Warm-up

Sunday, March 1, 2015

Solve the following quadratic. Use any method you like.

$1.\ f(x) = x^2 - 4x - 21$

Divide using synthetic division.

$2.\ (x^3 - 2x^2 - 14x - 5) \div (x + 3)$

$3.\ (x^4 - 3x^2 + 2x + 1) \div (x + 2)$

Objectives

Use the remainder Theorem to find the remainder of a division problem without having to perform any division.

Use the remainder Theorem to determine if a linear binomial is a factor of a given polynomial.

Homework

Packet Page 8; 1-5 all
Packet Page 9; Left column only, 1-5 all

Check your homework

Exercises

What is the quotient and remainder of the following polynomials?

11. $(x^3 - 2x + 8) \div (x + 2)$
$x^2 - 2x + 2$, R 4

12. $(12x^3 - 71x^2 + 57x - 10) \div (x - 5)$
$12x^2 - 11x + 2$, R 0

13. $(3x^4 + x^3 - 6x^2 - 9x + 12) \div (x + 1)$
$3x^3 - 2x^2 - 4x - 5$, R 17

14. $(2x^3 - 15x + 23) \div (x - 2)$
$2x^2 + 4x - 7$, R 9

15. $(x^3 + x + 10) \div (x + 2)$
$x^2 - 2x + 5$, R 0

16. $(x^4 - 12x^3 - 18x^2 + 10) \div (x + 4)$
$x^3 - 16x^2 + 46x - 184$, R 746

Surprise Quiz!

Divide using synthetic division.

1. $(8x^3 - 55x^2 + 44x - 12) \div (x - 6)$

2. $(x^3 - 9n - 3) \div (n + 3)$

Use synthetic division to determine if $x - 3$ is a factor of $f(x) = 6x^3 - 5x^2 + 4x - 17$

3	**6**	**-5**	**4**	**-17**
		18	**39**	**129**
	6	**13**	**43**	**112**

Now just for grins evaluate $f(3)$.

$$f(3) = 6(3)^3 - 5(3)^2 + 4(3) - 17 \ \color{red}{\mathbf{=112}}$$

One more polynomial tool for you

The Remainder Theorem

If $f(x)$ is a polynomial in x then the remainder when dividing $f(x)$ by $x - a$ is $f(a)$

So what the heck does that mean?

I can do a couple of things without even having to do any division!

Example 1 pp8

Find the remainder of $(3x^3 + 4x^2 - 5x + 3) \div (x + 2)$

Evaluate the divisor at $x = -2.$

$$3(-2)^3 + 4(-2)^2 - 5(-2) + 3 = 5$$

Check by synthetic division

-2	**3**	**4**	**-5**	**3**
		-6	**4**	**2**
	3	**-2**	**-1**	**5**

Example 2 pp8

Find the remainder of $(2x^3 - 5x^2 + x - 3) \div (x - 1)$

Evaluate the divisor at $x = 1$.

$$2(1)^3 - 5(1)^2 + (1) - 3 = -5$$

Check by synthetic division

1	**2**	**-5**	**1**	**-3**
		2	**-3**	**-2**
	2	**-3**	**-2**	**-5**

You do exercises 1-5 on pp8.

Check 1

Find the remainder of $(x^3 - 5x^2 + 6x - 4) \div (x - 2)$

Evaluate the divisor at $x = 2$.

$$2^3 - 5(2)^2 + 6(2) - 4 = -4$$

Check by synthetic division

2	**1**	**-5**	**6**	**-4**
		2	**-6**	**0**
	1	**-3**	**0**	**-4**

We can also use the Remainder Theorem to find factors of polynomials.

Remember: When we divide a polynomial by a **factor** the remainder is always **zero**.

Is $(x - 3)$ a factor of $2x^3 - 3x^2 - 8x - 3$?

Method 1: Check by synthetic division

3	**2**	**-3**	**-8**	**-3**
		6	**9**	**3**
	2	**3**	**1**	**0**

Method 2: Remainder Theorem

$$2(3)^3 - 3(3)^2 - 8(3) - 3 = 0$$

You do exercises 1-5 on pp9. Left side only.

Factoring Cubic Functions, an introduction to finding ALL roots.

If $(x+1)$ is a factor of $x^3+6x^2+11x+6$, what are the remaining factors?

Some things to think about…

How many factors will this cubic have in total?

Three.

What should I do first?

Divide out the given factor.

What will be the degree of the quotient after dividing out the first factor?

2, a quadratic!

How can I find the remaining factors?

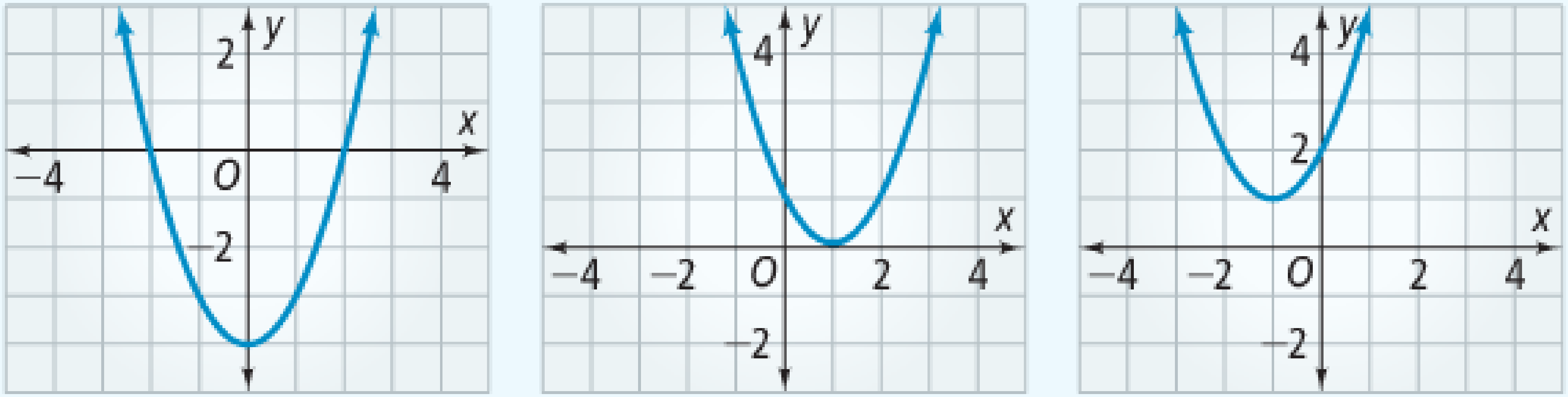
AC method, Quadratic Formula, Graphing! Oh My!

If $(x + 1)$ is a factor of $x^3 + 6x^2 + 11x + 6$, what are the remaining factors?

Did you get $(x + 1), (x + 2)\ and\ (x + 3)$?

Essential Understanding The degree of a polynomial equation tells you how many roots the equation has.

It is easy to see graphically that every polynomial function of degree 1 has a single zero, the *x*-intercept. However, there appear to be three possibilities for polynomials of degree 2. They correspond to these three graphs:



$y = x^2 - 4$	$y = x^2 - 2x + 1$	$y = x^2 + 2x + 2$
Two real zeros	One real zero	No real zeros

take note

Theorem **The Fundamental Theorem of Algebra**

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots, including multiple and complex roots.

So **$p(x) = x^3 + 4x^2 - 2$** has **3 roots**

So **$f(x) = x^4 + 3x^2 - 7$** has **4 roots**

So **$g(x) = 7x^{102} + 43x^{27} - x$** has **102 roots**

Let's play how many roots?

Show me with your fingers…

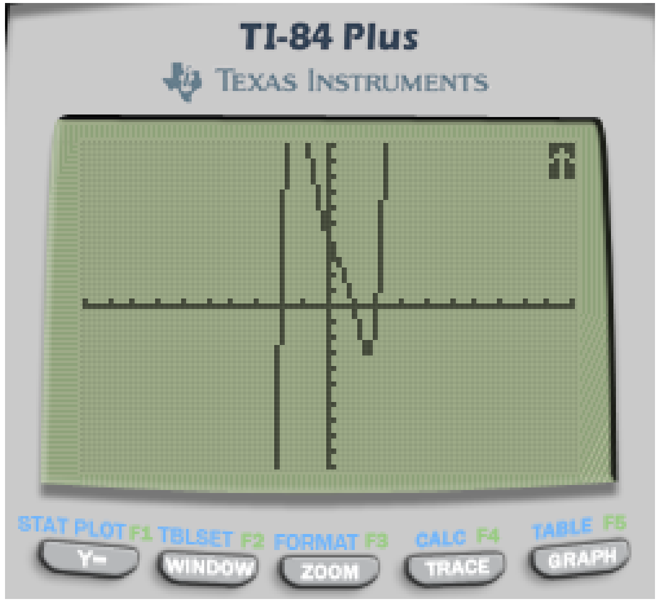
$f(x) = x^2 + 2$

$f(x) = 7x^5 + 4x^4 + 3x - 3$

$f(x) = x^2 + x^6 - 2$

So how do we find all these roots?

Find all the roots of $x^5 - x^4 - 3x^3 + 3x^2 - 4x + 4 = 0$



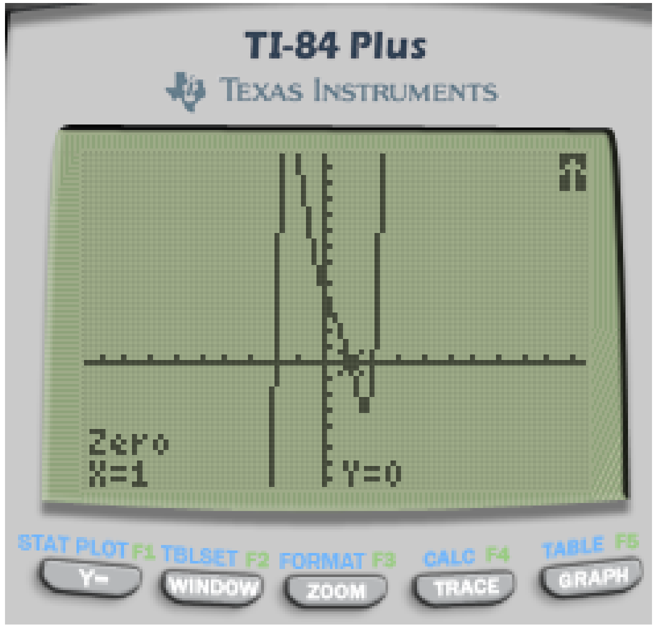
Enter the equation into your calculator and graph.

How many roots/zeros do you see?

According to the FTA, how many roots are there?

Use your calculator to find the three roots.

You should get x = -2, x = 1 and x = 2



But what about the other 2 roots?