Warm-up

Solve each of the following functions giving exact answers for the roots. Use any method you like. State the type of roots you found for each function.

1. $f(x) = x^2 - 4x - 5$

2. $f(x) = x^2 - 6x + 13$

3. $f(x) = 12x^2 - 2x - 2$

Objectives

Use the remainder Theorem to find the remainder of a division problem without having to perform any division.

Use the remainder Theorem to determine if a linear binomial is a factor of a given polynomial.

Homework

Packet Page 35 – 36; problems 8-15 and 25-30 all

Check your homework

Exercises

What is the quotient and remainder of the following polynomials?

11. $(x^3 - 2x + 8) \div (x + 2)$

$x^2 - 2x + 2$, R 4

12. $(12x^3 - 71x^2 + 57x - 10) \div (x - 5)$

$12x^2 - 11x + 2$, R 0

13. $(3x^4 + x^3 - 6x^2 - 9x + 12) \div (x + 1)$

$3x^3 - 2x^2 - 4x - 5$, R 17

14. $(2x^3 - 15x + 23) \div (x - 2)$

$2x^2 + 4x - 7$, R 9

15. $(x^3 + x + 10) \div (x + 2)$

$x^2 - 2x + 5$, R 0

16. $(x^4 - 12x^3 - 18x^2 + 10) \div (x + 4)$

$x^3 - 16x^2 + 46x - 184$, R 746

17. $(x^3 + x^2 - 14x - 27) \div (x + 3)$

$x^2 - 2x - 8$, R -3

19. $(x^3 - 15) \div (x - 1)$

$x^2 + x + 1$, R -14

21. $(3x^3 - 70x + 2) \div (x - 5)$

$3x^2 + 15x + 5$, R 27

25. $P(x) = 5x^3 - 12x^2 + 2x + 1, a = 3$

$P(3) = 34$

27. $P(x) = x^3 + 6x^2 - 2, a = 3$

$P(3) = 79$

29. $P(x) = x^3 - 412, a = 8$

$P(8) = 100$

Use synthetic division to determine if $x - 3$ is a factor of $f(x) = 6x^3 - 5x^2 + 4x - 17$

3	**6**	**-5**	**4**	**-17**
		18	**39**	**129**
	6	**13**	**43**	**112**

Now just for grins evaluate $f(3)$.

$$f(3) = 6(3)^3 - 5(3)^2 + 4(3) - 17 \quad = \mathbf{112}$$

One more polynomial tool for you

The Remainder Theorem

If $f(x)$ is a polynomial in x then the remainder on dividing $f(x)$ by $x - a$ is $f(a)$

So what the heck does that mean?

I can do a couple of things without even having to do any division!

For example…

Find the remainder of $(x^3 - 5x^2 + 6x - 4) \div (x - 2)$

Evaluate the divisor at $x = 2$.

$$2^3 - 5(2)^2 + 6(2) - 4 = -4$$

Check by synthetic division

2	1	-5	6	-4
		2	-6	0
	1	-3	0	-4

You do Exercises 2-5 on the left side of the MATHEMATICS SUPPORT CENTER page in your packet.

Yet another use for the Remainder Theorem...

Is $(x-1)$ a factor of $f(x) = x^3 + 2x^2 - 2x - 1$?

In terms of a remainder, what would be a characteristic of a factor?

The remainder of the division would result in zero for a remainder.

Instead of dividing, we can evaluate $f(1)$. If the result is zero then $(x-1)$ is a factor.

$f(1) = 1^3 + 2(1)^2 - 2(1) - 1 = 0$ so $(x-1)$ is a factor.

You do first set of Exercises 2-5 on the right side of the MATHEMATICS SUPPORT CENTER page in your packet.

Factoring Cubic Functions, an introduction to finding ALL roots.

If $(x + 1)$ is a factor of $x^3 + 6x^2 + 11x + 6$, what are the remaining factors?

Some things to think about…

How many factors will this cubic have in total?

Three.

What should I do first?

Divide out the given factor.

What will be the degree of the quotient after dividing out the first factor?

2, a quadratic!

How can I find the remaining factors?

AC method, Quadratic Formula, Graphing! Oh My!

If $(x + 1)$ is a factor of $x^3 + 6x^2 + 11x + 6$, what are the remaining factors?

Did you get $(x + 1), (x + 2)\ and\ (x + 3)$?

Time to play ***Stump Your Neighbor***!

On a separate sheet of paper, create a fully factorable cubic function.

HINT: create your three factors, multiply them together and simplify. Put in standard Polynomial form.

Give your cubic polynomial to your partner. (DON'T SHOW THEM THE FACTORS).

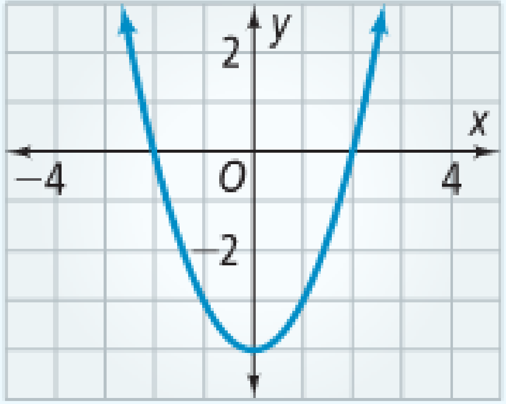
They will do the same for you.

See if you can find all three factors of the polynomial.

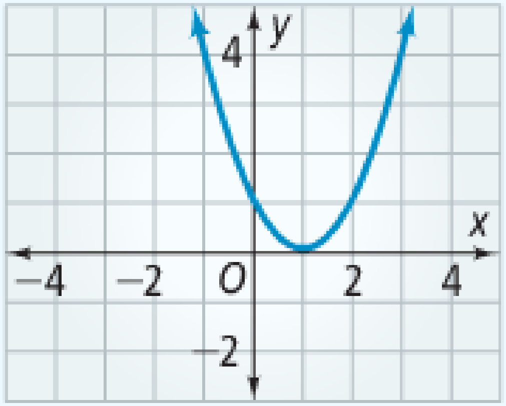
Oooo but let's make this interesting….
we need interesting factors….

Essential Understanding The degree of a polynomial equation tells you how many roots the equation has.

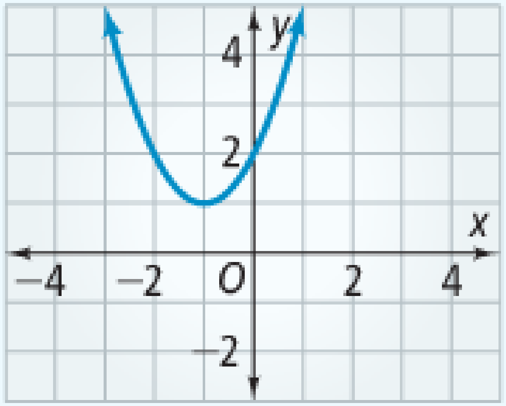
It is easy to see graphically that every polynomial function of degree 1 has a single zero, the x-intercept. However, there appear to be three possibilities for polynomials of degree 2. They correspond to these three graphs:



$y = x^2 - 4$
Two real zeros



$y = x^2 - 2x + 1$
One real zero



$y = x^2 + 2x + 2$
No real zeros

take note

Theorem **The Fundamental Theorem of Algebra**

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots, including multiple and complex roots.

So $p(x) = x^3 + 4x^2 - 2$ has **3 roots**

So $f(x) = x^4 + 3x^2 - 7$ has **4 roots**

So $g(x) = 7x^{102} + 43x^{27} - x$ has **102 roots**

Let's play how many roots?

Show me with your fingers…

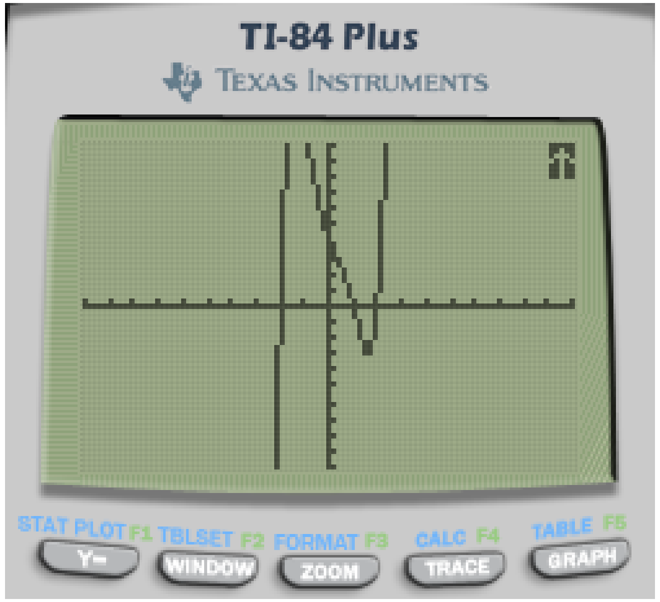
$f(x) = x^2 + 2$

$f(x) = 7x^5 + 4x^4 + 3x - 3$

$f(x) = x^2 + x^6 - 2$

So how do we find all these roots?

Find all the roots of $x^5 - x^4 - 3x^3 + 3x^2 - 4x + 4 = 0$



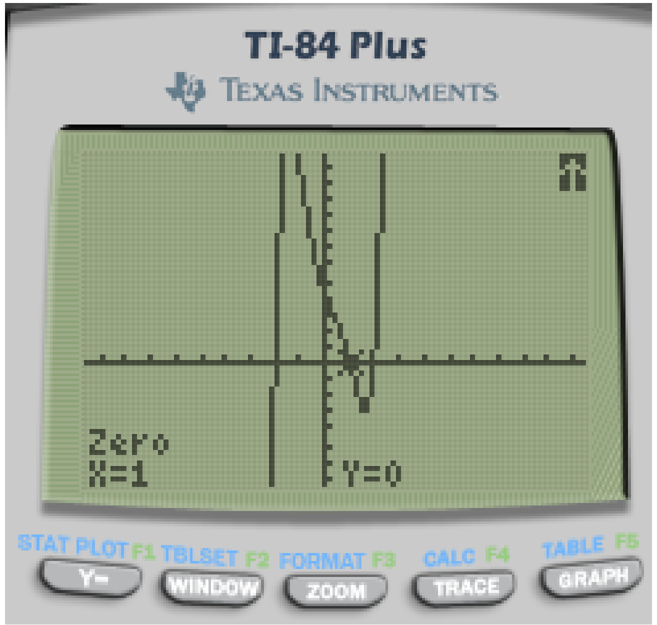
Enter the equation into your calculator and graph.

How many roots/zeros do you see?

According to the FTA, how many roots are there?

Use your calculator to find the three roots.

You should get x = -2, x = 1 and x = 2



But what about the other 2 roots?