WARM UP

1. Put the equation $m = t^r$ in logarithmic form.

2. Write the expression $\log x - 3\log y + 5\log z$ as a single log statement.

3. Find the inverse of the function $y = 3x^2 + 7$

4. Find the inverse of the function $y = 7^{x+2} + 5$

Objectives

- Construct the Unit Circle
- Use the Unit Circle to find trig values for the common angles on the circle.

Homework

- Complete the Unit Circle Worksheets

Homework Review

1. $(-3, 4)$

$\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$,

$\csc\theta = \frac{5}{4}$, $\sec\theta = -\frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

2. $(12, -5)$

$\sin\theta = -\frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = -\frac{5}{12}$,

$\csc\theta = -\frac{13}{5}$, $\sec\theta = \frac{13}{12}$, $\cot\theta = -\frac{12}{5}$

3. $(-2, -1)$

$\sin\theta = -\frac{\sqrt{5}}{5}$, $\cos\theta = -\frac{2\sqrt{5}}{5}$, $\tan\theta = \frac{1}{2}$,

$\csc\theta = -\sqrt{5}$, $\sec\theta = -\frac{\sqrt{5}}{2}$, $\cot\theta = 2$

4. $(\sqrt{5}, 2)$

$\sin\theta = \frac{2}{3}$, $\cos\theta = \frac{\sqrt{5}}{3}$, $\tan\theta = \frac{2\sqrt{5}}{5}$,

$\csc\theta = \frac{3}{2}$, $\sec\theta = \frac{3\sqrt{5}}{5}$, $\cot\theta = \frac{\sqrt{5}}{2}$

5. A hiker is standing on one bank of a river. A tree stands on the opposite bank, which is 750 ft away. A line from the top of the tree to the ground at the hiker's feet makes an angle of 12° with the ground. How tall is the tree?

about 159 ft

Homework Review

7. In $\triangle ABC$, $\angle C$ is a right angle and $\tan A = \frac{2}{3}$. Draw a diagram and find each value in fraction form and in decimal form. Round your answer to the nearest tenth, if necessary.

a. $\cos A$
$\frac{3\sqrt{13}}{13}$; 0.8

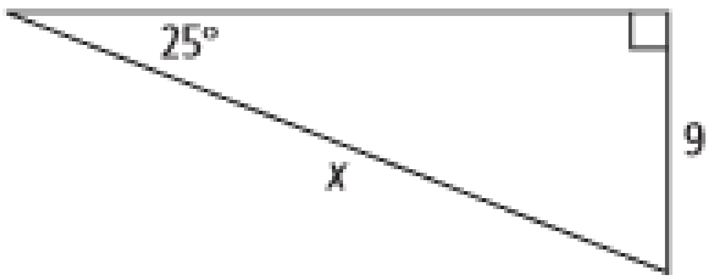
b. $\tan B$
$\frac{3}{2}$; 1.5

c. $\sin A$
$\frac{2\sqrt{13}}{13}$; 0.6

d. $\cot B$
$\frac{2}{3}$; 0.7

e. $\sec A$
$\frac{\sqrt{13}}{3}$; 1.2

f. $\csc B$
$\frac{\sqrt{13}}{3}$; 1.2

9.


21.3

Homework Review

11. $f = 8, e = 15$
$d = 17; \angle F = 28.1°; \angle E = 61.9°$

12. $f = 1, e = 2$
$d = 2.2; \angle F = 26.6°; \angle E = 63.4°$

13. $f = 2, e = 1$
$d = 2.2; \angle F = 63.4°; \angle E = 26.6°$

14. $f = 1, d = 500$
$e = 500; \angle F = 0.1°; \angle E = 89.9°$

15. $d = 21, e = 8$
$f = 19.4; \angle F = 67.6°; \angle E = 22.4°$

16. $e = 5, f = 1$
$d = 5.1; \angle F = 11.3°; \angle E = 78.7°$

17. You are designing several access ramps. What angle would each ramp make with the ground, to the nearest 0.1°?

a. 20 ft long, rises 16 in. 3.8°

b. 8 ft long, rises 8 in. 4.8°

c. 12 ft long, rises 6 in. 2.4°

d. 30 ft long, rises 32 in. 5.1°

e. 4 ft long, rises 6 in. 7.1°

f. 6 ft long, rises 14 in. 11.0°

Sketch a right triangle with θ as the measure of one acute angle. Find the other five trigonometric ratios of θ.

18. $\cos\theta = \frac{4}{11}$ $\sin\theta = \frac{\sqrt{105}}{11}, \tan\theta = \frac{\sqrt{105}}{4}, \cot\theta = \frac{4\sqrt{105}}{105}, \csc\theta = \frac{11\sqrt{105}}{105}, \sec\theta = \frac{11}{4}$

19. $\sin\theta = \frac{7}{12}$ $\cos\theta = \frac{\sqrt{95}}{12}, \tan\theta = \frac{7\sqrt{95}}{95}, \cot\theta = \frac{\sqrt{95}}{7}, \csc\theta = \frac{12}{7}, \sec\theta = \frac{12\sqrt{95}}{95}$

20. $\csc\theta = \frac{14}{6}$ $\sin\theta = \frac{3}{7}, \cos\theta = \frac{2\sqrt{10}}{7}, \tan\theta = \frac{3\sqrt{10}}{20}, \cot\theta = \frac{2\sqrt{10}}{3}, \sec\theta = \frac{7\sqrt{10}}{20}$

21. $\cos\theta = \frac{9}{16}$ $\sin\theta = \frac{5\sqrt{7}}{16}, \tan\theta = \frac{5\sqrt{7}}{9}, \cot\theta = \frac{9\sqrt{7}}{35}, \csc\theta = \frac{16\sqrt{7}}{35}, \sec\theta = \frac{16}{9}$

22. $\sin\theta = 0.45$ $\cos\theta \approx 0.893, \tan\theta \approx 0.504, \cot\theta \approx 1.98, \csc\theta \approx 2.22, \sec\theta \approx 1.12$

23. $\sec\theta = 7.6$ $\sin\theta \approx 0.991, \cos\theta \approx 0.132, \tan\theta \approx 7.53, \csc\theta \approx 1.01, \cot\theta \approx 0.133$

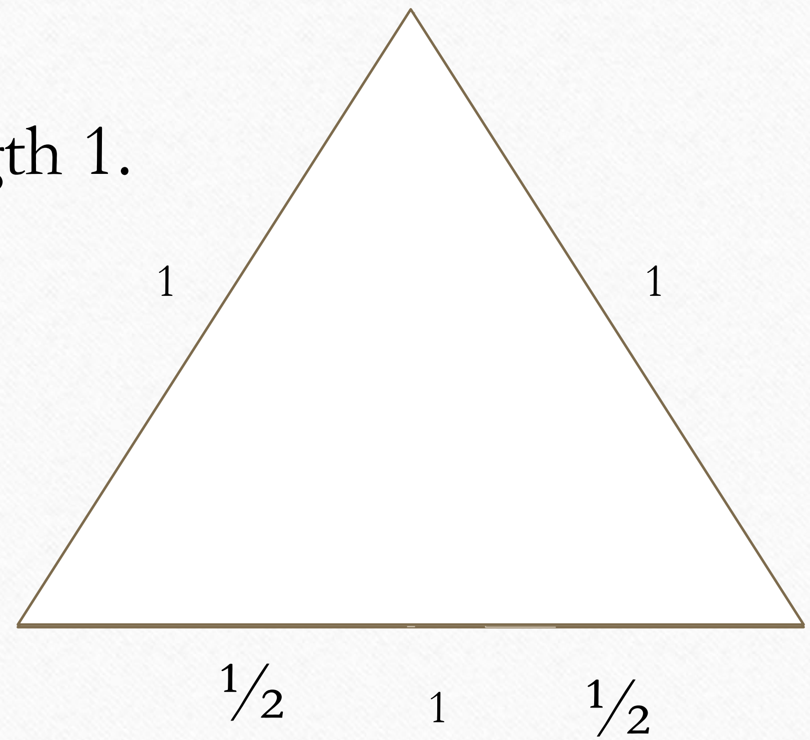
$30° - 60° - 90°$ Triangles revisited

Start with an equilateral triangle of side length 1.

Drop an angle bisector and we have two 30-60-90 triangles

To find the length of x, use the Pythagorean theorem.

$$\boldsymbol{a^2 + b^2 = c^2}$$



$30° - 60° - 90°$ Triangles revisited

$a^2 + b^2 = c^2$

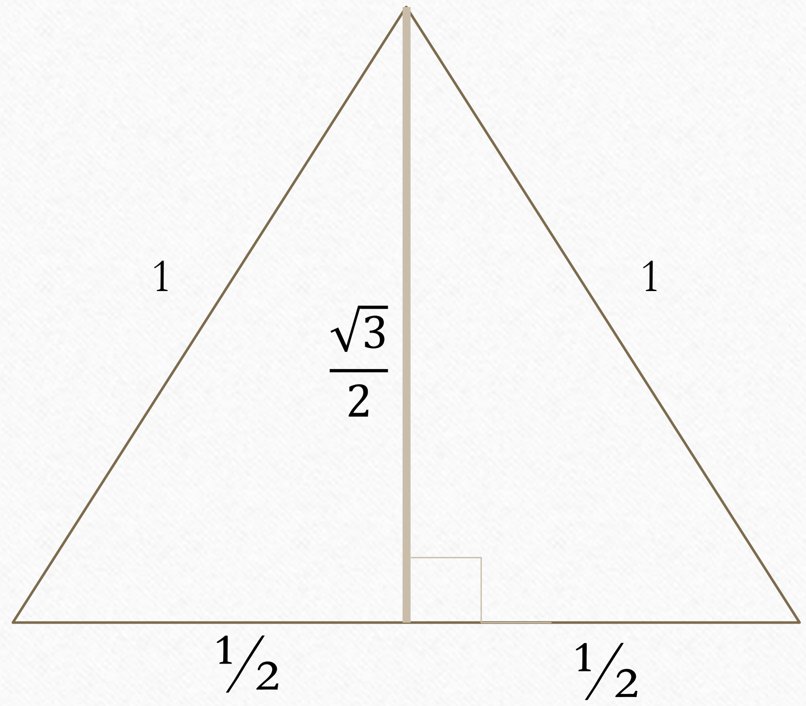
$$x^2 + \left(\frac{1}{2}\right)^2 = 1^2$$

$$x^2 + \frac{1}{4} = 1$$

$$x^2 = 1 - \frac{1}{4}$$

$$x^2 = \frac{3}{4}$$

$$x = \frac{\sqrt{3}}{2}$$



45° – 45° – 90° Isosceles Right Triangle Revisited

Use the Pythagorean Theorem to find *x* again.

$$a^2 + b^2 = c^2$$

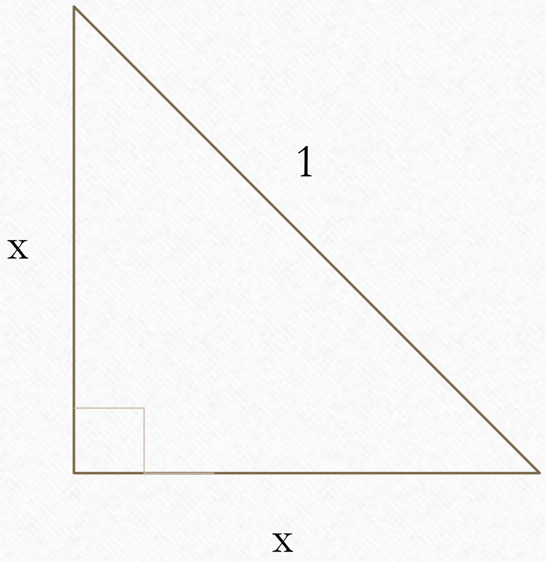
$$x^2 + x^2 = 1^2$$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$x = \frac{1}{\sqrt{2}}$$

$$x = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$$



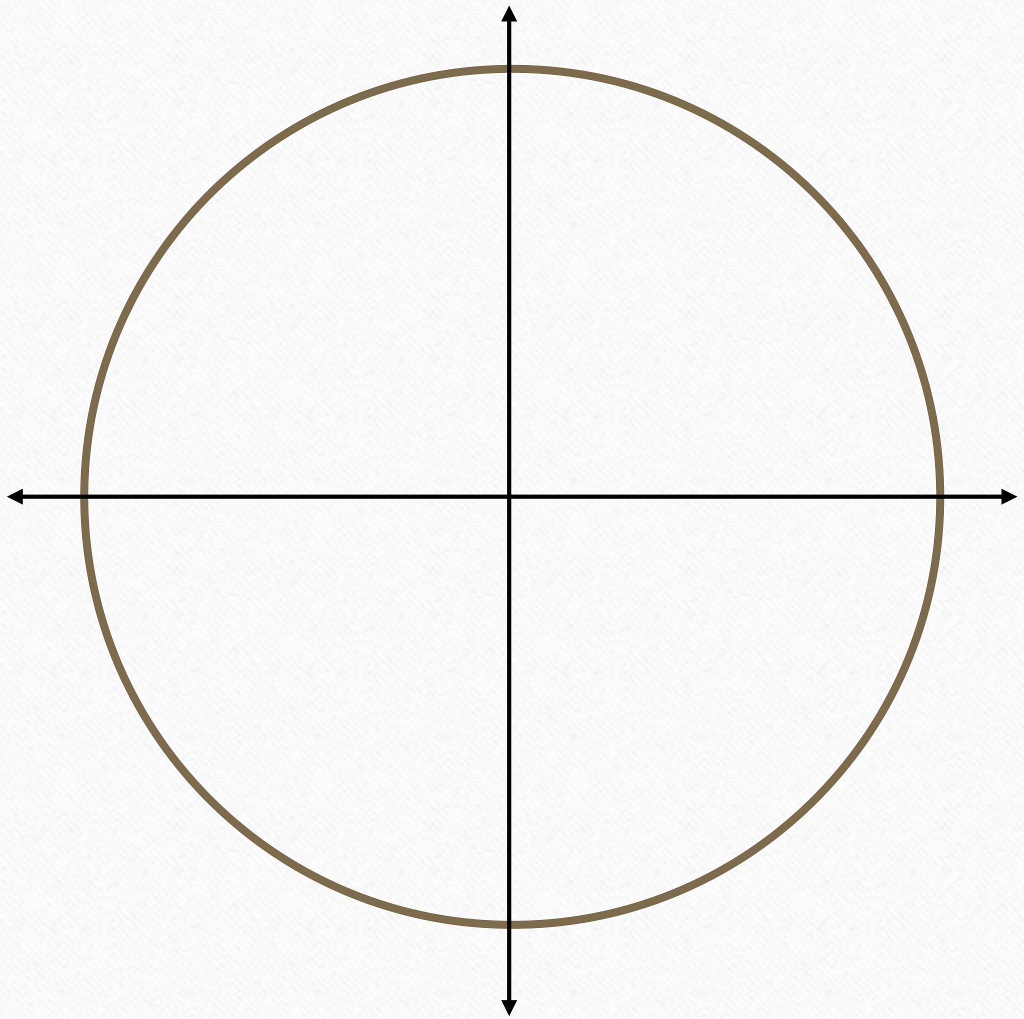
The Unit Circle

It's a circle

The radius is 1

It lives on the x/y coordinate plane.

Its center is the origin



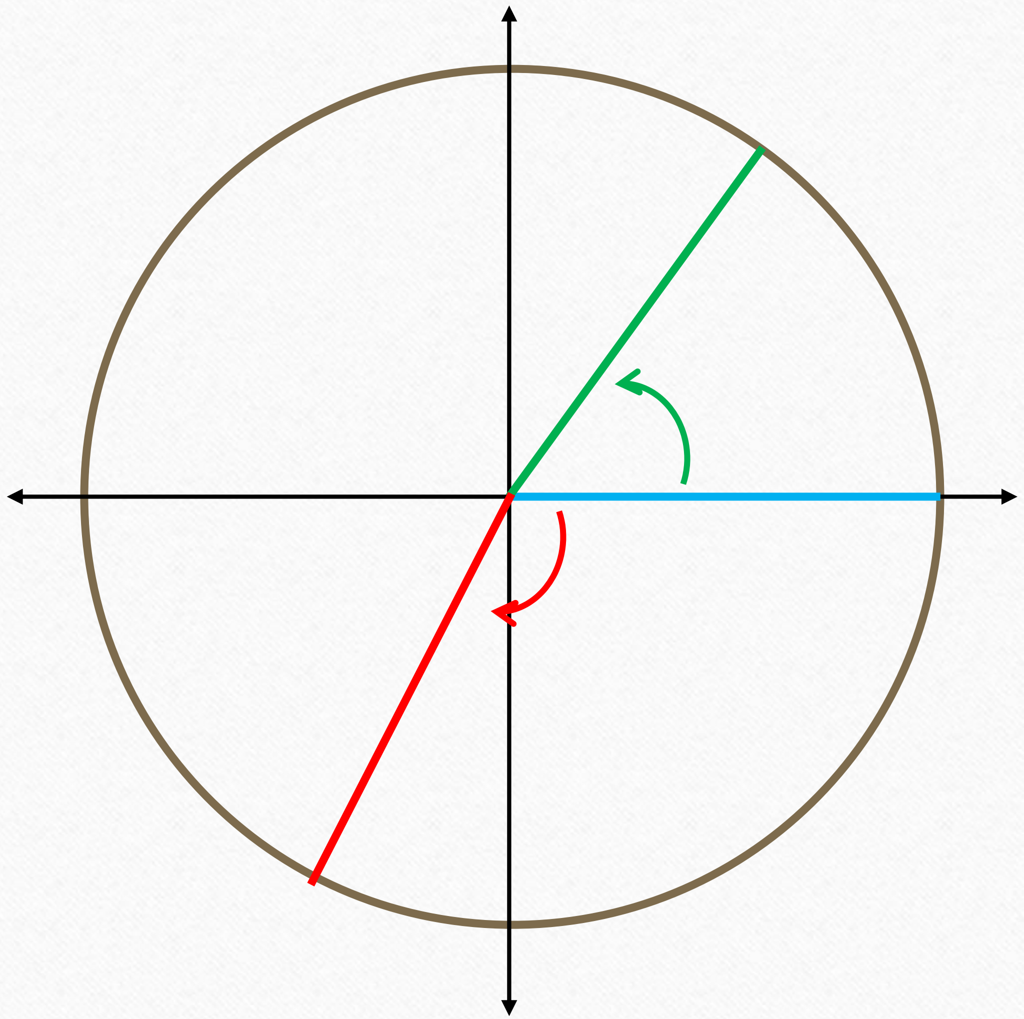
Angles are measured from the origin.

The **initial side** of angles starts at the x axis.

Positive angles sweep in a **counter clockwise** direction.

Negative angles sweep in a **clockwise** direction.

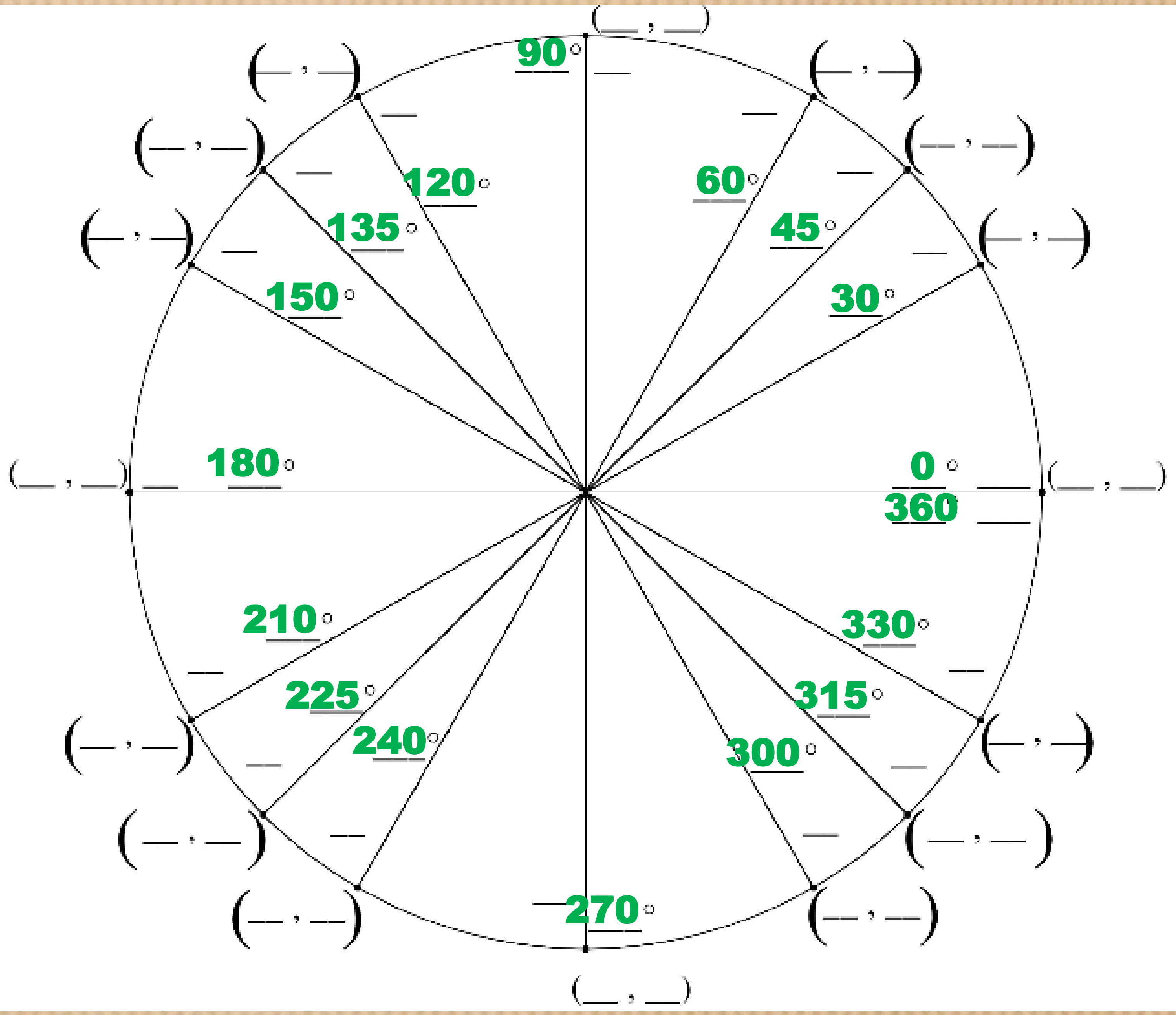
The Unit Circle



Let's start filling in the Unit Circle.

Start with the degree measures…

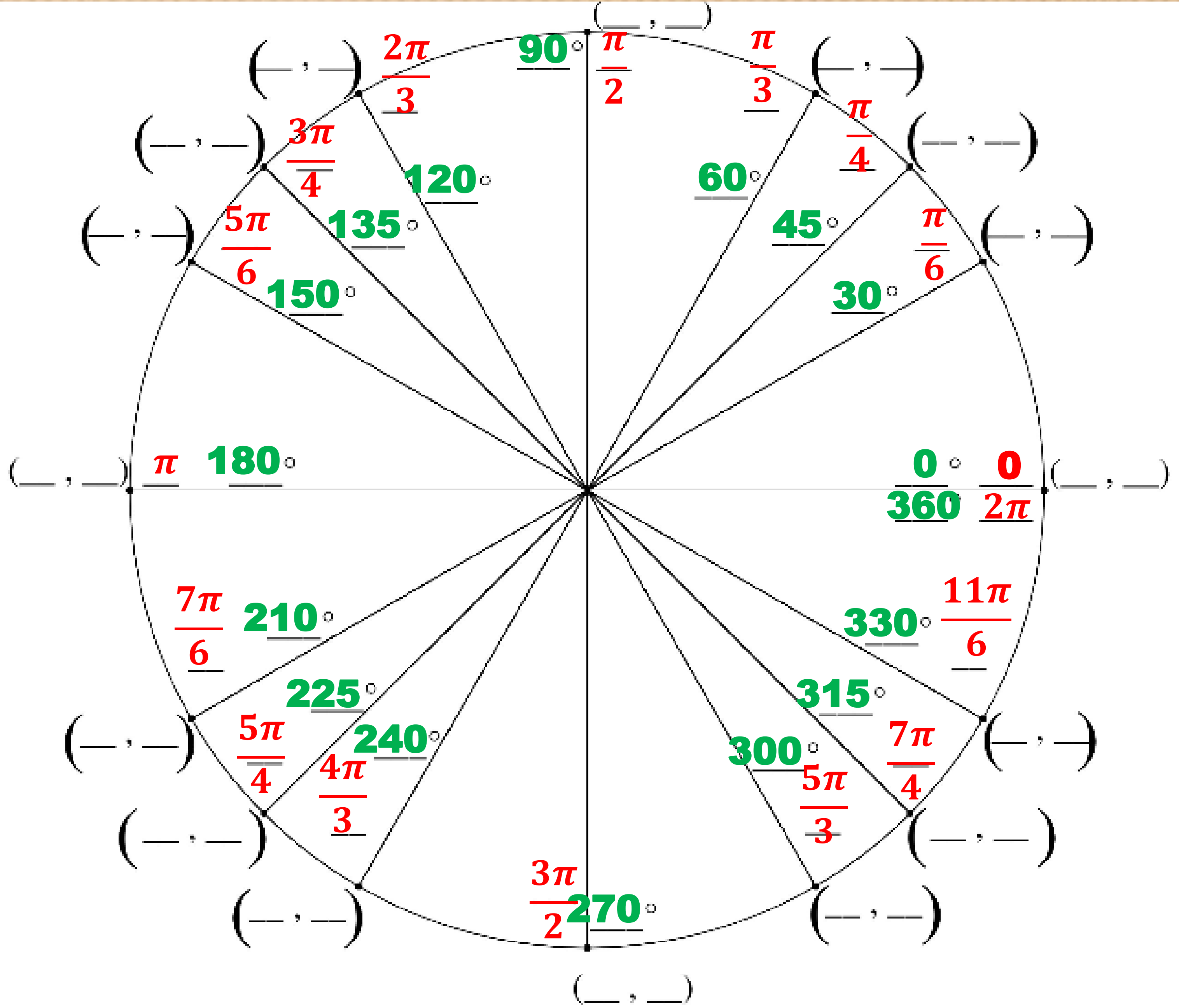
We focus on the common angles, 30, 45, 60 and 90.



Radian Measure

Another way to measure angles is in terms of the circumference.

Since the radius of this circle is 1, the circumference is equal to 2π



Converting between radian and degree measure

To convert a degree measure to a radian measure multiply by $\frac{\pi}{180}$

To convert a radian measure to a degree measure multiply by $\frac{180}{\pi}$

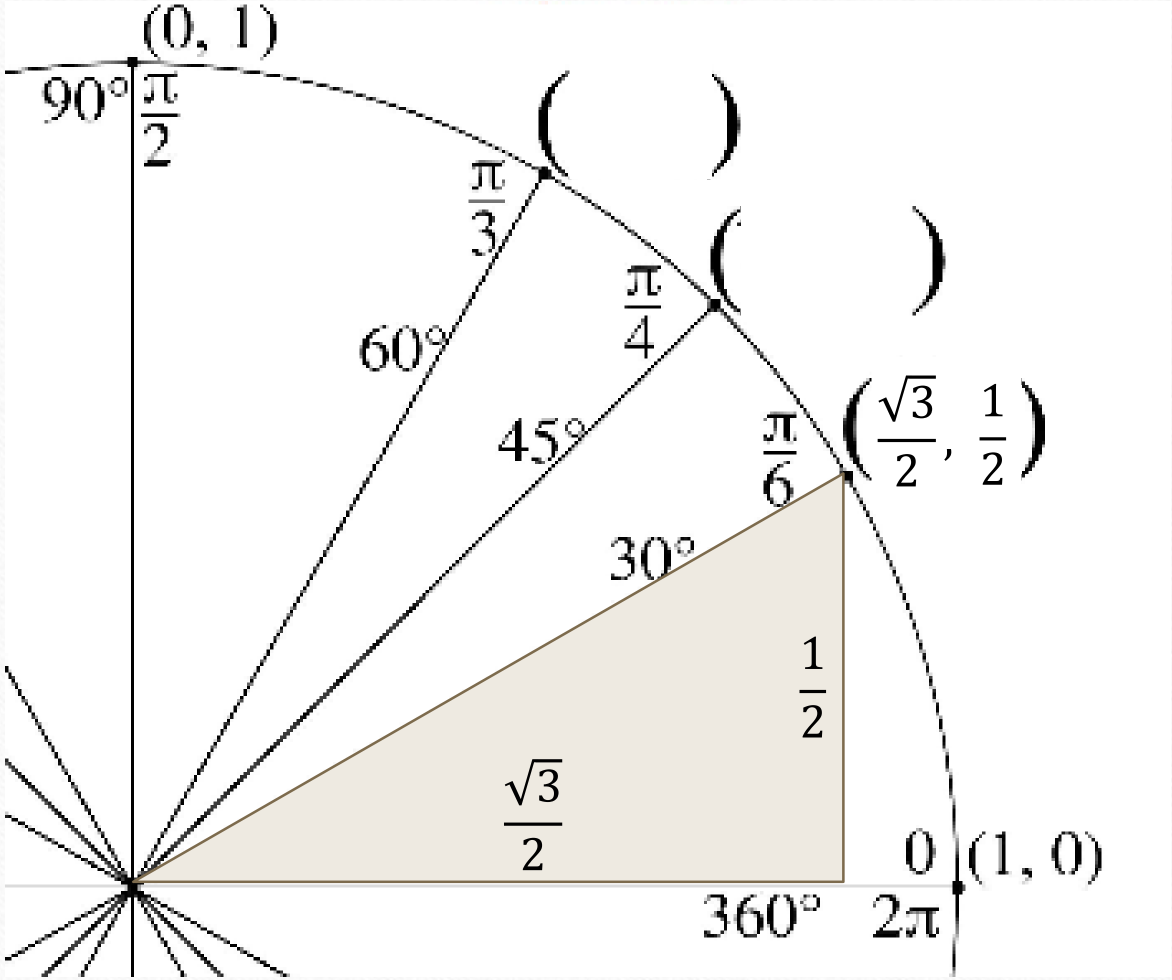
Convert $60°$ to radians.

$$60\left(\frac{\pi}{180}\right) = \frac{\pi}{3}$$

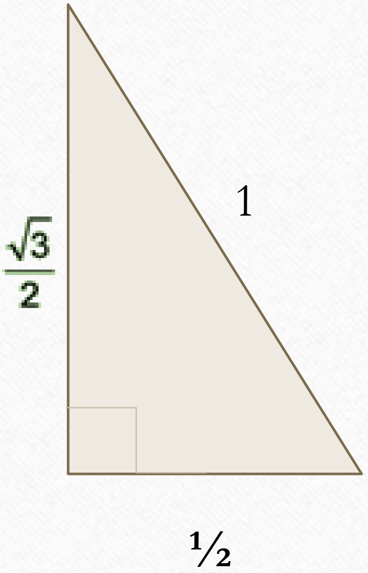
Convert $\frac{5\pi}{3}$ radians to degrees.

$$\frac{5\pi}{3}\left(\frac{180}{\pi}\right) = 300$$

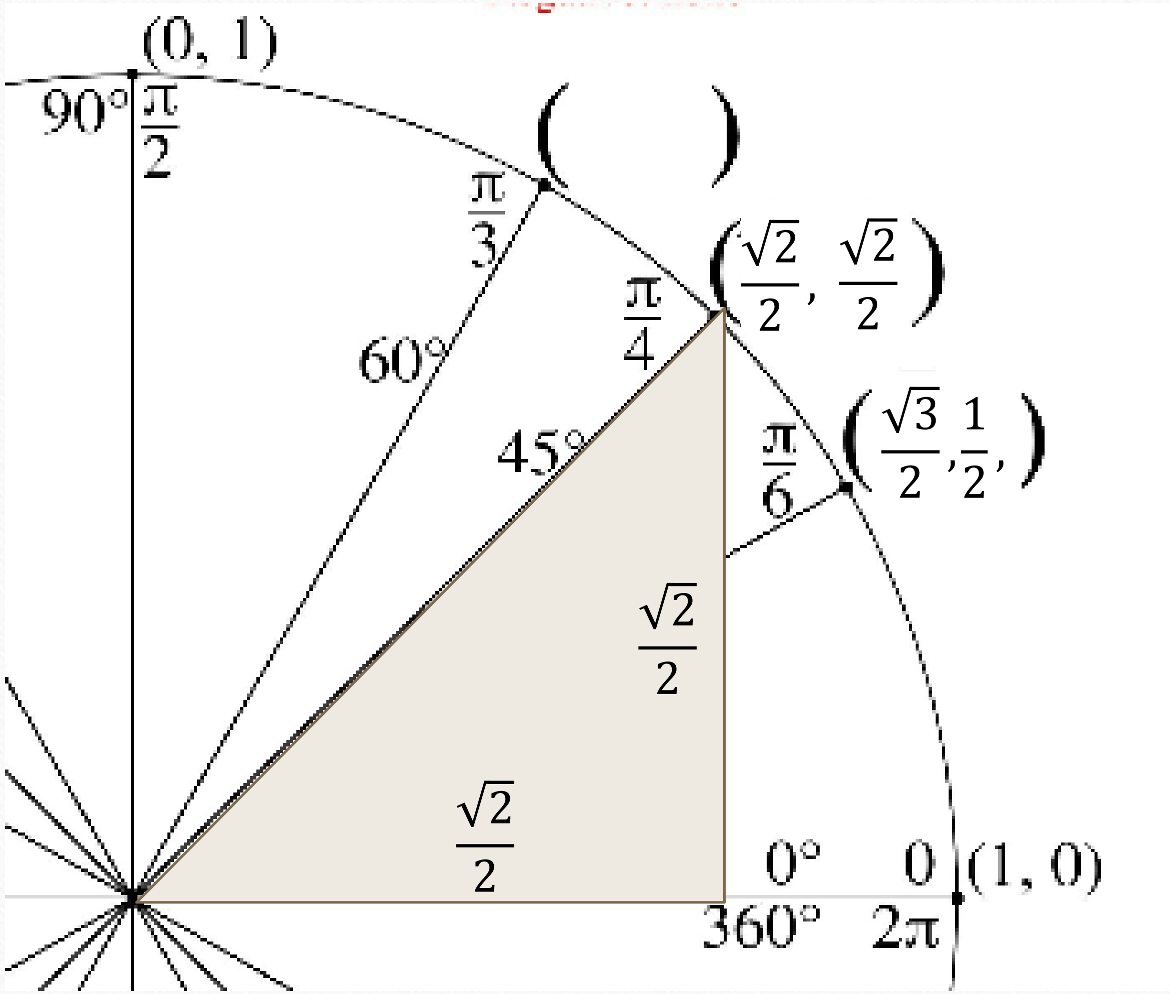
Sine and Cosine on the Unit Circle



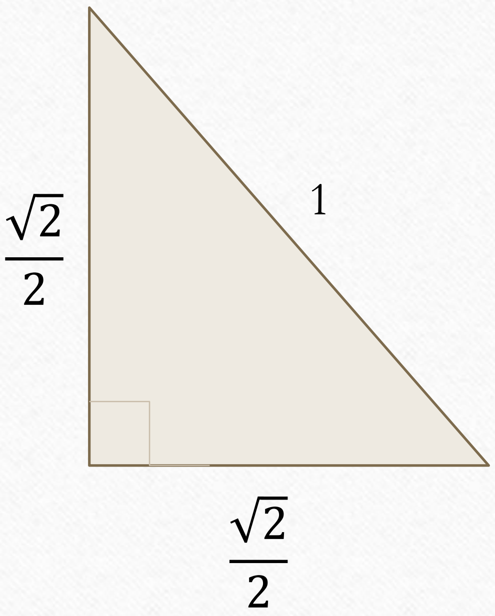
Remember the
30-60-90 Triangle



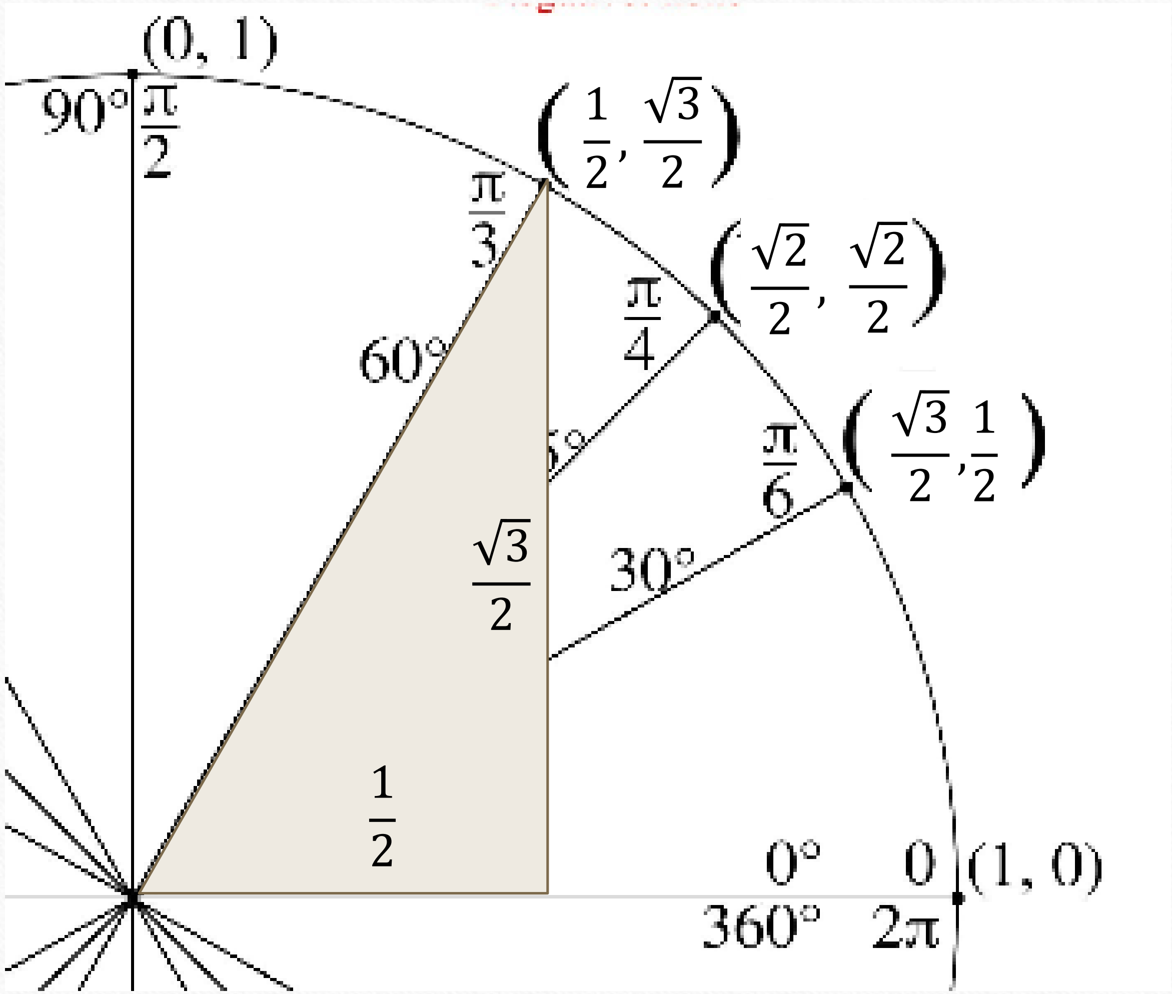
Sine and Cosine on the Unit Circle



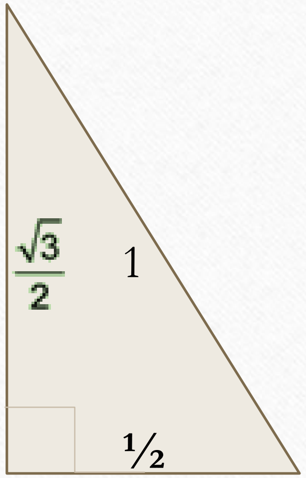
Remember the
45-45-90 Triangle



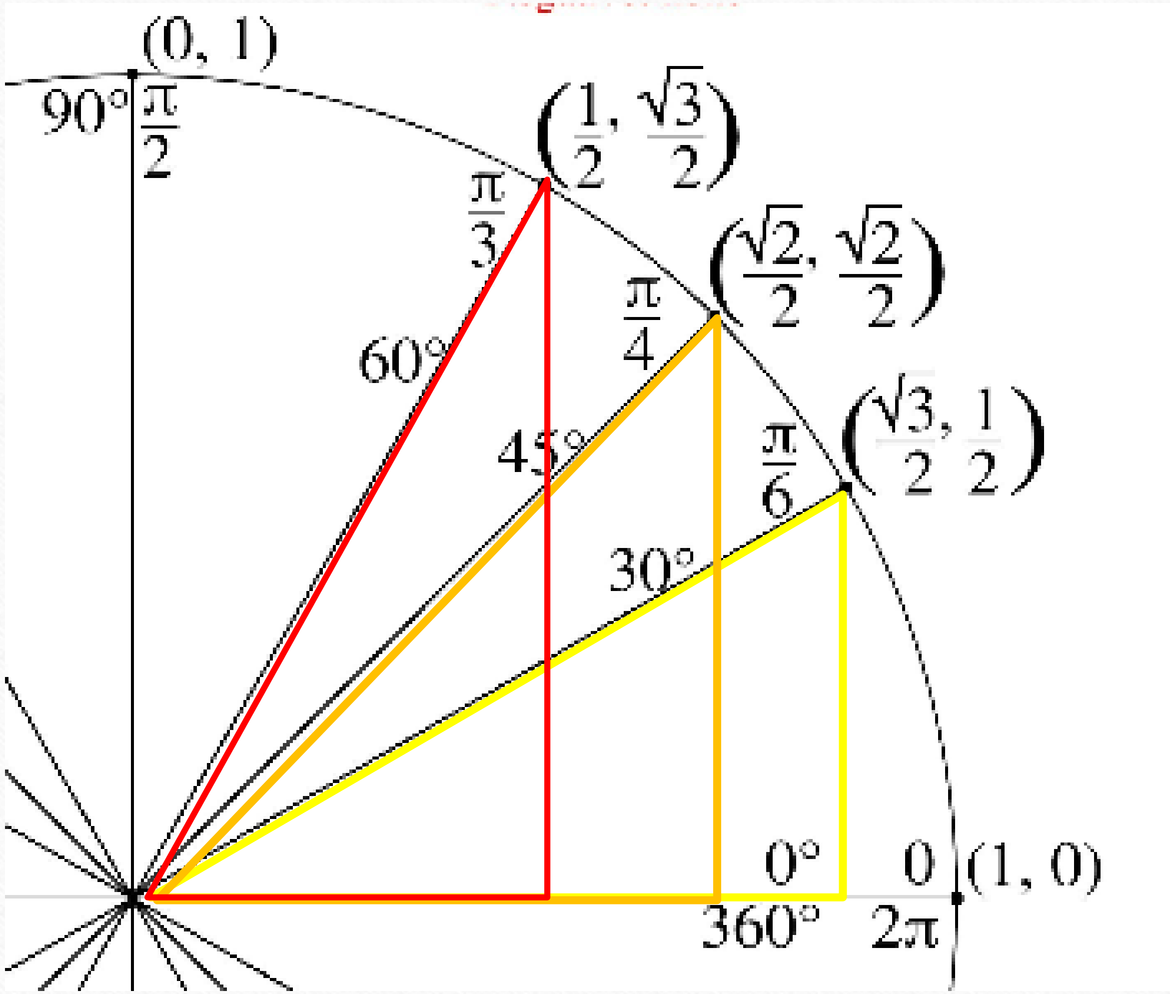
Sine and Cosine on the Unit Circle



Remember the
30-60-90 Triangle



Sine and Cosine on the Unit Circle



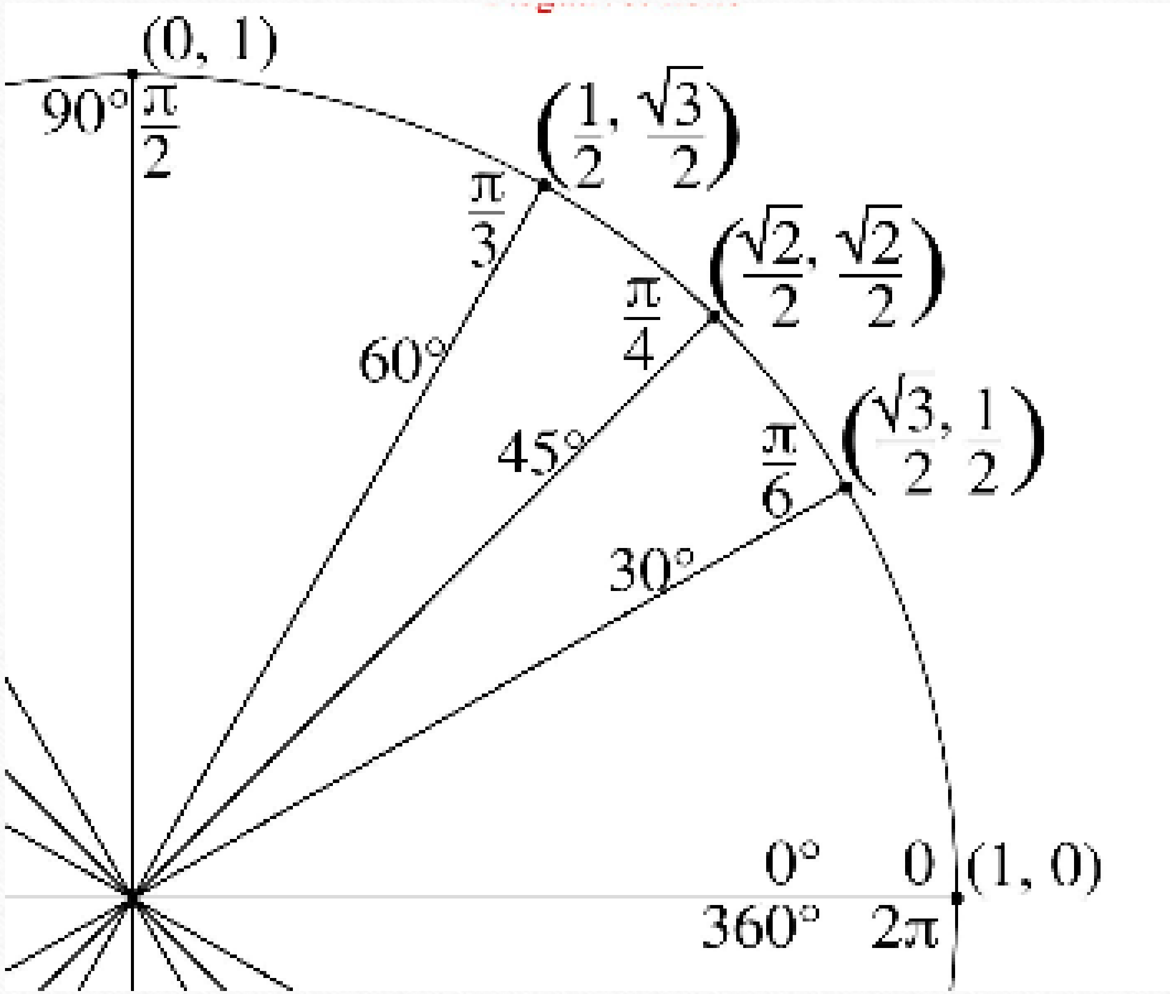
Remember the radius on the unit circle is equal to 1.

Therefore the **cosine** of any angle on the unit circle is equal to the **x coordiante** of the point on the circle.

Sine is equal to the **y coordinate** of the point on the circle.

(cos,sin)

Sine and Cosine on the Unit Circle



Find the cosine of 60°

$$\cos(60) = \frac{1}{2}$$

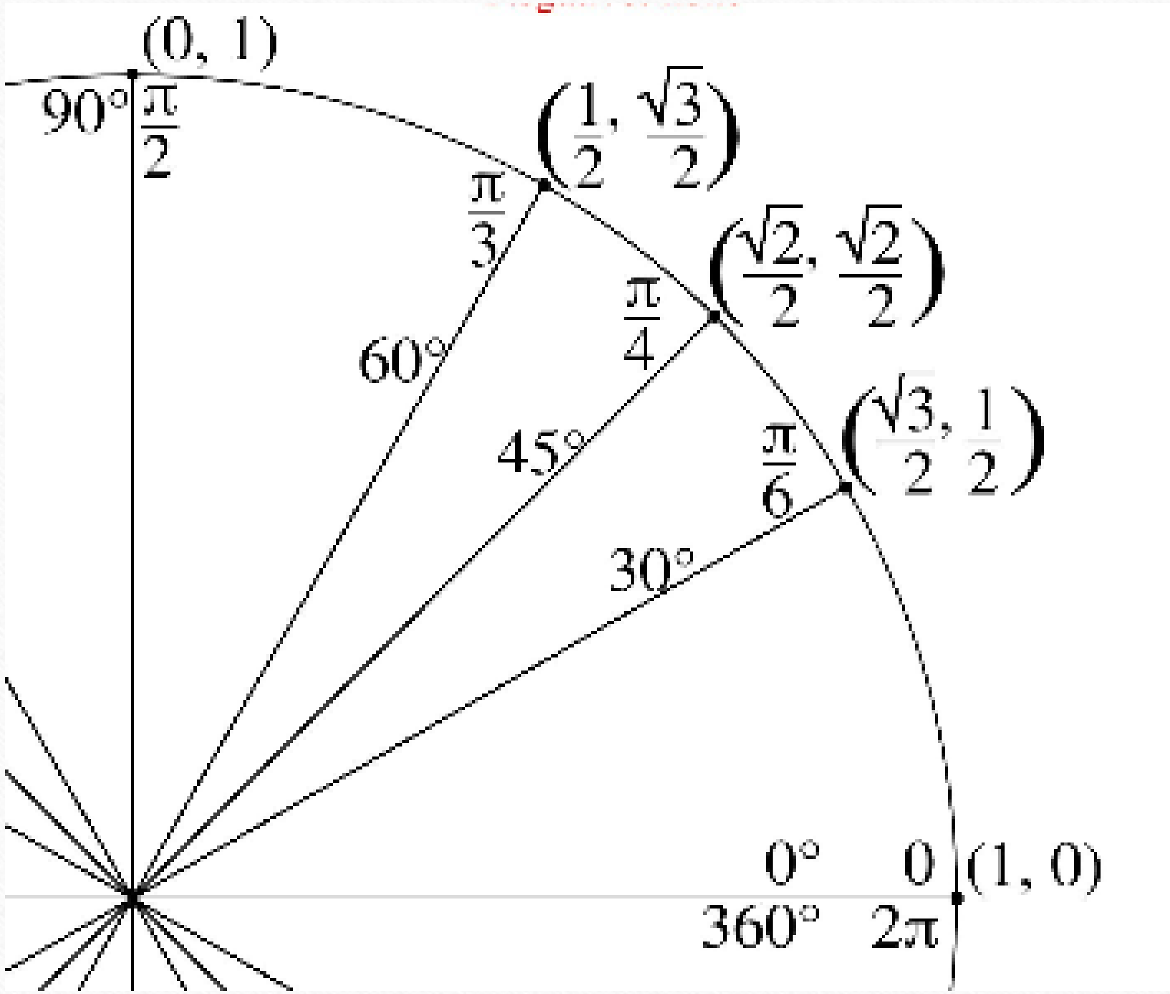
Find the sine of 60°

$$\sin(60) = \frac{\sqrt{3}}{2}$$

Find the sine of $\frac{\pi}{4}$

$$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

TANGENT on the Unit Circle



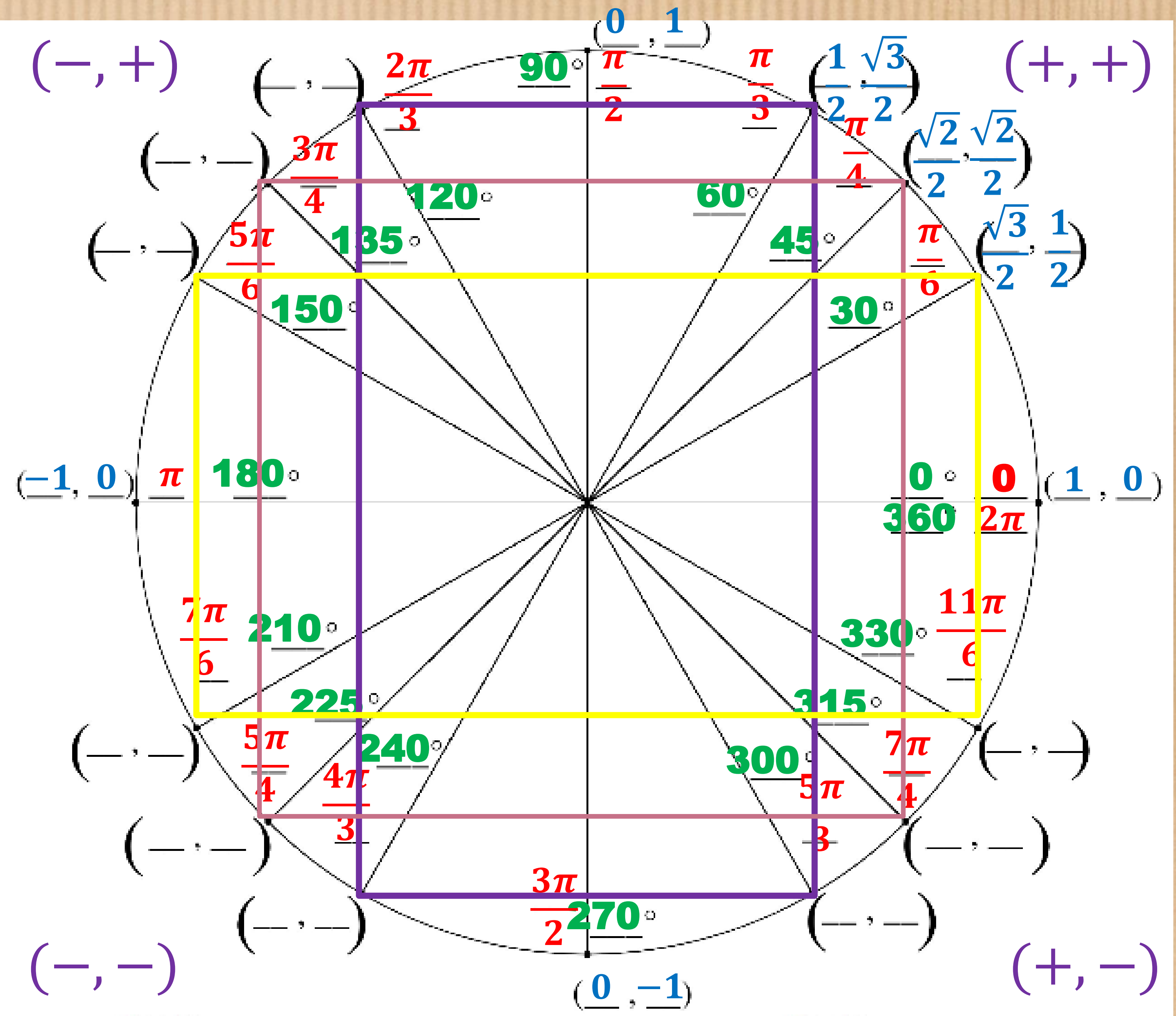
$$\text{Tan} = \frac{Sin}{Cos}$$

Find the tangent of 45°

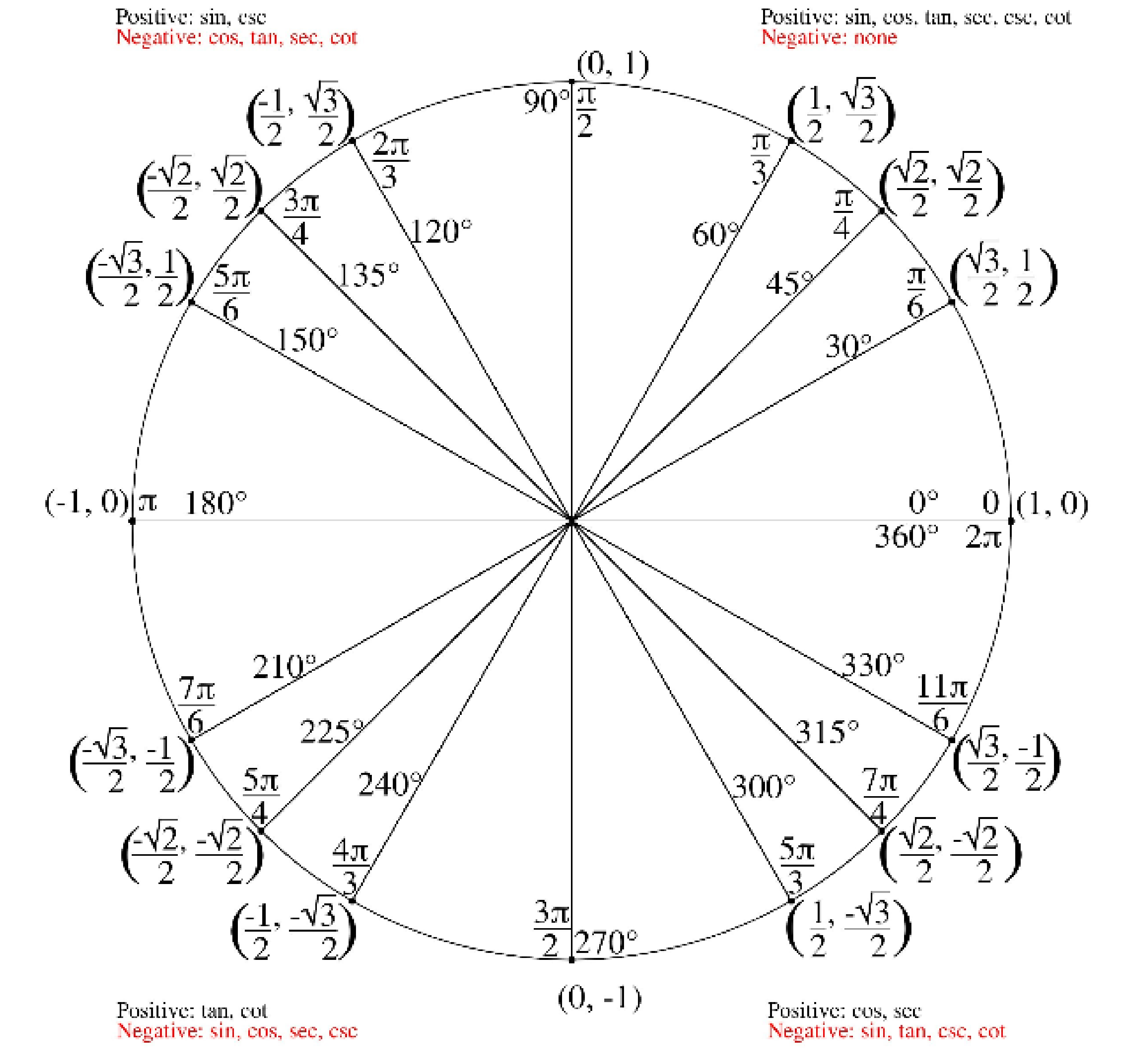
$$\tan 45 = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

Use patterns to fill in the rest.

Follow the boxes. The coordinates of the points are the same. Just change the signs to match the quadrant.



Your completed unit circle should look like this…



Use your unit circle to find the following

1) $\sin(90^\circ) =$

2) $\cos\left(\frac{\pi}{4}\right) =$

3) $\sin\left(\frac{5\pi}{4}\right) =$

4) $\cos\ 135^\circ\ =$

5) $\tan\left(\frac{5\pi}{4}\right) =$

6) $\tan(180^\circ) =$

Finish working on the worksheet