WARM UP

Sunday, October 19, 2014

1. Simplify $(2 + 2i)(1 - i)$

2. Solve $(2 + xi) - (y - i) = 7 + 14i$

3. What is the next term in the sequence
   a.) 5, 15, 25, …
   b.) Y, W, U, S, …

10
9
8
7
6
5
4
3
2
1

Objectives

Write a formula for an Arithmetic Sequence

Find the sum of an Arithmetic Series

Evaluate a series in Summation notation

Homework

WBP 239: 11-19 odd, 33, 46 b-d

WBP 247: 1, 3, 5, 8, 10, 24, 26

Homework Check

Sequence

An ordered list of numbers. Each number is a **term** of the sequence

The numbers in the sequence follow a certain pattern or **rule**.

For example 0, 10, 20, 30, 40, 50, 60, 70, … is a sequence.

Each term is found by adding 10 to the previous term.

Defining the RULE for a Sequence

127, 140, 153, 166, …

$a_1, \quad a_2, \quad a_3, \quad a_4, \ldots$ *Check out the notation we will use for each term*

First we have to determine if we actually have a sequence. We need to find the **Common Difference**.

Subtract each term from the previous term.

$$a_2 - a_1 = 140 - 127 = 13$$

$$a_3 - a_2 = 153 - 140 = 13$$

$$a_4 - a_3 = 166 - 153 = 13$$

Yes, we have an **Arithmetic Sequence** and the **Common Difference** is 13.

Defining the RULE for a Sequence

Explicit

In general terms, an Arithmetic Sequence with a starting value $\boldsymbol{a}$ and common difference $\boldsymbol{d}$ is a sequence of the form

$$a, a + d, a + 2d, a + 3d, \dots$$

127, 140, 153, 166, …

Common Difference 13

Remember our sequence?

$$127, 127 + 13, 127 + 2(13), 127 + 3(13), \dots$$

An **explicit definition** of a sequence has the form

$$a_n = a + (n - 1)d, \text{ for } n \geq 1$$

Where $\boldsymbol{a}$ is the first term and $\boldsymbol{d}$ is the common difference

Our rule is now $a_n = 127 + (n - 1)13, \text{ for } n \geq 1$

Explicit

So what can we do with this rule?

$$a_n = 127 + (n-1)13, \text{ for } n \geq 1$$

Answer questions like "What's the 10th term in the sequence 127, 140, 153, 166…?"

$$a_{10} = 127 + (10-1)13$$
$$a_{10} = 127 + (9)13$$
$$a_{10} = 127 + 117$$
$$a_{10} = 244$$

YouDo

Pp 239

Determine if the following sequences are arithmetic. If so find the **explicit definition** of the sequence and calculate the 11th term.

4.) 3, 8, 13, 18

Common Difference is 5. $a_n = 3 + (n-1)5 = 5n - 2$

$a_{11} = 3 + (11-1)5 = 5(11) - 2 = 53$

10.) 11, 13, 17, 25

No, the difference between 17 and 25 is **8** and the difference between 11 and 13 is **2**.

Defining the RULE for a Sequence

Recursive

There is another way to write the rule for a sequence. A **recursive definition** has two parts.

Initial Condition $a_1 = a$

Recursive Formula $a_{n+1} = a_n + d$, for $n \geq 1$

So for our sequence 127, 140, 153, 166, … with a common difference of 13, the recursive definition is

Initial Condition $a_1 = a$

Recursive Formula $a_{n+1} = a_n + 13$, for $n \geq 1$

Not very useful… ask me why you need to know this.

Problem?

As a part-time home health care aide, you are paid a weekly salary plus a fixed fuel fee for every patient you visit. You receive $240 in a week that you visit 1 patient. You receive $250 in a week that you visit 2 patients. How much will you receive if you visit 12 patients in 1 week?

How many terms in the sequence of payments are you given? **2**

What is the common difference between them? $\mathbf{250 - 240 = 10}$

Write the explicit definition for this sequence of payments.

$$a_n = 240 + (n - 1)10, \text{ for } n \geq 1$$

Use your rule to find the payment for a 12 patient week.

$$a_{12} = 240 + (12 - 1)10 = 350$$

Definitions

A **Sequence** is an ordered list of numbers.

$$4, 8, 12, 16, 20$$

A **Series** is a sum of the terms of a Sequence

$$4 + 8 + 12 + 16 + 20 = 60$$

A **Finite Series** has a first and a last term

$$4 + 8 + 12 + 16 + 20$$

An **Infinite Series** continues without end.

$$4 + 8 + 12 + 16 + 20 \ldots$$

Formula

The sum S_n of a **finite** arithmetic series $a_1 + a_2 + a_3 + \cdots + a_n$ is given by the formula

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Where a_1 is the first term, a_n is the n^th^ (last) term and n is the number of terms.

For example:

Find the sum of the series $11 + 13 + 15 + 17 + 19 + 21 + 23$

$$S_7 = \frac{7}{2}(11 + 23)$$

$$S_7 = 119$$

YouDo

1.) Find the sum of the series $1 + 3 + 5 + 7 + 9$

$$S_5 = \frac{5}{2}(1 + 9)$$

$$S_5 = 25$$

Tricky!

What is the sum of the following finite arithmetic series?

$$4 + 9 + 14 + 19 + 24 + \cdots + 99$$

What do we know?

First term 4 Last Term 99 Formula $S_n = \frac{n}{2}(4 + 99)$

What we don't know is the number of terms, n.

Step 1: Find the **common difference**. $9 - 4 = 5$

Step 2: Use the **common difference** to find the number of terms.

$$n = \frac{last\ term\ - first\ term}{common\ difference} + 1 \qquad n = \frac{99 - 4}{5} + 1 = 20$$

Now we have everything we need to find the sum.

$$S_{20} = \frac{20}{2}(4 + 99) = 1030$$

YouDo

What is the sum of the following finite arithmetic series?

$$5 + 8 + 11 + \cdots + 26$$

What do we know?

First term **5** | Last Term **26** | Formula $\boldsymbol{S_n = \frac{n}{2}(5 + 26)}$

What we don't know is the number of terms, n.

Step 1: Find the **common difference.** $\mathbf{8 - 5 = 3}$

Step 2: Use the **common difference** to find the number of terms.

$$n = \frac{last\ term\ - first\ term}{common\ difference} + 1 \qquad \boldsymbol{n = \frac{26 - 5}{3} + 1 = 8}$$

$$\boldsymbol{S_8 = \frac{8}{2}(5 + 26) = 124}$$

Notation

A series can be represented in a compact form called **Summation** notation (or sigma notation).

To represent "the summation from 1 to 4 of 3n we would write

$$\sum_{n=1}^{4} 3n = 3(1) + 3(2) + 3(3) + 3(4)$$

$$= 3 + 6 + 9 + 12$$

$$= 30$$

N is always an integer and in incremented by 1

Finding the sum of a series in Summation notation

Notation

Find the sum of the finite series $\sum_{n=1}^{40} (3n - 8)$

Look at the formula for a Finite Arithmetic Series. We have everything we need!

$$S_n = \frac{n}{2}(a_1 + a_n)$$

1. How many terms? $n = 40$
2. Find the first term $a_1 = 3(1) - 8 = -5$
3. Find the last term $a_{40} = 3(40) - 8 = 112$
4. Fill in the formula

$$S_{40} = \frac{40}{2}(-5 + 112)$$

$$S_{40} = 2140$$

YouDo

Find the sum of the finite series $\sum_{n=1}^{4}(n-1)$

Look at the formula for a
Finite Arithmetic Series.
We have everything we need!

$$S_n = \frac{n}{2}(a_1 + a_n)$$

1. How many terms? $n = 4$
2. Find the first term $a_1 = (1) - 1 = 0$
3. Find the last term $a_4 = (4) - 1 = 3$
4. Fill in the formula

$$S_4 = \frac{4}{2}(0 + 3)$$

$$S_4 = 6$$

Write the following series in summation notation

$$4 + 8 + 12 + 16$$

1. Find the common difference $d = 4$

2. Find the first term $a_1 = 4$

3. Use the explicit formula for a sequence and simplify.

$$a_n = a + (n-1)d$$

$$a_4 = 4 + (n-1)4$$

$$a_4 = 4 + 4n - 4$$

$$a_4 = 4n$$

4. Fill in what you know

$$\sum_{n=1}^{4} 4n$$

YouDo

Write the following series in summation notation

$$1 + 11 + 21 + 31 + 41 + 51 + 61$$

1. Find the common difference $d = 10$

2. Find the first term $a_1 = 1$

3. Use the explicit formula for a sequence and simplify.

$$a_n = a + (n - 1)d$$
$$a_7 = 1 + (n - 1)10$$
$$a_7 = 10n - 9$$

4. Fill in what you know

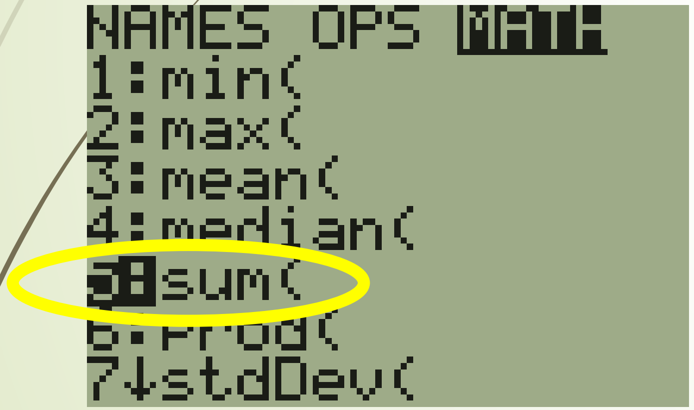
$$\sum_{n=1}^{7} 10n - 9$$

What can you do on you calculator?

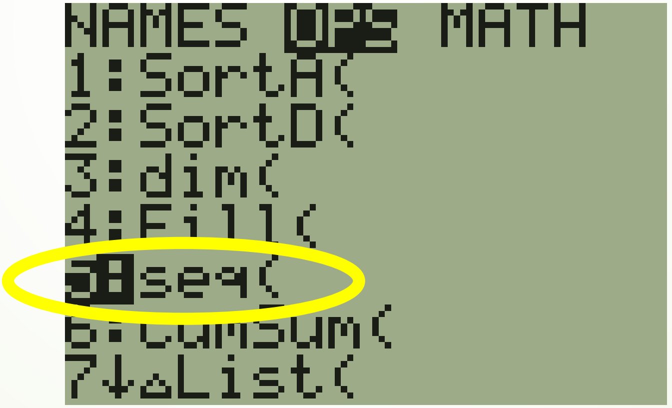
You can find the sum of a sequence on your graphing calculator!

Look at problem 23 on page 247 $\sum_{n=1}^{15} n+3$

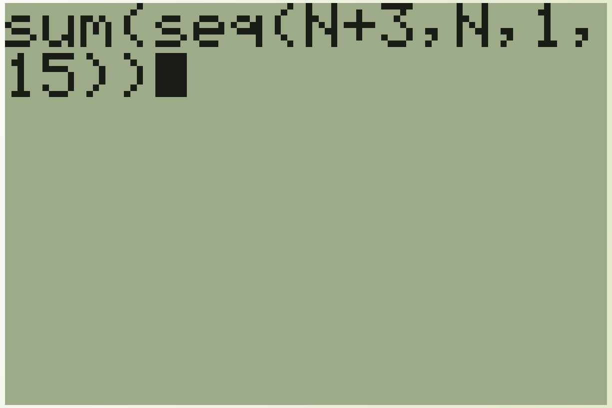
2^{nd} STAT MATH 5: sum

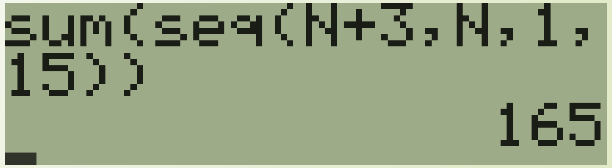


2^{nd} STAT OPS seq



Expression, variable, start, end))





What can you do on you calculator?

YouDo

You can find the sum of a sequence on your graphing calculator!

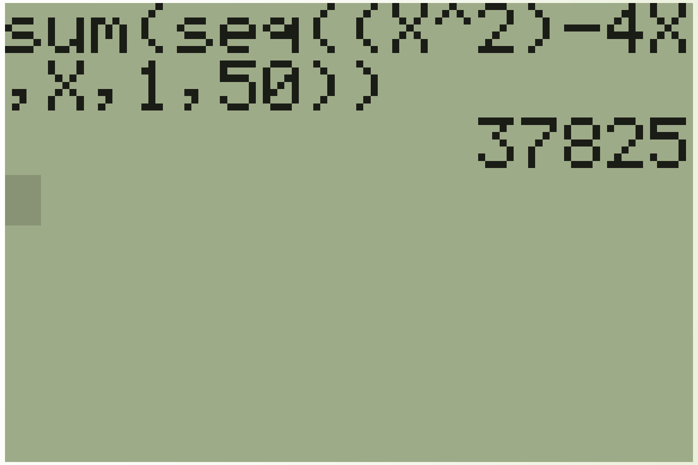
Look at problem 27 on page 247

$$\sum_{n=1}^{50} n^2 - 4n$$

2nd STAT MATH 5: sum

2nd STAT OPS seq

Expression, variable, start, end))



Work on your homework