Warm-up

Wednesday, March 04, 2015

1. Divide using synthetic division $(x^4 + 2x - 1) \div (x + 3)$

2. Use the remainder theorem to determine if $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

3. Is $(x - 3)$ a factor of $x^3 - 27$?

4. Solve by graphing $y = 2x^3 - 3x - 2$.

Objectives

Recognize a difference of squares and be able to factor it.

Recognize a sum or difference of cubes and be able to factor it.

Homework

Packet Page 10 and 11, all problems

Look at packet page 6.

Today we're going to look at some special cases for factoring polynomials.

The good news is we just have to recognize the pattern and *(sort of)* fill in the blanks.

take note

Concept Summary Polynomial Factoring Techniques

Techniques	Examples
Factoring out the GCF Factor out the greatest common factor of all the terms.	$15x^4 - 20x^3 + 35x^2$ $= 5x^2(3x^2 - 4x + 7)$
Quadratic Trinomials For $ax^2 + bx + c$, find factors with product ac and sum b.	$6x^2 + 11x - 10$ $= (3x - 2)(2x + 5)$
Perfect Square Trinomials $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 10x + 25 = (x + 5)^2$ $x^2 - 10x + 25 = (x - 5)^2$
Difference of Squares $a^2 - b^2 = (a + b)(a - b)$	$4x^2 - 15 = (2x + \sqrt{15})(2x - \sqrt{15})$
Factoring by Grouping $ax + ay + bx + by$ $= a(x + y) + b(x + y)$ $= (a + b)(x + y)$	$x^3 + 2x^2 - 3x - 6$ $= x^2(x + 2) + (-3)(x + 2)$ $= (x^2 - 3)(x + 2)$
Sum or Difference of Cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$8x^3 + 1 = (2x + 1)(4x^2 - 2x + 1)$ $8x^3 - 1 = (2x - 1)(4x^2 + 2x + 1)$

Let's start with a Difference of Squares?

Look at the expression $x^2 - 16$

The first term is a perfect square: $x^2 = (x)(x)$

What do you notice about the second term?

Also a perfect square: $16 = (4)^2 = (4)(4)$

We call this a difference of squares the second term is subtracted from the first and each term can be expressed as a perfect square.

To factor, follow the following pattern: $a^2 - b^2 = (a+b)(a-b)$

In this example $a = x$ and $b = 4$

$$x^2 - 16 = x^2 - 4^2 = (x+4)(x-4)$$

Difference of Squares

$$a^2 - b^2 = (a+b)(a-b)$$

Look at packet page 10.

1. $16n^2 - 9$

Take the square root of the 1st term.

$$a = \sqrt{16n^2} = 4n$$

Take the square root of the 2nd term.

$$b = \sqrt{9} = 3$$

Follow the pattern

$$(4n+3)(4n-3)$$

2. $4m^2 - 25$

Take the square root of the 1st term.

$$a = \sqrt{4m^2} = 2m$$

Take the square root of the 2nd term.

$$b = \sqrt{25} = 5$$

Follow the pattern

$$(2m+5)(2m-5)$$

7. $n^4 - 100$

$$a = \sqrt{n^4} = n^2$$

$$b = \sqrt{100} = 10$$

$$(n^2+10)(n^2-10)$$

The other two special cases are the Sum and Difference of Cubes.

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

This time we take the cube root of each term to determine the values for a and b. Then we follow the pattern.

Sum and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\boxed{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$$

Factor $x^3 - 8$

$$a = \sqrt[3]{x^3} = x$$

$$b = \sqrt[3]{8} = 2$$

$$\begin{aligned} x^3 - 8 &= (x - 2)(x^2 + x2 + 2^2) \\ &= (x - 2)(x^2 + 2x + 4) \end{aligned}$$

Sum and Difference of Cubes

$$\boxed{a^3 + b^3 = (a+b)(a^2 - ab + b^2)}$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Factor $8x^3 + 27$

$$a = \sqrt[3]{8x^3} = 2x$$
$$b = \sqrt[3]{27} = 3$$

$$\begin{aligned} 8x^3 + 27 &= (2x+3)((2x)^2 - 2x(3) + 3^2) \\ &= (2x+3)(4x^2 - 6x + 9) \end{aligned}$$

Factor and then solve.

1. $8x^3 - 27 = 0$

2. $x^3 + 64 = 0$