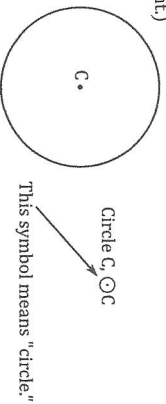


# Circles

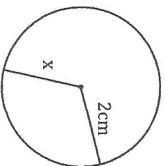
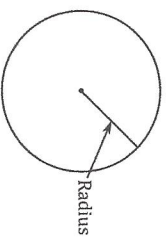
Round and round we go, and where we'll stop? Well, we won't stop I guess...

**Circle:** We all know what they look like, but what is it? Well, the overly technical definition of a circle is the set of points equidistant from a single point. Huh? In other words, that nice round shape you see is a bunch of points that are all exactly the same distance away from the center. This is circle C. (You name circles by their center point.)



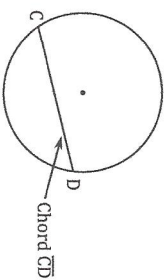
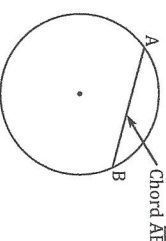
Pretty cool huh? Let's look at some more properties of circles....

**Radius:** The radius is the distance that is from the center of the circle to the edge. It is the same length everywhere in the circle, because it is that distance from the center that makes the circle from its definition above. (The radius is also half the length of the diameter.) All radii (plural form...more than one...not radiuses) from the same circle are equal.



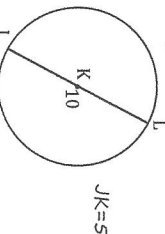
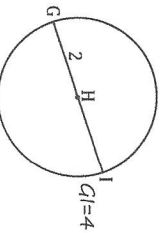
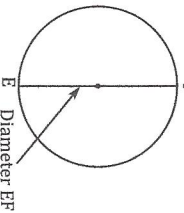
Ex. 1. Find the value of x.

Answer:  
 $x=2\text{cm}$  because all radii in a circle are equal.



**Chord:** A chord is a line drawn inside the circle that touches the circle at its endpoints.

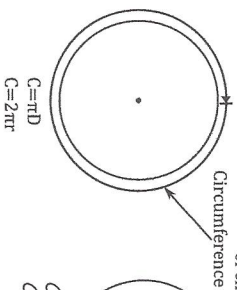
**Diameter:** is a special chord that goes through the center of the circle. It is also twice as long as the radius.



Ex. 2. Find the length of diameter GI if  $GH=2$ .

Ex. 3. Find the length of radius JK if  $KL=10$ .

**Circumference:** Ah pie, delicious apple or cherry hot out of the oven. No wait, not that kind of pie! This one has no "e". The circumference is the distance around the circle. Its distance is  $\pi D$  and because  $D=2r$  then  $2\pi r$  is also very popular for the distance of the circumference. Its measurement is 360°.



$$C = \pi D$$

$$C = 2\pi r$$

These steps seem to be out of order. Don't worry, multiplication is commutative. This is how it's usually done.

$$C = \pi D$$

$$C = 7\pi \approx 21.99$$

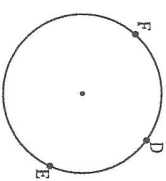
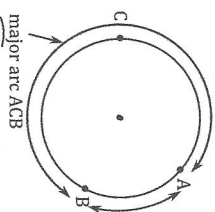
$$C = 2\pi r$$

$$C = 2\pi(7)$$

$$C = 1.8TT \approx 56.55$$

A note about  $\pi$  "pi".  $\pi$  is actually the ratio of the circumference to the diameter  $C/D$  and is approximately equal to 3.1415926535897923846264338327950288419716939937...and never ever ends. Very interesting... Some people have memorized this out thousands of digits. (Get a job, right?) and I think there is a super computer still working on it to see if it will ever end. Some texts and places don't even want you to multiply by pi, hence "r" as in the answer to example 4. This is called "in terms of pi". Some people want you to write it out. For the exercises that follow you should do both so you are used to both. For pi, just round to 3.14159. That's usually plenty accurate unless you're measuring amoeba's or splitting atoms.

An **Arc** is simply part of the circumference. A **Minor Arc** is the short one. The **Major Arc** is the long one. Always use 3 points when naming a major arc to separate it from the minor arc.



Ex. 6. Find  $\widehat{DEF}$  if  $\widehat{DE}=127^\circ$ .

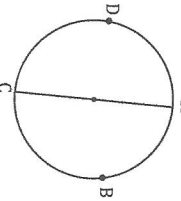
$$\widehat{DEF} + \widehat{DE} = 360^\circ$$

$$\widehat{DE} + 127^\circ = 360^\circ$$

$$-127^\circ \quad -127^\circ$$

$$\widehat{DE} = 233^\circ$$

A **Semi-Circle** is an arc that intercepts a diameter. The diameter actually cuts the circle in half so the semi-circle is always  $360^\circ/2$  or  $180^\circ$ . By the way, "intercept" is a funny word. All it means is that the ends of the arc are the same as the ends of the diameter or whatever is intercepting the arc. (We'll have angles intercept arcs in the next section. Won't that be fun!)



Arc  $\widehat{ABC}$  and  $\widehat{CDA}$  both measure  $180^\circ$ .

Okay, get your wheels and lets practice!

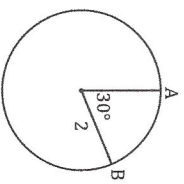


# Finding Lengths of Arcs and Areas of Sectors

Doesn't that sound like fun?

Well okay, maybe not to you, but these problems are fun. Let's take a look...

Find the length (not the measure) of  $\widehat{AB}$ .



This is the measure of the part of the circle we want over the total measure of the circle.

Step 1. Find the fraction of the circle's circumference that  $\widehat{AB}$  represents. How? Like this:...

$$\frac{30^\circ}{360^\circ} = \frac{1}{12}$$

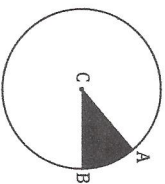
Step 2. Find the circumference of the circle...  $C = 2\pi r$   
 $C = 4\pi r$

Step 3. Multiply the circumference by the fraction... and presto!

$$4\pi * \frac{1}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$$

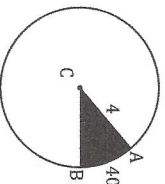
$$\widehat{AB} = \frac{\pi}{3} \approx 1.05$$

Next, let's look at Sectors. What the heck is a sector? A sector is a portion of the area of a circle enclosed by two radii and an arc. Huh? Well if you had a pie and you cut a piece out of it that piece would be a sector of the pie. Like this...



The shaded area in the diagram is a sector. Sector CAB is a sector of  $\odot C$ .

Find the area of sector CAB.



Step 1. Find the fraction of the circle that sector CAB represents. How? Like this:...

$$\frac{40^\circ}{360^\circ} = \frac{1}{9}$$

Step 2. Find the area of the entire circle...

$$A = \pi r^2$$

$$A = \pi 4^2$$

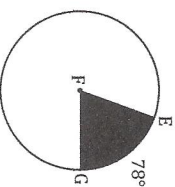
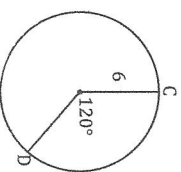
$$A = 16\pi$$

Step 3. Multiply the area by the fraction... and presto!

$$16\pi * \frac{1}{9} = \frac{16\pi}{9}$$

$$m\widehat{AB} = \frac{16\pi}{9} \approx 5.59u^2$$

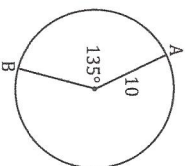
You try! Find the length of  $\widehat{CD}$ .



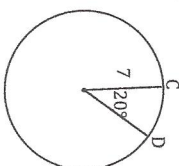
You try! Find the area of sector EFC.

Calculate the length of each arc...

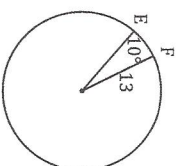
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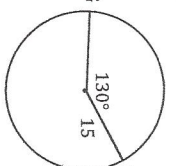
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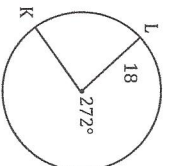
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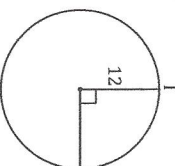
4.



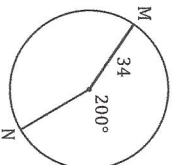
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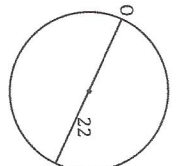
6.



7.



8.

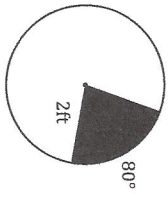


Bubble all the correct answers from above. Don't bubble incorrect answers.

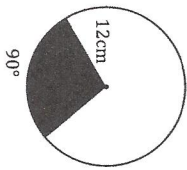
- 88.56  
  23.56  
  34.03  
  2.55  
  85.45  
  76.39  
  118.68  
  2.27  
  25.34  
  37.48  
  69.12  
  49.57  
  18.85  
  2.44

Find the area of each shaded sector...

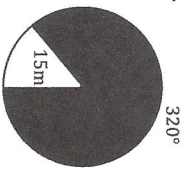
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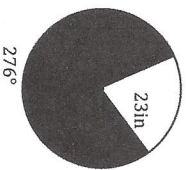
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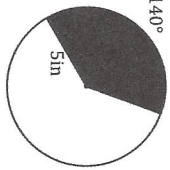
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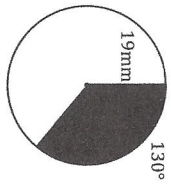
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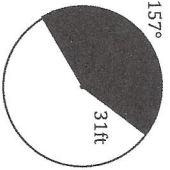
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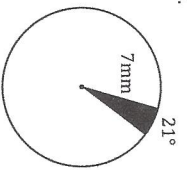
12.



14.



16.



Bubble all the correct answers from above. Don't bubble incorrect answers.

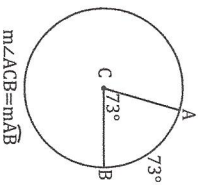
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  1274.13  
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  575.34  
  67.09  
  45.78  
  698.87  
  31.50  
  113.1  
  30.54  
  1265.75
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# Central Angles and Inscribed Angles

What's in a circle...

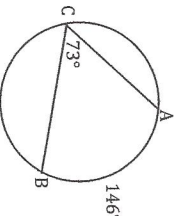
Angles! That's what... Okay, here are some really cool angles that go inside circles. They are actually pretty easy but can get tricky at times. Let's take a look.

**Central Angles:** A central angle is an angle that has its vertex at the center of a circle and extends to the edge of the circle. Like the picture. Its measurement is equal to the measurement of its included arc. (The arc that is in the interior of the circle.) This makes sense because in order to measure an angle, we basically draw an arc between the rays with our protractor, and see how many degrees it is. Study the picture.



$$m\angle ACB = m\widehat{AB}$$

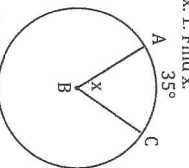
An Inscribed angle is an angle that has its vertex on the edge of the circle extending inward to the opposite edge of the circle. Its measurement is half its included arc, and consequently the included arc is twice the inscribed angle. Like this...



$$146^\circ \quad m\angle ACB = 1/2 m\widehat{AB}$$

$$m\widehat{AB} = 2m\angle ACB$$

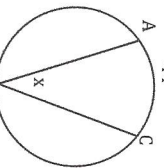
Ex. 1. Find x.



$$m\angle B = m\widehat{AC}$$

$$x = 35^\circ$$

Ex. 2. Find x.

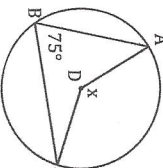


$$2(m\angle B) = m\widehat{AC}$$

$$\frac{2x}{2} = \frac{44}{2}$$

$$x = 22^\circ$$

Ex. 3. Find x.



$$2(m\angle B) = m\widehat{AC}$$

$$2(75^\circ) = m\widehat{AC}$$

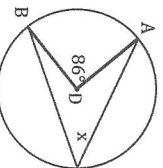
$$150^\circ = m\widehat{AC}$$

$$m\angle D = m\widehat{AC}$$

$$x = 150^\circ$$

Notice on this one x ended up being 2(75°).

Ex. 4. Find x.



$$m\angle D = m\widehat{AB}$$

$$86^\circ = m\widehat{AB}$$

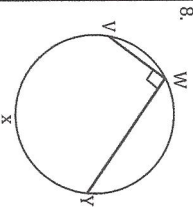
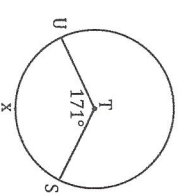
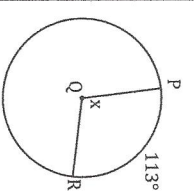
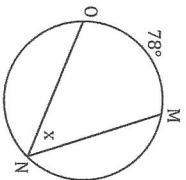
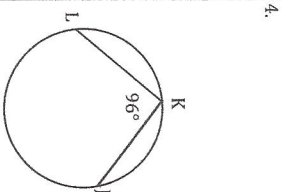
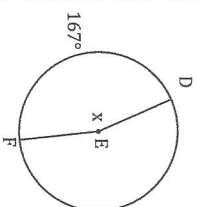
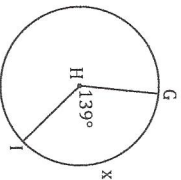
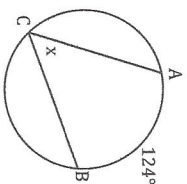
$$x = 1/2 m\widehat{AB}$$

$$x = 1/2(86^\circ)$$

$$x = 43^\circ$$

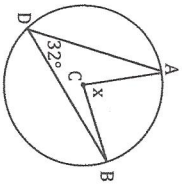
Notice on this one x ended up being 1/2(86°).

For these.. find the value of x. Show all your work...

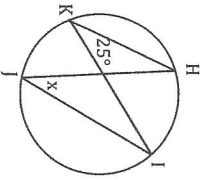


Bubble all the correct answers from above. Don't bubble incorrect answers.

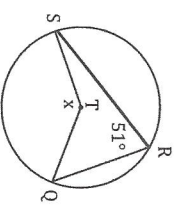
- 352° ○ 90° ○ 39° ○ 171° ○ 192° ○ 124° ○ 69.5° ○ 180° ○ 139° ○ 83.5° ○ 226° ○ 113° ○ 167° ○ 62°



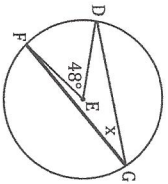
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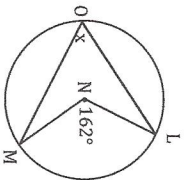
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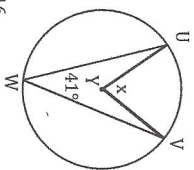
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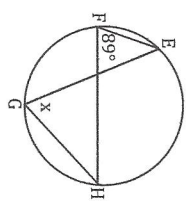
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14.

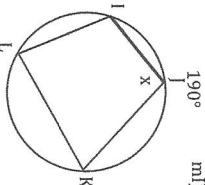


16.



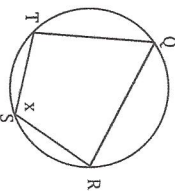
Bubble all the correct answers from above. Don't bubble incorrect answers.

- 162° 
  102° 
  64° 
  25° 
  48° 
  81° 
  82° 
  41° 
  89° 
  81° 
  24° 
  162° 
  51° 
  131°

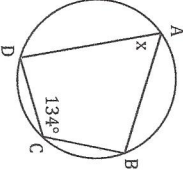


17.  $m\widehat{JK} = 190^\circ$

First find  $m\widehat{KLI}$ .  
 $m\widehat{JK} + m\widehat{KLI} = 360^\circ$   
 $m\widehat{JK} + m\widehat{KLI} = 360^\circ$   
 $190^\circ + m\widehat{KLI} = 360^\circ$   
 $-190^\circ$   
 $m\widehat{KLI} = 170^\circ$   
 Then find  $m\angle I$ .  
 $m\angle J = 1/2 m\widehat{KLI}$   
 $x = 1/2(170^\circ)$   
 $x = 85^\circ$

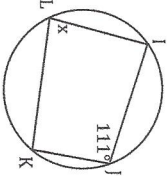


21.

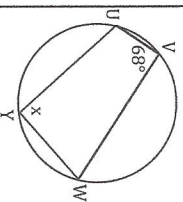
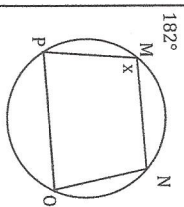


19.  $m\widehat{RS} = 122^\circ$

23.

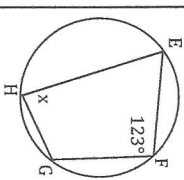


18.  $m\widehat{PMN} = 182^\circ$

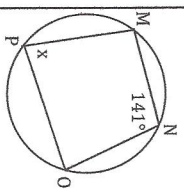


20. First find  $m\widehat{UYW}$

22.



24.



Bubble all the correct answers from above. Don't bubble incorrect answers.

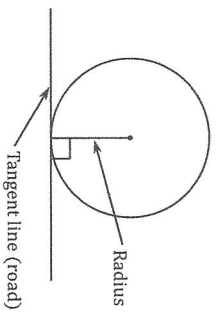
- 114° 
  56° 
  69° 
  57° 
  47° 
  119° 
  231° 
  46° 
  39° 
  85° 
  99° 
  162° 
  136° 
  224°



## Tangent Radius Theorem

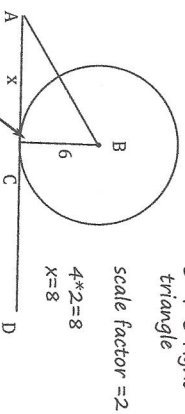
Keep your wheels on the road.

This theorem has a lot to do with wheels and the road. How 'bout that? If you have a tangent line, the road, and a radius that intersects it, the wheel, then the radius and tangent are perpendicular at the point of tangency. Do you have any idea how many words I'm saving you from reading?



That's it... let's look at some examples. Like the radius chord theorem, problems that use this concept can vary. All you need to know is that a tangent and the radius are perpendicular at the point of tangency.

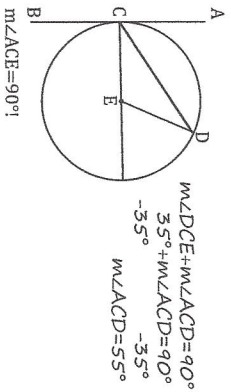
Ex. 1. Find  $x$  if  $AB=10$ .



3-4-5 right triangle  
scale factor = 2  
 $4 * 2 = 8$   
 $x = 8$

This angle measures  $90^\circ$  so this is a right triangle.

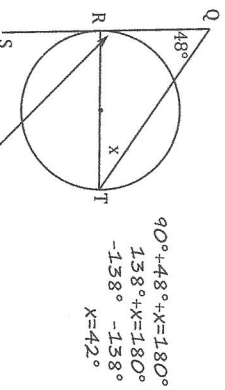
Ex. 2.  $m\angle DCE = 35^\circ$ . What is  $m\angle ACD$ ?



$m\angle DCE + m\angle ACD = 90^\circ$   
 $35^\circ + m\angle ACD = 90^\circ$   
 $-35^\circ$   
 $m\angle ACD = 55^\circ$

$m\angle ACB = 90^\circ$

Ex. 3. Find  $x$ .



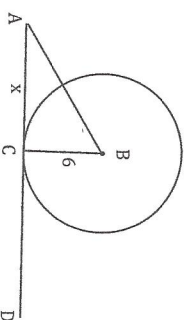
$90^\circ + 48^\circ + x = 180^\circ$   
 $138^\circ + x = 180^\circ$   
 $-138^\circ$   
 $x = 42^\circ$

This angle measures  $90^\circ$  so this is a right triangle.

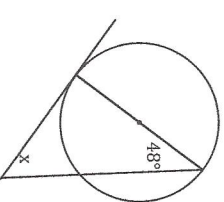
Great! Let's practice....

Find the missing length or angle for each.

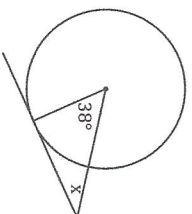
1.  $AB=9$ . Find  $x$ .



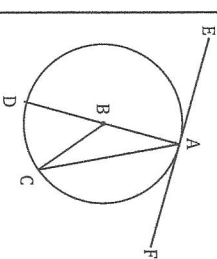
3. Find  $x$ .



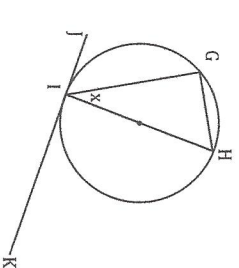
2. Find  $x$ .



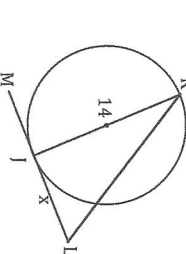
4.  $m\angle BAC = 23^\circ$ . What is  $m\angle CAF$ ?



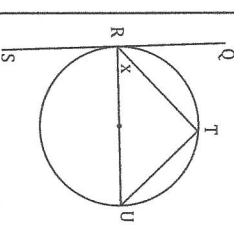
5.  $m\angle IGC = 65^\circ$ . What is  $m\angle GHI$ ?



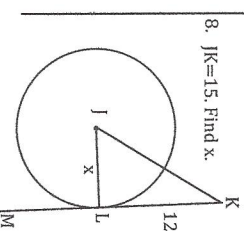
7.  $KL=19$ . Find  $x$ .



6.  $\angle QRT = 37^\circ$ . What is  $m\angle TRU$ ?



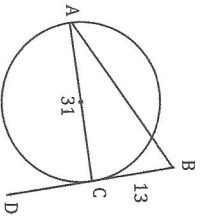
8.  $JK=15$ . Find  $x$ .



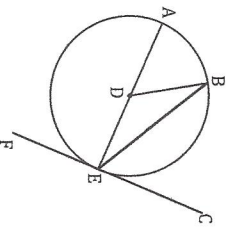
Bubble all the correct answers from above. Don't bubble incorrect answers.

- 09  2.85  25  17  42  67  52  53  6.71  7.5  45  16.43  10  45

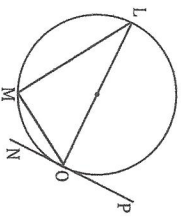
9. Find AB.



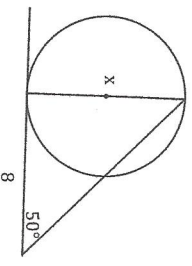
11.  $m\widehat{BE}=149^\circ$  and  $m\angle CEB=83^\circ$ . Find  $m\angle DBE$ .



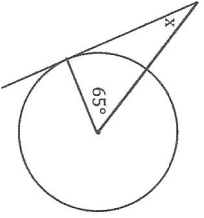
13.  $m\angle NOM=32^\circ$ . Find  $m\angle MLO$ .



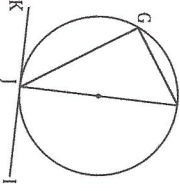
15.



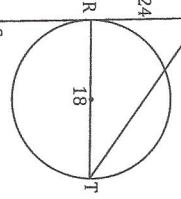
10.



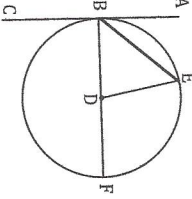
12.  $m\angle KIG=71^\circ$ . Find  $m\angle GHJ$ . (Hint: What is  $m\angle HGJ$ ?) H



14. Find QT.



16.  $m\widehat{BE}=84^\circ$  and  $m\angle ABE=45^\circ$ . Find  $m\angle BED$ .

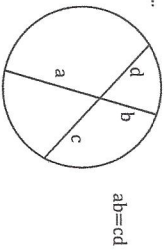


Bubble all the correct answers from above. Don't bubble incorrect answers.

- 24  
  71  
  14  
  51  
  9.53  
  40  
  30  
  25  
  4.35  
  34.78  
  53  
  32  
  33.62  
  8

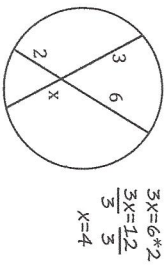
# Intersecting Chord Theorem and Intersecting Tangents Theorem

Have we crossed paths before? This one is really easy, but really important. The Intersecting Chord Theorem says that if two chords intersect (that's cross each other) inside a circle, then the products of their segments are equal. Huh? A picture's worth a thousand words. So, instead of boring you with one-thousand words...

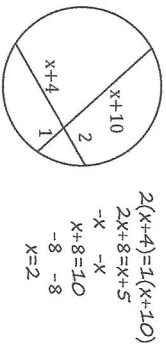


Here are a couple of examples:

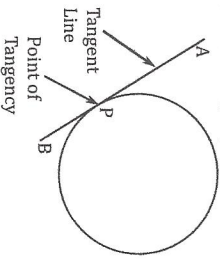
Ex. 1. Find x.



Ex. 2. Solve for x.

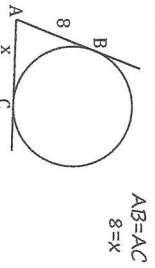


A Tangent is a line that intersects the exterior of the circle in exactly one spot. This spot is called the Point of Tangency.

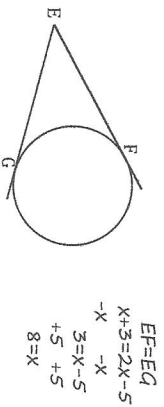


Let's look at examples of this...

Ex. 3.  $AB=8$  Find AC.

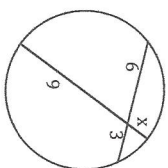
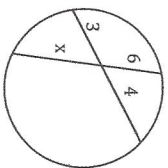


Ex. 4.  $EF=x+3$ ,  $EG=2x-5$ . Solve for x.

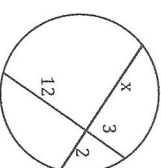


Let's practice...

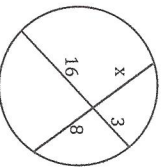
Find x...



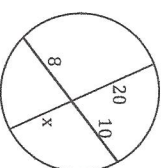
3.



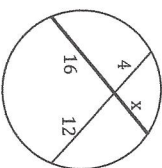
4.



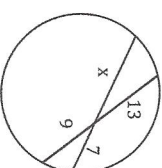
5.



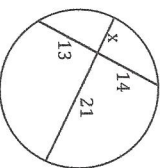
6.



7.

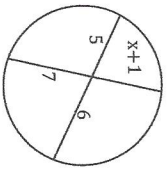


8.

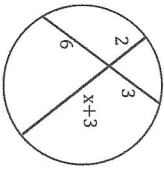


Bubble all the correct answers from above. Don't bubble incorrect answers.

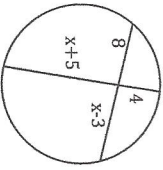




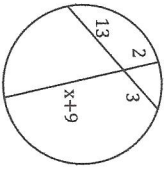
9.



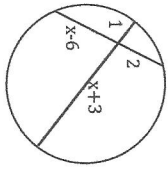
11.



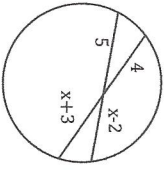
13.



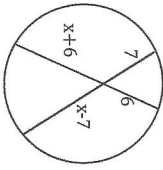
15.



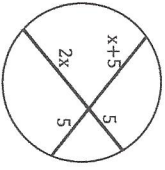
10.



12.

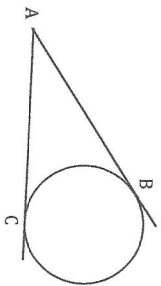


14.

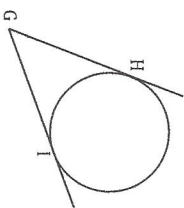


16.

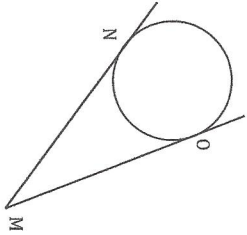
Answer each question...  
17.  $AB=10$ . Find  $AC$ .



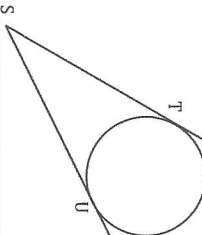
19.  $GH=15$ . Find  $GI$ .



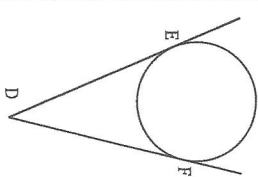
21.  $MN=x+5$ ,  $MO=2x-11$ . Solve for  $x$ .



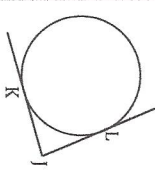
23.  $ST=2x+12$ ,  $SU=3x-8$ . Solve for  $x$ .



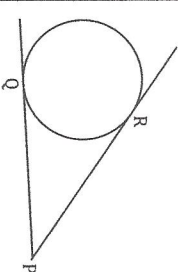
18.  $DE=12$ . Find  $DF$ .



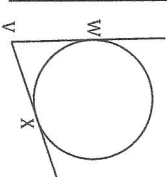
20.  $JK=4$ . Find  $JL$ .



22.  $PQ=x+7$ ,  $PR=2x-9$ . Solve for  $x$ .



24.  $VW=x+27$ ,  $VX=4x+13$ . Solve for  $x$ .



Bubble all the correct answers from above. Don't bubble incorrect answers.

- 22  11  6  8  13.5  10.5  15  5  4.39  85  3.29  58  93  4.67

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Bubble all the correct answers from above. Don't bubble incorrect answers.

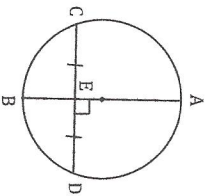
- 15  16  11  17  4  3.67  12  10  5  19  4.67  20  16  12

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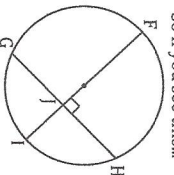
# VI Diameter Chord Theorem

It's half and perpendicular....

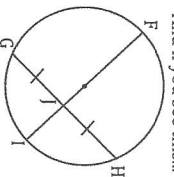
No seriously, it is. If you have a diameter and a chord and the diameter is perpendicular to the chord, the diameter bisects the chord. In addition, it also bisects the two arcs that intercept the chord. Here's another thousand words...



AB is a diameter and  $AB \perp CD$ . That means that AB bisects CD and so...  $CE = DE$ . It also means that  $m\widehat{CB} = m\widehat{DB}$  and  $m\widehat{CA} = m\widehat{DA}$ .



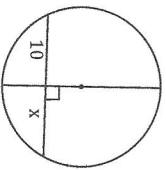
So if you see this... It means...  $GJ = HJ$ ,  $\angle FJH = \angle IJH$ ,  $\angle IJG$  and  $m\widehat{FH} = m\widehat{IH}$  and  $m\widehat{FG} = m\widehat{FH}$ .



And if you see this... It means...  $FJ \perp GH$  and so...  $\angle FJH = \angle IJH$ ,  $\angle IJG$  and  $m\widehat{GH} = m\widehat{IH}$  and  $m\widehat{FI} = m\widehat{FH}$ .

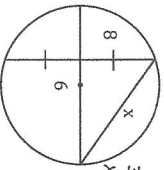
Problems that use this theorem are quite varied, but they usually involve right triangles and things that are bisected.... Let's look at a few before we practice...

Ex. 1. Find x.



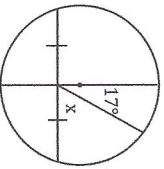
$$10 = x$$

Ex. 2. Find x.



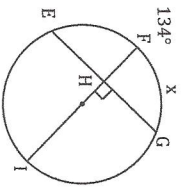
$$3-4-5 \text{ right triangle} \\ x = 10$$

Ex. 3. Find x.



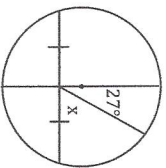
$$17^\circ + x = 90^\circ \\ -17^\circ \quad -17^\circ \\ x = 73^\circ$$

Ex. 4.  $m\widehat{FG} = 134^\circ$ . Find  $m\widehat{EG}$ .

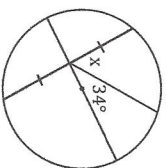


$$134^\circ / 2 = m\widehat{FG} \\ 67^\circ = m\widehat{FG}$$

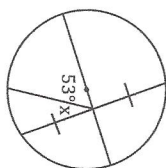
9.



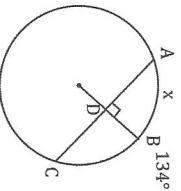
11.



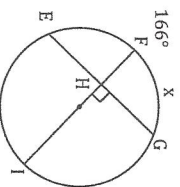
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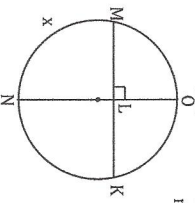
12.  $m\widehat{BC} = 134^\circ$ . Find  $m\widehat{AB}$ .



13.  $m\widehat{FG} = 166^\circ$ . Find  $m\widehat{EG}$ .

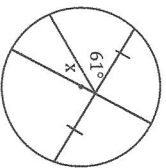


14.  $m\widehat{OK} = 184^\circ$ . Find  $m\widehat{MN}$ .

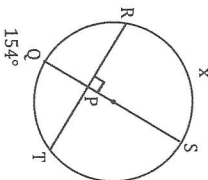


Hint: Find  $m\widehat{MO}$ , then  $m\widehat{MO} + m\widehat{MN} = 180^\circ$ .

15.



16.  $m\widehat{QS} = 154^\circ$ . Find  $m\widehat{SR}$ .



Bubble all the correct answers from above. Don't bubble incorrect answers.

- 87  88  89  90  91  92  93  94  95  96  97  98  99  100  101  102  103  104  105  106  107  108  109  110