Warm Up

Factor the following expressions

1. $121r^2 - 1$

2. $a^2 + 11a + 18$

3. $16b^2 + 60b - 100$

Quadratic Formula

Objectives

Solve Quadratic Expressions Using the Quadratic Formula.

Use the Discriminant to determine the number and type of roots for a quadratic function.

Homework

Packet Pages 69-70: 1-6 and 8-18 even
Packet Pages 79-80: 2-20 even

Homework Questions

Exercises

Solve each equation by factoring. Check your answers.

1. $x^2 - 10x + 16 = 0$ 2, 8
2. $x^2 + 2x = 63$ −9, 7
3. $x^2 + 9x = 22$ −11, 2
4. $x^2 - 24x + 144 = 0$ 12
5. $2x^2 = 7x + 4$ $-\frac{1}{2}$, 4
6. $2x^2 = -5x + 12$ −4, $\frac{3}{2}$
7. $x^2 - 7x = -12$ 3, 4
8. $2x^2 + 10x = 0$ −5, 0
9. $x^2 + x = 2$ −2, 1
10. $3x^2 - 5x + 2 = 0$ $\frac{2}{3}$, 1
11. $x^2 = -5x - 6$ −3, −2
12. $x^2 + x = 20$ −5, 4

Homework Questions

Exercises

Solve the equation by graphing. Give each answer to at most two decimal places.

13. $x^2 = 5$ −2.24, 2.24

14. $x^2 = 5x + 1$ −0.19, 5.19

15. $x^2 + 7x = 3$ −7.41, 0.41

16. $x^2 + x = 5$ −2.79, 1.79

17. $x^2 + 3x + 1 = 0$ −2.62, −0.38

18. $x^2 = 2x + 4$ −1.24, 3.24

19. $3x^2 - 5x + 9 = 8$ 0.23, 1.43

20. $4 = 2x^2 + 3x$ −2.35, 0.85

21. $x^2 - 6x = -7$ 1.59, 4.41

22. $-x^2 = 8x + 8$ −6.83, −1.17

Quadratic Formula

THE QUADRATIC FORMULA

Yet another method for solving quadratic equations of the form $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

Let’s sing!

Quadratic Formula

Solve **$x^2 + 4x = 12$** using the **quadratic formula**.

Step 1: Put the equation in standard form. $x^2 + 4x - 12 = 0$

Step 2: Find the values of a, b, and c. $a = 1, b = 4, c = -12$

Step 3: Substitute a, b, and c into the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-12)}}{2(1)}$$

Step 4: Simplify

$$x = \frac{-4 \pm \sqrt{16 + 48}}{2} = \frac{-4 \pm \sqrt{64}}{2} = \frac{-4 \pm 8}{2} = \frac{-4}{2} \pm \frac{8}{2} = -2 \pm 4$$

Step 5: Determine final answer(s) $-2 + 4 = \mathbf{2}\ and\ -2 - 4 = \mathbf{-6}$

Quadratic Formula

Solve $2x^2 + 3x = 9$ using the **quadratic formula**.

Step 1: Put the equation in standard form.

Step 2: Find the values of a, b, and c.

Step 3: Substitute a, b, and c into the formula

Step 4: Simplify

Step 5: Determine final answer(s)

Quadratic Formula

Solve $2x^2 - x = 4$ using the **quadratic formula**.

Step 1: Put the equation in standard form.

Step 2: Find the values of a, b, and c.

Step 3: Substitute a, b, and c into the formula

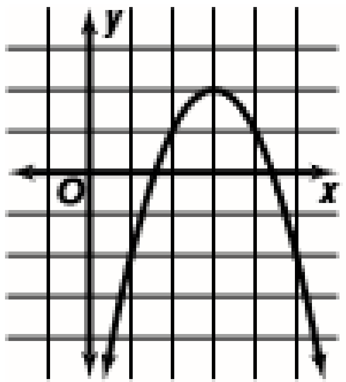
Step 4: Simplify

Step 5: Determine final answer(s)

Quadratic Formula

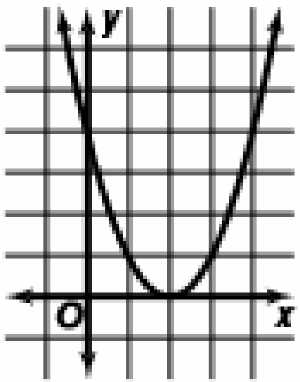
Examples **Describe the real roots of the quadratic equations whose related functions are graphed below.**

a.



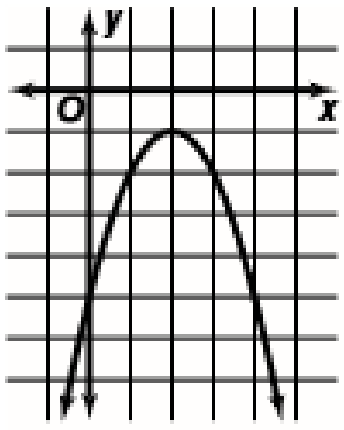
The parabola crosses the x-axis twice. One root is between 1 and 2, and the other is between 4 and 5.

b.



Since the vertex of the parabola lies on the x-axis the function has one distinct root, 2.

c.



This parabola does not intersect the x-axis, so there are no real roots. The solution set is ∅.

The **discriminant** of a quadratic equation tells us how many solutions (roots) exist for a given quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$	If $b^2 - 4ac = 0$	If $b^2 - 4ac < 0$
2 real roots	1 repeated real root	no real roots

Quadratic Formula

Lets look at $-3x^2 + 7x = 2$

Put in standard form; $3x^2 - 7x + 2 = 0$

The discriminant is $b^2 - 4ac$.

For this equation, $a = 3$, $b = -7$ and $c = 2$

The discriminant for this equation is $(-7)^2 - 4(3)(2) = 25$.

What does that tell us?

Since $25 > 0$ there are **two real roots.**

Factored form: $(3x - 1)(x - 2) = 0$, roots are $\mathbf{x = {}^{1}/_{3}}$ and $\mathbf{x = 2}$

Lets look at $x^2 + 4x + 4 = 0$

The discriminant is $b^2 - 4ac$.

For this equation, a = 1, b = 4 and c = 4

The discriminant for this equation is $4^2 – 4(1)(4) = 0$.

What does that tell us?

Since 0 = 0 there is **one real repeated root**.

Factored form: $(x + 2)(x + 2) = 0$, one repeated root, **$x = -2$**

Quadratic Formula

Lets look at $3x^2 - 4x + 10 = 0$

The discriminant is $b^2 - 4ac$.

For this equation, $a = 3$, $b = -4$ and $c = 10$

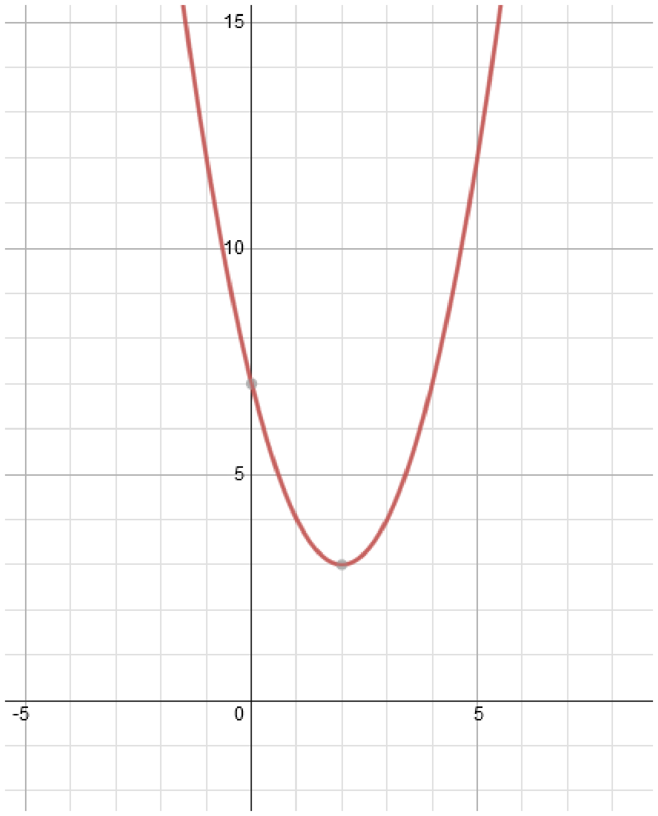
The discriminant for this equation is $(-4)^2 – 4(3)(10) =$ **-104**.

What does that tell us?

Since $-104 < 0$ there are no real roots!

Using the quadratic formula causes us to take the square root of a negative.

What does this mean “no real roots”?



Graphically we know it means that there are **no x intercepts**.

Algebraically this means we have **complex roots**.

Let’s talk about *imaginary numbers*.

$$\sqrt{-1} = i$$

Home for imaginary numbers…

Remember getting your hand slapped when you tried to take the square root of a negative number?

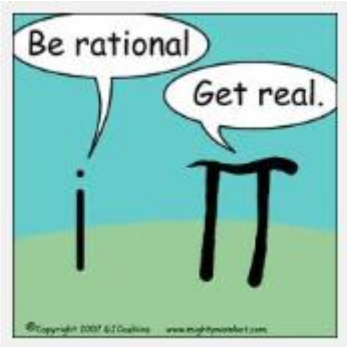
Well thanks to imaginary numbers, you'll never have to worry about that again.

Now we'll starting thinking like this…

$$\sqrt{-4} = \sqrt{(-1)(4)} = 2\sqrt{-1}$$

But we're going to let the symbol i represent $\sqrt{-1}$ and write

$$\sqrt{-4} = 2i$$



Remember, i is a number like π and e

π is the ratio between circumference and diameter shared by all circles.

e is the base rate of growth shared by all continually growing processes.

i is the is the square root of negative 1.

We can perform mathematical operations on i.

$$i^0 = 1$$

$$i^1 = \sqrt{-1} = i$$

$$i^2 = (\sqrt{-1})(\sqrt{-1}) = -1$$

$$i^3 = (i^2)(i) = (-1)(i) = -i$$

Simplifying numbers using i

Write $\sqrt{-18}$ using the imaginary unit.

Simplifying numbers using i

Write $\sqrt{-25}$ using the imaginary unit.

Write $\sqrt{-12}$ using the imaginary unit.

Write $\sqrt{-7}$ using the imaginary unit.


But wait!
There's more!
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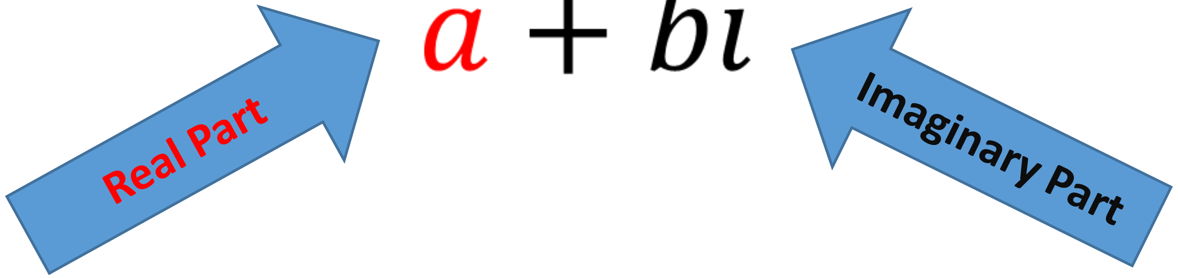
Complex Numbers, *not so complex*

$$a + bi$$



a and b are real numbers

If $a = 0$ the number is $a + bi$ is a **pure imaginary number**

If $a \neq 0$ and $b = 0$ the number is $a + bi$ is a **real number**

If $a \neq 0$ and $b \neq 0$ the number is $a + bi$ is a **complex number**

A little math humor.

After having dinner together a one mathematician asked the other “How was your meal”?

“Great but $\frac{\sqrt{-1}}{8}$.”

“Yeah,” said the other, “I over 8 too.”

Adding and subtracting Complex Numbers

Simplify $(4 - 3i) + (-4 + 3i)$

Simplify $(5 - 3i) - (-2 + 4i)$

Find Each sum or difference

1. $(7 - 2i) + (-3 + i)$

2. $(1 + 5i) - (3 - 2i)$

3. $(8 + 6i) - (8 - 6i)$

4. $(-3 + 9i) + (3 + 9i)$

Multiplying Complex Numbers

Simplify $(3\mathrm{i})(-5+2\mathrm{i})$

Simplify $(4+3\mathrm{i})(-1-2\mathrm{i})$

Find Each Product

1. $(7\mathrm{i})(3\mathrm{i})$

2. $(2-3\mathrm{i})(4+5\mathrm{i})$

3. $(-4+5\mathrm{i})(-4-5\mathrm{i})$

What did you notice about the last problem?

$$(-4 + 5i)(-4 - 5i)$$

The product was a real number.

Both terms in each factor are identical.

The sign (or operation) in each factor is the opposite of the other factor.

We call these **conjugates**. In this case, they are **complex conjugates**.

Dividing Complex Numbers

Simplify $\dfrac{2+3i}{1-4i}$

Multiply numerator and denominator by the complex conjugate of the denominator, $1 + 4\mathrm{i}$.

Substitute -1 for i^2

Dividing Complex Numbers

Simplify $\frac{9+12i}{3i}$

Multiply numerator and denominator by the complex conjugate of the denominator, $-3i$.

Substitute -1 for i^2

Find Each Quotient

1. $\frac{5-2i}{3+4i}$

2. $\frac{4-i}{6i}$

3. $\frac{8-7i}{8+7i}$

Let's look at this one again. Use the quadratic formula to find the roots of this equation.

$$3x^2 - 4x + 10 = 0$$

Complex roots always come in pairs!

Quadratic Formula

Find the discriminant of each equation and determine the number of real solutions.

1. $-x^2 + 2x - 9 = 0$

2. $x^2 + 17x + 4 = 0$

3. $x^2 - 6x + 9 = 0$

Quadratic Formula

To Turn In …

Solve the following equations using the Quadratic Formula.

1. $x^2 + 12x + 35 = 0$

2. $2x^2 + 3 = 7x$