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Reteaching

Complex Numbers

- A *complex number* consists of a real part and an imaginary part. It is written in the form $a + bi$, where a and b are real numbers.
- $i = \sqrt{-1}$ and $i^2 = (\sqrt{-1})(\sqrt{-1}) = -1$
- When adding or subtracting complex numbers, combine the real parts and then combine the imaginary parts.
- When multiplying complex numbers, use the Distributive Property or FOIL.

Problem

What is $(3 - i) + (2 + 3i)$?

$$(3 - i) + (2 + 3i)$$

$$= \textcircled{3} - \boxed{i} + \textcircled{2} + \boxed{3i}$$

Circle real parts. Put a square around imaginary parts.

$$= (3 + 2) + (-1 + 3)i$$

Combine.

$$= 5 + 2i$$

Simplify.

Problem

What is the product $(7 - 3i)(-4 + 9i)$?

Use FOIL to multiply:

$$(7 - 3i)(-4 + 9i) = 7(-4) + 7(9i) + (-3i)(-4) + (-3i)(9i)$$

$$(7 - 3i)(-4 + 9i)$$

$$= -28 + 63i + 12i - 27i^2$$

$$\text{First} = 7(-4)$$

$$= -28 + 75i - 27i^2$$

$$\text{Outer} = 7(9i)$$

You can simplify the expression by substituting -1 for i^2 .

$$\text{Inner} = (-3i)(-4)$$

$$(7 - 3i)(-4 + 9i) = -28 + 75i - 27(-1)$$

$$\text{Last} = (-3i)(9i)$$

$$= -1 + 75i$$

Exercises

Simplify each expression.

1. $2i + (-4 - 2i)$

2. $(3 + i)(2 + i)$

3. $(4 + 3i)(1 + 2i)$

4. $3i(1 - 2i)$

5. $3i(4 - i)$

6. $3 - (-2 + 3i) + (-5 + i)$

7. $4i(6 - 2i)$

8. $(5 + 6i) + (-2 + 4i)$

9. $9(11 + 5i)$

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Reteaching (continued)

Complex Numbers

- The *complex conjugate* of a complex number $a + bi$ is the complex number $a - bi$.
- $(a + bi)(a - bi) = a^2 + b^2$
- To divide complex numbers, use complex conjugates to simplify the denominator.

Problem

What is the quotient $\frac{4+5i}{2-i}$?

$$\frac{4+5i}{2-i}$$

$$= \frac{4+5i}{2-i} \cdot \frac{2+i}{2+i}$$

$$= \frac{8+4i+10i+5i^2}{(2-i)(2+i)}$$

$$= \frac{8+4i+10i+5i^2}{2^2+1^2}$$

$$= \frac{8+14i+5(-1)}{4+1}$$

$$= \frac{3+14i}{5}$$

$$= \frac{3}{5} + \frac{14}{5}i$$

The complex conjugate of $2 - i$ is $2 + i$.

Multiply both numerator and denominator $2 + i$.

Use FOIL to multiply the numerators.

Simplify the denominator. $(a + bi)(a - bi) = a^2 + b^2$

Substitute -1 for i^2 .

Simplify.

Write as a complex number $a + bi$.

Exercises

Find the complex conjugate of each complex number.

10. $1 - 2i$

11. $3 + 5i$

12. i

13. $3 - i$

14. $2 + 3i$

15. $-5 - 2i$

Write each quotient as a complex number.

16. $\frac{3i}{1-2i}$

17. $\frac{6}{3+5i}$

18. $\frac{2+2i}{i}$

19. $\frac{2+5i}{3-i}$

20. $\frac{-4-i}{2+3i}$

21. $\frac{6+i}{-5-2i}$

4-8**Practice**

Form G

Complex Numbers

Simplify each number by using the imaginary number i .

1. $\sqrt{-49}$

2. $\sqrt{-144}$

3. $\sqrt{-7}$

4. $\sqrt{-10}$

5. $\sqrt{-8}$

6. $\sqrt{-48}$

Simplify each expression.

10. $(-2 + 3i) + (5 - 2i)$

11. $(-5 + 3i) - (-8 + 2i)$

14. $(4 - 3i)(-5 + 4i)$

15. $(2 - i)(-3 + 6i)$

16. $(5 - 3i)(5 + 3i)$

17. $(-1 + 3i)^2$

18. $(4 - i)^2$

19. $(-2i)(5i)(-i)$

20. $(6 - \sqrt{-16}) + (-4 + \sqrt{-25})$

21. $(-2 + \sqrt{-9}) + (-1 - \sqrt{-36})$

24. $3i(2 + 2i)$

27. $(5 + \sqrt{-1})(2 - \sqrt{-36})$

26. $(2 + \sqrt{-4})(-1 + \sqrt{-9})$

Solve each equation. Check your answer.

32. $x^2 + 64 = 0$

33. $3x^2 + 27 = 0$

38. $x^2 + 2x + 5 = 0$

39. $-x^2 + 2x - 10 = 0$

40. $2x^2 - 3x + 5 = 0$

41. $-4x^2 + 6x - 3 = 0$

42. $3x^2 + 2x + 5 = 0$

43. $2x^2 - 2x + 7 = 0$

