(2 + i)$
   $5 + 5i$

3. $(4 + 3i)(1 + 2i)$
   $-2 + 11i$

4. $3i(1 - 2i)$
   $6 + 3i$

5. $3i(4 - i)$
   $3 + 12i$

6. $3 - (-2 + 3i) + (-5 + i)$
   $-2i$

7. $4i(6 - 2i)$
   $8 + 24i$

8. $(5 + 6i) + (-2 + 4i)$
   $3 + 10i$

9. $9(11 + 5i)$
   $99 + 45i$

Homework Review

Exercises

Find the complex conjugate of each complex number.

10. $1 - 2i$ $1 + 2i$

11. $3 + 5i$ $3 - 5i$

12. i $-i$

13. $3 - i$ $3 + i$

14. $2 + 3i$ $2 - 3i$

15. $-5 - 2i$ $-5 + 2i$

Write each quotient as a complex number.

16. $\frac{3i}{1 - 2i}$ $-\frac{6}{5} + \frac{3}{5}i$

17. $\frac{6}{3 + 5i}$ $\frac{9}{17} - \frac{15}{17}i$

18. $\frac{2 + 2i}{i}$ $2 - 2i$

19. $\frac{2 + 5i}{3 - i}$ $\frac{1}{10} + \frac{17}{10}i$

20. $\frac{-4 - i}{2 + 3i}$ $-\frac{11}{13} + \frac{10}{13}i$

21. $\frac{6 + i}{-5 - 2i}$ $-\frac{32}{29} + \frac{7}{29}i$

Remember Imaginary #s?

Discriminant?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$

> 0	Two real roots
$= 0$	One real repeated root
< 0	No real roots, Two complex roots

$i = \sqrt{-1}$

$i^2 = -1$

Complex Roots

Sind the solutions to the equation $x^2 - 3x + 5 = 0$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{-11}}{2}$$

$$x = \frac{3 \pm i\sqrt{11}}{2}$$

$$x = \frac{3}{2} + \frac{\sqrt{11}}{2}i \qquad x = \frac{3}{2} - \frac{\sqrt{11}}{2}i$$

Complex Roots

Sind the solutions to the equation $x^2 - 10x + 34 = 0$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(34)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{-36}}{2}$$

$$x = \frac{10 \pm 6i}{2}$$

$x = 5 + 3i$ $\qquad$ $x = 5 - 3i$

Complex Roots

Sind the solutions to the equation $3x^2 - 4x + 10 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(10)}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{-104}}{6}$$

$$x = \frac{4 \pm 4i\sqrt{26}}{6}$$

$$x = \frac{2}{3} + \frac{2\sqrt{26}}{3}i \qquad x = \frac{2}{3} - \frac{2\sqrt{26}}{3}i$$

Remember complex roots always come in pairs!

Converting to vertex form.

The **standard form** of a quadratic function is

$$f(x) = ax^2 + bx + c$$

Step 1: Determine h using the formula $-\frac{b}{2a}$.

Step 2: Find k by evaluating the function at the h value found in step 1.

Step 3: Use a from the original standard form equation in the vertex form.

The **vertex form** of a quadratic function is

$$f(x) = a(x - h)^2 + k$$

Converting to vertex form.

For example, convert $f(x) = 2x^2 - 8x + 1$ into vertex form.

Step 1: Determine h using the formula $-\frac{b}{2a}$.

$$h = -\frac{-8}{2(2)} = 4$$

Step 2: Find k by evaluating the function at the h value found in step 1.

$$k = f(4) = 2(4)^2 - 8(4) + 1 = 1$$

Step 3: Use a from the original standard form equation in the vertex form.

$$f(x) = 2(x - 4)^2 + 1$$

Complete the Square

Now let's do it the long way. ☺

Complete the square to put the equation $y = x^2 - 6x + 14$ in vertex form.

$$y - 14 = x^2 - 6x$$

1. Move the constant to the other side of the equation.

$$y - 14 + \left(\frac{6}{2}\right)^2 = x^2 - 6x + \left(\frac{6}{2}\right)^2$$

$$y - 14 + 9 = x^2 - 6x + 9$$

$$y - 5 = x^2 - 6x + 9$$

2. Determine the value that will complete the square. $\left(\frac{b}{2}\right)^2$ Add it to both sides and simplify.

$$y - 14 = (x - 3)^2$$

3. Factor the trinomial. (It's a perfect square!)

$$y = (x - 3)^2 + 14$$

4. Move the constant back. Tah Dah! Vertex Form

Complete the Square

Put $y = 2x^2 + 16x - 4$ in vertex form.

$$y + 4 = 2x^2 + 16x$$

1. Move the constant to the other side of the equation.

$$y + 4 = 2(x^2 + 8x \qquad)$$

2. If $a > 0$, factor out of remaining terms.

$$y + 4 + 2\left(\frac{8}{2}\right)^2 = 2(x^2 + 8x + \left(\frac{8}{2}\right)^2)$$

3. Determine the value that will complete the square. $\left(\frac{b}{2}\right)^2$ Add it to both sides and simplify.

$$y + 36 = 2(x^2 + 8x + 16)$$

$$y + 36 = 2(x + 4)^2$$

4. Factor the trinomial. (It's a perfect square)

$$y = 2(x + 4)^2 - 36$$

5. Move the constant back. Tah Dah! Vertex Form

Complete the Square

Put $y = x^2 + 4x - 6$ in vertex form.

Put $y = 9x^2 - 18x - 27$ in vertex form.

Quiz Practice

1. Factor $x^2 - x - 72$

2. Solve $9x^2 - 6x + 1 = 0$ by factoring

3. Solve $2x^2 + 1 = 5 - 7x$ using the quadratic formula

4. Solve $3x^2 - 5x + 9 = 8$ by graphing