Tuesday, October 7, 2014

Factor the following expressions completely

1. $2m^2 + 46m - 48$
2. $3y^2 - 20y + 12$
3. $8x^3 + 125$

10 9 8 7 6 5 4 3 2 1

Objectives Simplify rational expressions

Multiply and Divide rational expressions

Identify any restrictions that apply to a given rational expression

Homework Workbook page 223, 1-23 odd

Homework…

So what is a rational function?

First let's answer the question "What is a rational number?"

A rational number is any number that can be written as a fraction.

For example 1.5 is a rational number because $1.5 = \frac{3}{2}$.

Which means 1.5 can be written as the ratio 3/2.

More examples…

Number	As a Fraction
5	5/1
1.75	7/4
.001	1/1000
-0.1	-1/10
0.111...	1/9

So what is a rational function?

A rational function is a function that can be written as the ratio of two polynomial functions.

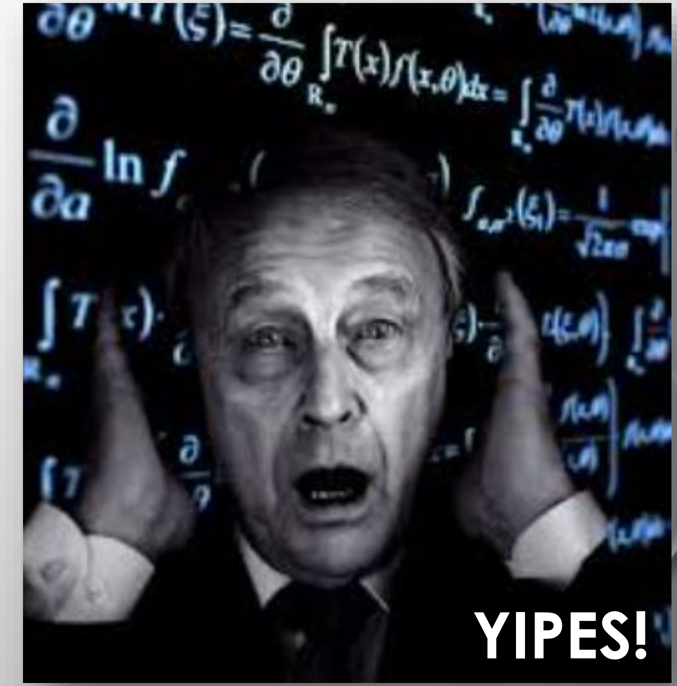
Some examples

$$f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$$

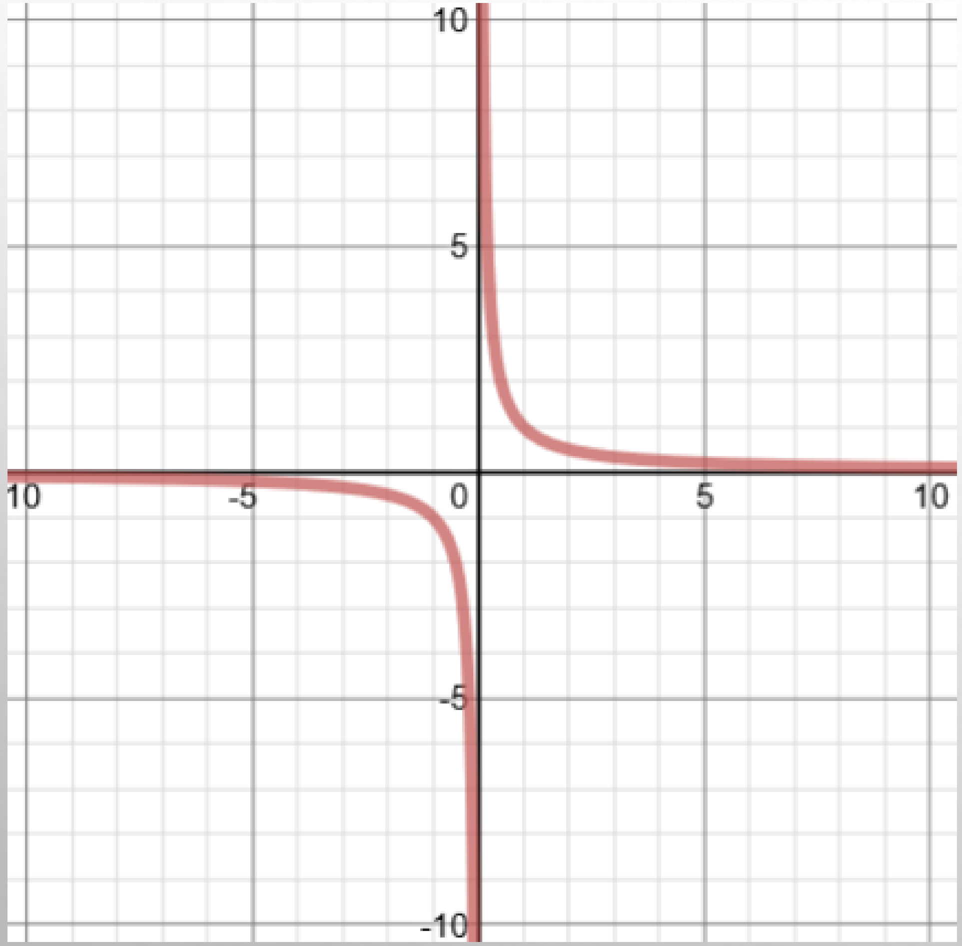
$$f(x) = \frac{1}{3x^2 + 3x - 18}$$

$$f(x) = \frac{x-2}{x-4}$$

$$f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$$



The parent function of **rational functions** is the inverse function



$$f(x) = \frac{1}{x}$$

We can't divide by zero. So x can never equal zero. Therefore there is a **vertical asymptote** at the line $\boldsymbol{x = 0}$

Since there is no number by which we can divide 1 and get zero, there will never be a function value of zero.

The bigger the x value we have the smaller function value we create. Therefore there is a **horizontal asymptote** at the line $\boldsymbol{y = 0}$

Not as bad as they look

We just have to remember a few things…

Factoring

GCF
AC Method
Swing Method

And what's the one thing you can't do with a fraction?

Really bad things happen when you…

DIVIDE BY ZERO!

Simplifying Fractions

Remember how we simplify regular fractions?

$$\mathbf{\frac{35}{45} = \frac{(7)(\cancel{5})}{(9)(\cancel{5})} = \frac{7}{9}}$$

1. Factor
2. Cancel
3. Simplify

We do the same thing with rational functions.

$$\frac{35x + 70}{35x}$$

1. Factor
2. Cancel

$$= \frac{\not{35}(x + 2)}{\not{35}x}$$

3. Simplify

$$= \frac{x + 2}{x}$$

A more complicated example.

$$\frac{x^2 + 10x + 16}{x^2 + 6x + 8}$$

1. Factor
2. Cancel

$$= \frac{(x+8)\cancel{(x+2)}}{(x+4)\cancel{(x+2)}}$$

3. Simplify

$$= \frac{(x+8)}{(x+4)}$$

Simplifying Fractions
You try, page 223

6. $\frac{3x^2-12}{x^2-x-6}$

1. Factor $=\frac{3(x^2-4)}{(x-3)(x+2)}$

2. Cancel $=\frac{3(x-2)\cancel{(x+2)}}{(x-3)\cancel{(x+2)}}$

3. Simplify $=\frac{3(x-2)}{(x-3)}$

8. $\frac{x^2+13x+40}{x^2-2x-35}$

1. Factor
2. Cancel

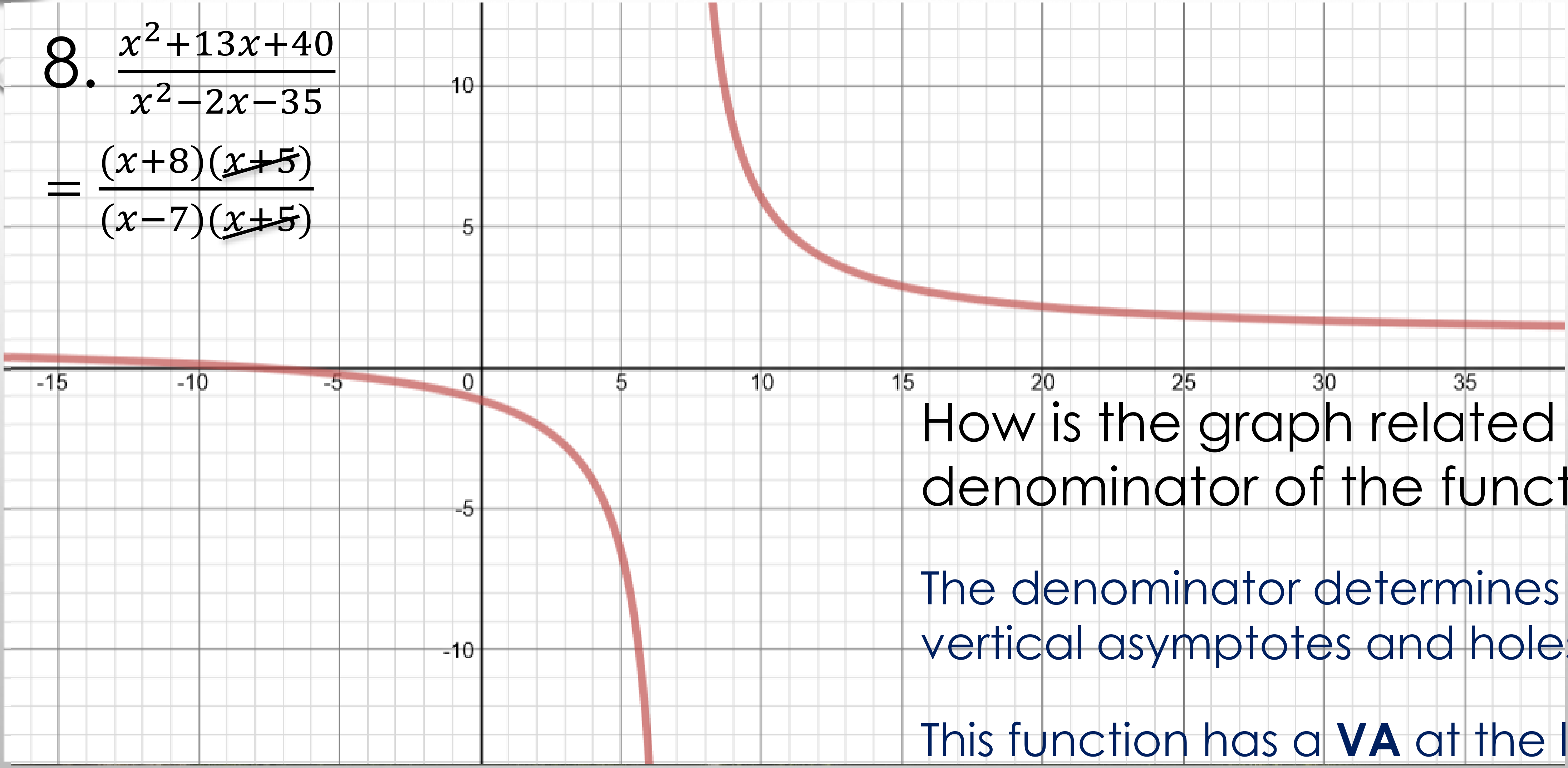
$=\frac{(x+8)\cancel{(x+5)}}{(x-7)\cancel{(x+5)}}$

3. Simplify $=\frac{(x+8)}{(x-7)}$

Let's look at the graph of the example we did.

8. $\frac{x^2+13x+40}{x^2-2x-35}$

$= \frac{(x+8)\cancel{(x+5)}}{(x-7)\cancel{(x+5)}}$



How is the graph related to the denominator of the function?

The denominator determines where vertical asymptotes and holes occur.

This function has a **VA** at the line $x = 7$ and a **hole** at $x = 5$.

8. $\frac{x^2+13x+40}{x^2-2x-35}$

$= \frac{(x+8)\cancel{(x+5)}}{(x-7)\cancel{(x+5)}}$

Restrictions:

$x \neq 7, x \neq -5$

We restrict the domain of rational functions by excluding x values that will cause the original denominator to be equal to zero.

- Look at the factored denominator.
- Set each factor equal to zero and solve for x.
- Your restrictions are these values of x.

State the restrictions on x.

6. $\frac{3x^2-12}{x^2-x-6}$

$=\frac{3(x^2-4)}{(x-3)(x+2)}$

$=\frac{3(x-2)\cancel{(x+2)}}{(x-3)\cancel{(x+2)}}$

$=\frac{3(x-2)}{(x-3)}$

Restrictions:
$\boldsymbol{x \neq 3, x \neq -2}$

4. $\frac{7x-28}{x^2-16}$

$=\frac{7(x-4)}{(x+4)(x-4)}$

$=\frac{7}{(x+4)}$

Restrictions:
$\boldsymbol{x \neq 4, x \neq -4}$

2. $\frac{2y}{y^2+6y}$

$=\frac{2y}{y(y+6)}$

$=\frac{2}{(y+6)}$

Restrictions:
$\boldsymbol{x \neq 0, x \neq -6}$

Remember how we multiply fractions?

$$\frac{3}{4} \times \frac{5}{2}$$

$$\frac{35}{21} \times \frac{120}{40} = \frac{5 \times 7}{3 \times 7} \times \frac{3 \times 4 \times 2 \times 5}{4 \times 2 \times 5}$$

$$= \frac{\cancel{5} \times \cancel{7} \times \cancel{3} \times \cancel{4} \times \cancel{2} \times 5}{\cancel{3} \times \cancel{7} \times \cancel{4} \times \cancel{2} \times \cancel{5}}$$

$$= 5$$

We do the same thing when we multiply rational functions.

Multiplication

10. $\frac{2x+4}{10x} \cdot \frac{15x^2}{x+2}$

$= \frac{\cancel{2}\cancel{(x+2)}}{\cancel{(2)}\cancel{(5)}\cancel{x}} \cdot \frac{(3)\cancel{(5)}\cancel{(x)}(x)}{\cancel{x+2}}$

$= 3x$

Restrictions:

$x \neq 0 \; and \; x \neq -2$

Fully factor all numerators and denominators.

Cancel common factors.

Simplify into one expression

State restrictions

Multiplication

$$14.\ \frac{x-2}{(x+2)^2} \cdot \frac{x+2}{2x-4} = \frac{x-2}{(x+2)(x+2)} \cdot \frac{x+2}{2(x-2)}$$

Fully factor all numerators and denominators.

Cancel common factors.

$$= \frac{1}{2(x+2)}$$

Simplify into one fraction

State restrictions

Restrictions:

$x \neq 2\ and\ x \neq -2$

Multiplication

You try

$$16.\ \frac{y^2-2y}{y^2+7y-18}\cdot\frac{y^2-81}{y^2-11y+18}=\frac{y\cancel{(y-2)}}{\cancel{(y+9)}\cancel{(y-2)}}\cdot\frac{\cancel{(y-9)}\cancel{(y+9)}}{\cancel{(y-9)}(y-2)}$$

$$=\frac{y}{y-2}$$

Restrictions:

$x\neq -9, x\neq 2, x\neq 9$

Remember how we divide fractions?

$$\frac{3}{4} \div \frac{5}{2} = \frac{3}{4} \times \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$$

KCF

Not this guy!

Keep the first fraction
Change to multiply
Flip the second fraction

Yep, we do the same thing with rational functions.

Division

18. $\dfrac{6x+6}{7} \div \dfrac{4x+4}{x-2}$

$$= \frac{6x+6}{7} \times \frac{x-2}{4x+4}$$

$$= \frac{6(x+1)}{7} \times \frac{x-2}{4(x+1)}$$

$$= \frac{\cancel{(2)}(3)\cancel{(x+1)}}{7} \times \frac{x-2}{\cancel{(2)}(2)\cancel{(x+1)}}$$

$$= \frac{3(x-2)}{14}$$

Restrictions:

$x \neq 2, x \neq -1$

Division

You Try!

22. $\dfrac{x^2+10x+16}{x^2-6x-16} \div \dfrac{x+8}{x^2-64}$

$$= \frac{x^2+10x+16}{x^2-6x-16} \times \frac{x^2-64}{x+8}$$

$$= \frac{(x+2)(x+8)}{(x-8)(x+2)} \times \frac{(x+8)(x-8)}{x+8}$$

$$= (x+8)$$

Restrictions:

$x \neq -2, x \neq 8, x \neq -8$

WORK ON YOUR WORKSHEET