Polynomials

What does it mean to factor?

take note

Concept Summary Polynomial Factoring Techniques

Techniques	Examples
Factoring out the GCF Factor out the greatest common factor of all the terms.	$15x^4 - 20x^3 + 35x^2$ $= 5x^2(3x^2 - 4x + 7)$
Quadratic Trinomials For $ax^2 + bx + c$, find factors with product ac and sum b.	$6x^2 + 11x - 10$ $= (3x - 2)(2x + 5)$
Perfect Square Trinomials $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 10x + 25 = (x + 5)^2$ $x^2 - 10x + 25 = (x - 5)^2$
Difference of Squares $a^2 - b^2 = (a + b)(a - b)$	$4x^2 - 15 = (2x + \sqrt{15})(2x - \sqrt{15})$
Factoring by Grouping $ax + ay + bx + by$ $= a(x + y) + b(x + y)$ $= (a + b)(x + y)$	$x^3 + 2x^2 - 3x - 6$ $= x^2(x + 2) + (-3)(x + 2)$ $= (x^2 - 3)(x + 2)$
Sum or Difference of Cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$8x^3 + 1 = (2x + 1)(4x^2 - 2x + 1)$ $8x^3 - 1 = (2x - 1)(4x^2 + 2x + 1)$

When we factor, we're **dividing** the polynomial by what we are "factoring out"

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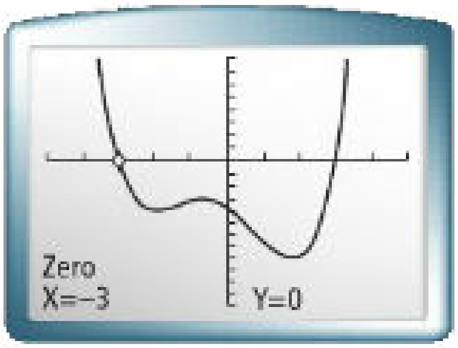
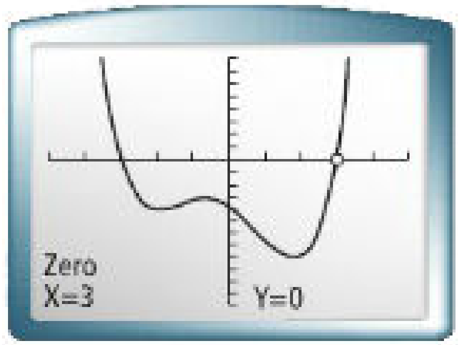
Polynomials

Finding All Zeros

What are the zeros of $f(x) = x^4 + x^3 - 7x^2 - 9x - 18$?

Step 1 Use a graphing calculator to find any real roots. The graph of $y = x^4 + x^3 - 7x^2 - 9x - 18$ shows real zeros at $x = -3$ and $x = 3$.





Step 2 Factor out the linear factors $x + 3$ and $x - 3$. Use synthetic division twice.

-3	1	1	-7	-9	-18
		-3	6	3	18
	1	-2	-1	-6	0

3	1	-2	-1	-6
		3	3	6
	1	1	2	0

$$x^4 + x^3 - 7x^2 - 9x - 18 = (x + 3)(x^3 - 2x^2 - x - 6)$$
$$= (x + 3)(x - 3)(x^2 + x + 2)$$

Step 3 Use the Quadratic Formula. Find the complex roots of $x^2 + x + 2 = 0$.

$a = 1, b = 1, c = 2$ Identify the values of *a*, *b*, and *c*.

$\frac{-1 \pm \sqrt{1^2 - 4(1)(2)}}{2(1)}$ Substitute.

$\frac{-1 \pm \sqrt{-7}}{2}$ Simplify.

The complex roots are $\frac{-1 + i\sqrt{7}}{2}$ and $\frac{-1 - i\sqrt{7}}{2}$.

Step 4 The four zeros of the function are -3, 3, $\frac{-1 + i\sqrt{7}}{2}$, and $\frac{-1 - i\sqrt{7}}{2}$.

By the Fundamental Theorem of Algebra, there can be no other zeros.

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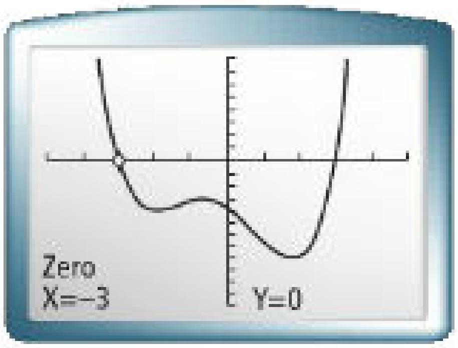

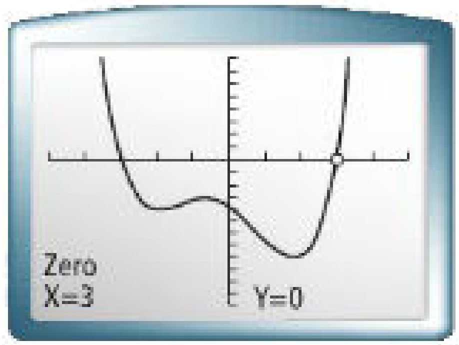

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