Warm-up

Friday, February 27, 2015

1. Use long division (no calculator) to find the answer to $672 \div 21$

2. What does your answer tell you about 21 and 672?

3. Factor the quadratic $x^2 + 14x + 49$

Objectives

Use Synthetic Division to divide one polynomial into another polynomial.

Determine if a polynomial is a factor of another polynomial

Homework

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A little Polynomial Review

take note

Key Concept **Standard Form of a Polynomial Function**

The **standard form of a polynomial function** arranges the terms by degree in descending numerical order.

A polynomial function $P(x)$ in standard form is

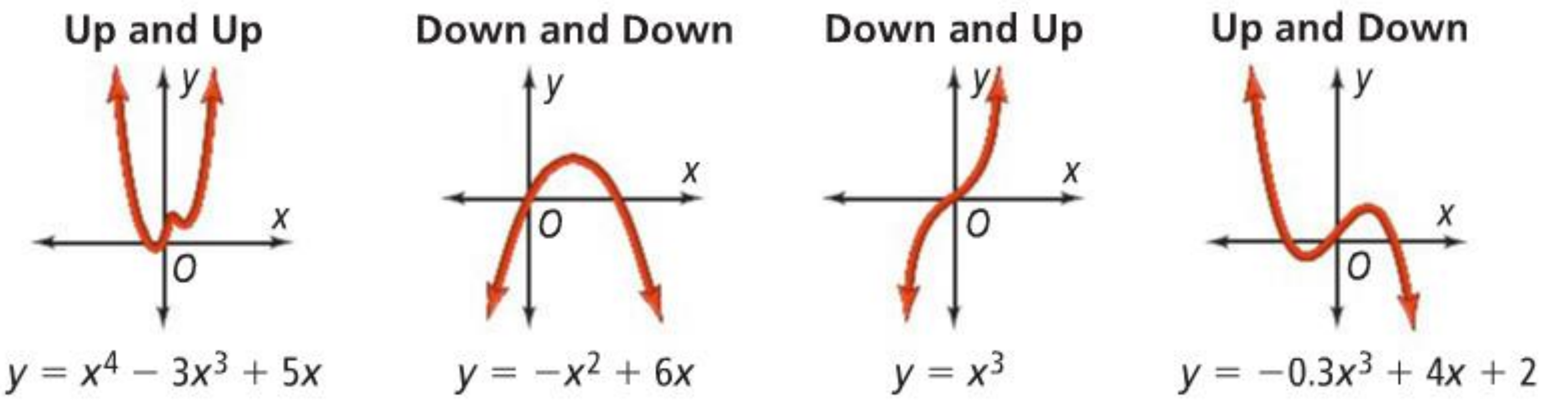
$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where n is a nonnegative integer and $a_n, \ldots, a_0$ are real numbers.

$$P(x) = 4x^3 + 3x^2 + 5x - 2$$

Cubic term | Quadratic term | Linear term | Constant term

The degree of a polynomial function affects the shape of its graph and determines the maximum number of **turning points**, or places where the graph changes direction. It also affects the **end behavior**, or the directions of the graph to the far left and to the far right.



End Behavior of a Polynomial Function of Degree *n* With Leading Term ax^n (Moving Away From the Origin)

	n Even	*n* Odd
a Positive	Up and Up	Down and Up
a Negative	Down and Down	Up and Down

If the highest degree is even, both ends point in the same direction. If the highest degree is odd, the ends of the graph point in opposite directions.

Look at 5-1 Polynomial Functions.

What is the classification of each polynomial by its degree? by its number of terms? What is its end behavior?

1. $8 - 6x^3 + 3x + x^3 - 2$

2. $15x^7 - 7$

3. $2x - 6x^2 - 9$

You've see most of these before…

take note

Concept Summary Polynomial Factoring Techniques

Techniques	Examples
Factoring out the GCF Factor out the greatest common factor of all the terms.	$15x^4 - 20x^3 + 35x^2$ $= 5x^2(3x^2 - 4x + 7)$
Quadratic Trinomials For $ax^2 + bx + c$, find factors with product ac and sum b.	$6x^2 + 11x - 10$ $= (3x - 2)(2x + 5)$
Perfect Square Trinomials $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 10x + 25 = (x + 5)^2$ $x^2 - 10x + 25 = (x - 5)^2$
Difference of Squares $a^2 - b^2 = (a + b)(a - b)$	$4x^2 - 15 = (2x + \sqrt{15})(2x - \sqrt{15})$
Factoring by Grouping $ax + ay + bx + by$ $= a(x + y) + b(x + y)$ $= (a + b)(x + y)$	$x^3 + 2x^2 - 3x - 6$ $= x^2(x + 2) + (-3)(x + 2)$ $= (x^2 - 3)(x + 2)$
Sum or Difference of Cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$8x^3 + 1 = (2x + 1)(4x^2 - 2x + 1)$ $8x^3 - 1 = (2x - 1)(4x^2 + 2x + 1)$

Problem 1 Writing a Polynomial in Factored Form

Plan

How do you write the factored form of a polynomial?
Write the polynomial as a product of factors. Make sure each factor cannot be factored any further.

What is the factored form of $x^3 - 2x^2 - 15x$?

$x^3 - 2x^2 - 15x = x(x^2 - 2x - 15)$ Factor out the GCF, x.

$= x(x - 5)(x + 3)$ Factor $x^2 - 2x - 15$.

Check $x(x - 5)(x + 3) = x(x^2 - 2x - 15)$ Multiply $(x - 5)(x + 3)$.

$= x^3 - 2x^2 - 15x$ ✔ Distributive Property

What is the factored from of $x^3 - x^2 - 12x$?

$$x^3 - x^2 - 12x = x\,(x+3)\,(x-4)$$

Remember the warm up?

Numerical long division and polynomial long division are similar.

Numerical Long Division		Polynomial Long Division	
32		$3x + 2$	
$21\overline{)672}$	21 divides into	$2x + 1\overline{)6x^2 + 7x + 2}$	$(2x + 1)$ divides into
$\underline{63}$	67 3 times	$\underline{6x^2 + 3x}$	$(6x^2 + 7x)$ $3x$ times
42	21 divides into	$4x + 2$	$(2x + 1)$ divides into
$\underline{42}$	42 2 times	$\underline{4x + 2}$	$(4x + 2)$ 2 times
0		0	

The remainder from each division above is 0, so 21 is a factor of 672 and $2x + 1$ is a factor of $6x^2 + 7x + 2$.

Exercises

Divide using polynomial long division.

1. $(3x^2 - 8x + 7) \div (x - 1)$

3. $(x^2 + 3x - 8) \div (x - 5)$

SYNTHETIC division is a nice shortcut for dividing polynomials!

Synthetic division was first modeled in the early 1800s by the Italian mathematician, Paolo Ruffini. This process was created to more efficiently perform long division between polynomials. Synthetic division is a form of shorthand mathematics, which allows you to work solely with the coefficients without having to worry about the variables. You can find more information here: http://www.purplemath.com/modules/synthdiv.htm

But…there's some fine print.

It only works when you divide by a linear factor. **(Degree of 1)**

AND you divide by the zero!

Use **synthetic** division to divide **$x^3 + 13x^2 + 46x + 48$** by **$x + 3$.** What is the **quotient** and what is the **remainder**?

This is example on page 7 of your packet.

Use **synthetic** division to divide **$x^3 + 3x^2 - 15$** by **$x + 5$.** What is the **quotient** and what is the **remainder**?

Use **synthetic** division to divide **$x^3 + x^2 - 10x + 8$** by **$x - 1$.** What is the **quotient** and what is the **remainder**?

Look at 5-4 Dividing Polynomials page 40.

Use synthetic division to find the quotient and remainder.

11. $(x^3 - 2x + 8) \div (x + 2)$

13. $(3x^4 + x^3 - 6x^2 - 9x + 12) \div (x + 1)$

To Turn in…

Use **synthetic** division to divide **$2x^4 + 23x^3 + 60x^2 - 125x - 500$** by **$x + 4$.** What is the **quotient** and what is the **remainder**?